# Heavy ion fusion reaction cross section: Analysis of the temperature dependence of the repulsive nuclear potential

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Using the Wong formula, fusion cross sections of $^{18}O + ^{63}Cu$, $^{18}O + ^{194}Pt$, $^{16}O + ^{208}Pb$, and $^{12}C + ^{141}Pr$ reactions are calculated with double-folding potentials (M3Y) and inclusion of the corrected temperature-dependent repulsive nuclear potential (M3Y + rep). Two different values of repulsive diffuseness are selected in calculations and effect of nuclear temperature on the repulsive nuclear potential, the effective potential, and three parameters of the Wong formula are investigated. Our results show that the inclusion of temperature-dependent repulsive nuclear potential increases the effective potential in interior regions, noticeably. However, in the near-barrier region, little enhancement of the effective potential causes small variations in the barrier height, position, and frequency. Better agreement between theory and experiment is observed by including the repulsive nuclear potential.

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## I. INTRODUCTION

The heavy ion fusion reaction is one of the well-known and interesting topics of nuclear physics [1–4]. In recent years, new modifications and corrections have been made in the nuclear potential to improve the theoretical calculation of the fusion cross section and relevant quantities [5–8].

In Refs. [9,10], the steep falloff of fusion cross sections in below and deep sub-barrier fusion reactions have been explained by adding a repulsive nuclear potential to the attractive one. The repulsive term has been determined through saturation properties of the nuclear matter. The fusion cross sections of $^{64}Ni + ^{64}Ni$, $^{58}Ni + ^{58}Ni$, and $^{64}Ni + ^{100}Mo$ reactions have been determined by the double-folding potential within the coupled-channels mechanism. Similarly, in Ref. [11], fusion reactions of the weakly bound projectile $^{9}Be$ with $^{27}Al$, $^{29}Si$, $^{208}Pb$, and $^{209}Bi$ targets have been studied using the sum of the attractive and repulsive double-folding nuclear potentials.

The initial energy of the projectile can be delivered to the compound nucleus causing its excitation into the continuum states. Therefore, the definition of nuclear temperature for a hot compound nucleus is meaningful and applicable for the equation of state. The fusion cross sections of $^{40}Ar + ^{40}Ca$, $^{28}Si + ^{40}Ca$, $^{35}Cl + ^{48}Ti$, and $^{40}Ar + ^{74}Ge$ reactions have been determined in Refs. [12,13] through coupled-channels calculation and using attractive and temperature-dependent repulsive double-folding nuclear potentials. The dependence of the repulsive potential on the temperature of the hot compound nucleus was originated from temperature dependence of the equation of state. Therefore, we tend to correct the temperature dependence of the double-folding repulsive nuclear potential and analyze the effect of temperature on the repulsive potential, the effective potential, the parameters of the Wong formula, and the fusion cross section. Moreover, the adequate value of the repulsive diffuseness may be determined through the comparative analysis of the calculated cross sections with different values of repulsive diffuseness. In Sec. II, the theoretical model for calculation of the fusion cross section by considering the temperature-dependent repulsive nuclear potential is introduced. The results are given in Sec. III, and concluding remarks are given in Sec. IV.

## II. THEORETICAL METHOD

The fusion cross section can be calculated from partial-wave expansion as

$$\sigma(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) T_l(E), \tag{1}$$

where $E$ is the center-of mass-energy ($E_{\mathrm{c.m.}}$), $k = \sqrt{\frac{2\mu E}{\hbar^2}}$ is the wave number, and $T_l$ is the transmission coefficient. The transmission coefficient depends on the effective interaction potential between the projectile and the target nuclei [3]. The effective nucleus-nucleus potential is written as

$$V_{\mathrm{eff}}(r) = V_N(r) + V_C(r), \tag{2}$$

where $V_N(r)$ is the attractive nuclear potential and $V_C(r)$ is the repulsive Coulomb potential. The nuclear and Coulomb

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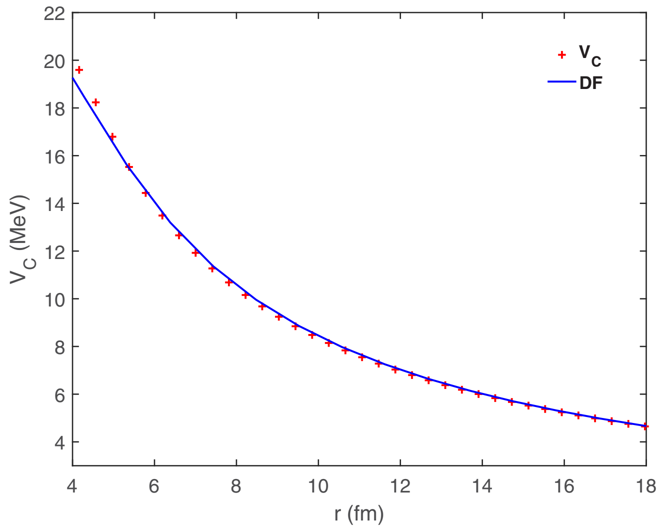


FIG. 1. Coulomb potential for the $^{18}$O + $^{63}$Cu reaction.

potentials can be calculated through the double-folding model [14],

$$V_{N(C)}(r) = \int d\vec{r}_1 d\vec{r}_2 \rho_p(\vec{r}_1) v(s) \rho_t(\vec{r}_2), \tag{3}$$

where $s = |\vec{r}_2 - \vec{r}_1 + \vec{r}|$ is the relative distance between the interacting nucleon pair. The effective nucleon-nucleon potential Yukawa (M3Y)-Paris-type interaction with zero-range exchange contribution and pointlike proton-proton potential are given, respectively, as [14]

$$v(s) = 11\,062\frac{e^{-4s}}{4s} - 2537.5\frac{e^{-2.5s}}{2.5s} - 592\delta(s), \tag{4}$$

$$v_c(s) = \frac{e^2}{4\pi\epsilon_0}\frac{1}{s}. \tag{5}$$

$\rho(r)$ is the matter (charge) density distributions of the nucleus which is given as the two-parameter Fermi distribution,

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[\frac{r-R_0}{a_0}\right]}, \tag{6}$$

where $R_0^{p(t)} = 1.07A_{p(t)}^{1/3}$ (fm). The superscripts $p$ and $t$ stand for projectile and target, respectively. The surface diffuseness parameter is $a_0 = 0.54$ (fm) [14]. The value of $\rho_0$ is determined through conservation of mass number and atomic number.

It is worthwhile to note that, as can be seen in Fig. 1, the calculated Coulomb potential with the double-folding method and uniform charge distribution approximately gives the same results in the near-barrier region. To reduce the calculation time, we have adopted the well-known uniform charge distribution relation of the Coulomb potential.

By assuming the potential barrier $V(r)$ as an inverted parabola, the Hill-Wheeler (HW) transmission coefficient [3,15,16] is obtained as

$$T_l(E) = \frac{1}{1 + \exp\left\{2\pi\left[V_b + \hbar^2 l(l+1)/2\mu R_b^2 - E\right]/\hbar\omega\right\}}, \tag{7}$$

where $V_b$, $R_b$, and $\omega$ are the barrier height, the barrier position, and the curvature of the potential, respectively. $V_l(r = R_b) = \frac{\hbar^2 l(l+1)}{2\mu R_b^2}$ is the repulsive centrifugal potential at the barrier position. The details of calculations have been given in Ref. [3].

By substitution of the HW transmission coefficient in partial-wave expansion and integrating over all $l$ (instead of summation), the analytic Wong formula of the fusion cross section is obtained [3,17]

$$\sigma(E) = \frac{\hbar\omega R_B^2}{2E}\ln\left[1 + \exp\left(\frac{2\pi}{\hbar\omega}(E - V_B)\right)\right]. \tag{8}$$

The double-folding method has been adopted to calculate the repulsive and attractive nuclear potentials. Consequently, the parameters of the Wong formula, barrier height, barrier position, and frequency have been determined using this method.

### Temperature-dependent nuclear potential

By adding a temperature-dependent repulsive nuclear potential term to the attractive one, the temperature-dependent effective potential

$V_{\text{eff}}(r,T) = V_N^{\text{attr.}}(r) + V_N^{\text{rep}}(r,T) + V_C(r)$ and, consequently, fusion cross section are obtained.

Using the zero-range repulsive nucleon-nucleon potential,

$$v_{\text{rep}}(s,T) = v_{\text{rep}}(T)\delta(s), \tag{9}$$

and matter density distribution with new diffuseness parameter, the repulsive nuclear potential can be calculated with the double-folding integration,

$$V_N^{\text{rep}}(r,T) = v_{\text{rep}}(T)\int d\vec{r}_1 d\vec{r}_2 \rho_p'(\vec{r}_1)\delta(s)\rho_t'(\vec{r}_2), \tag{10}$$

where $v_{\text{rep}}(T)$ is the temperature-dependent repulsive strength coefficient. $\rho_p'(\vec{r}_1)$ and $\rho_t'(\vec{r}_2)$ are density distributions of projectile and target nuclei with repulsive diffuseness parameter $a_{\text{rep}}$.

The factor $v_{\text{rep}}(T)$ can be determined through the relation among the nuclear temperature of the compound nucleus, the nuclear incompressibility, and the total nuclear potential at origin $r = 0$. Following the assumption of Refs. [9,10] to relate the energy increase in the compound system $\Delta U$ at the origin as a result of duplication of the saturation density $\rho_0$ at $r = 0$ to the total nuclear potential at the origin and including the temperature dependence of the equation of state,

$$\Delta U = V_N^{(\text{attr.})}(0) + V_N^{(\text{rep})}(0),$$
$$\Delta U = 2A_p[e_H(2\rho_0, T) - e_C(\rho_0)], \tag{11}$$

the factor $v_{\text{rep}}(T)$ is obtained from $V_N^{(\text{rep})}(0)$. In Eq. (11), $e(\rho)$ is the equation of state, and indices $H$ and $C$ stand for hot and cold nuclei. $A_p$ is the mass number of projectile. Based on the relation among $e_H$, $e_C$, and the excitation energy per nucleon $e_H = e_C + E'^*$ [18] and the relation between the equation of state and the incompressibility of nuclear matter $K$,

$$K = 18[e_C(2\rho_0) - e_C(\rho_0)], \tag{12}$$

TABLE I. Values of incompressibility parameter [25] and calculated level-density parameter of the compound nucleus.

| Fusion reaction | $K$ (MeV) | $a_{LD}$ (MeV$^{-1}$) |
|---|---|---|
| $^{18}O+{}^{63}Cu$ | 233.8 | 8.54 |
| $^{18}O+{}^{194}Pt$ | 220.6 | 20.8 |
| $^{16}O+{}^{208}Pb$ | 219.1 | 21.9 |
| $^{12}C+{}^{141}Pr$ | 224.6 | 15.17 |

the following temperature-dependent relation for $\Delta U$ is obtained as

$$\Delta U = \frac{A_p}{9}K + 2A_pE'^*, \tag{13}$$

where $E'^*$ is the excitation energy of the compound nucleus per nucleon. Equation (11) has been obtained using the approximate form of the equation of state as the parabolic expansion around $\rho_0$, $e(\rho) = e(\rho_0) + \frac{K}{18\rho_0^2}(\rho - \rho_0)^2$ [19]. By means of the definition of nuclear temperature $T$ [20], the excitation energy per nucleon is obtained as

$$E'^* = a'_{LD}T^2, \tag{14}$$

and, finally, $\Delta U$ is calculated as

$$\Delta U = 2A_p\left(\frac{K}{18} + a'_{LD}T^2\right), \tag{15}$$

where $a'_{LD} = \frac{a_{LD}}{A_{CN}}$. $A_{CN}$ is the mass number of the compound nucleus, and $a_{LD}$ is the level-density parameter of the compound nucleus. Using the Thomas-Fermi method [21], the level-density parameter is determined by calculation of the single-particle level density at Fermi energy $g(E_F)$,

$$a_{LD} = \frac{\pi^2}{6}\left[g_n(E_F^n) + g_p(E_F^p)\right]. \tag{16}$$

The details of calculations have been given in Refs. [22,23].

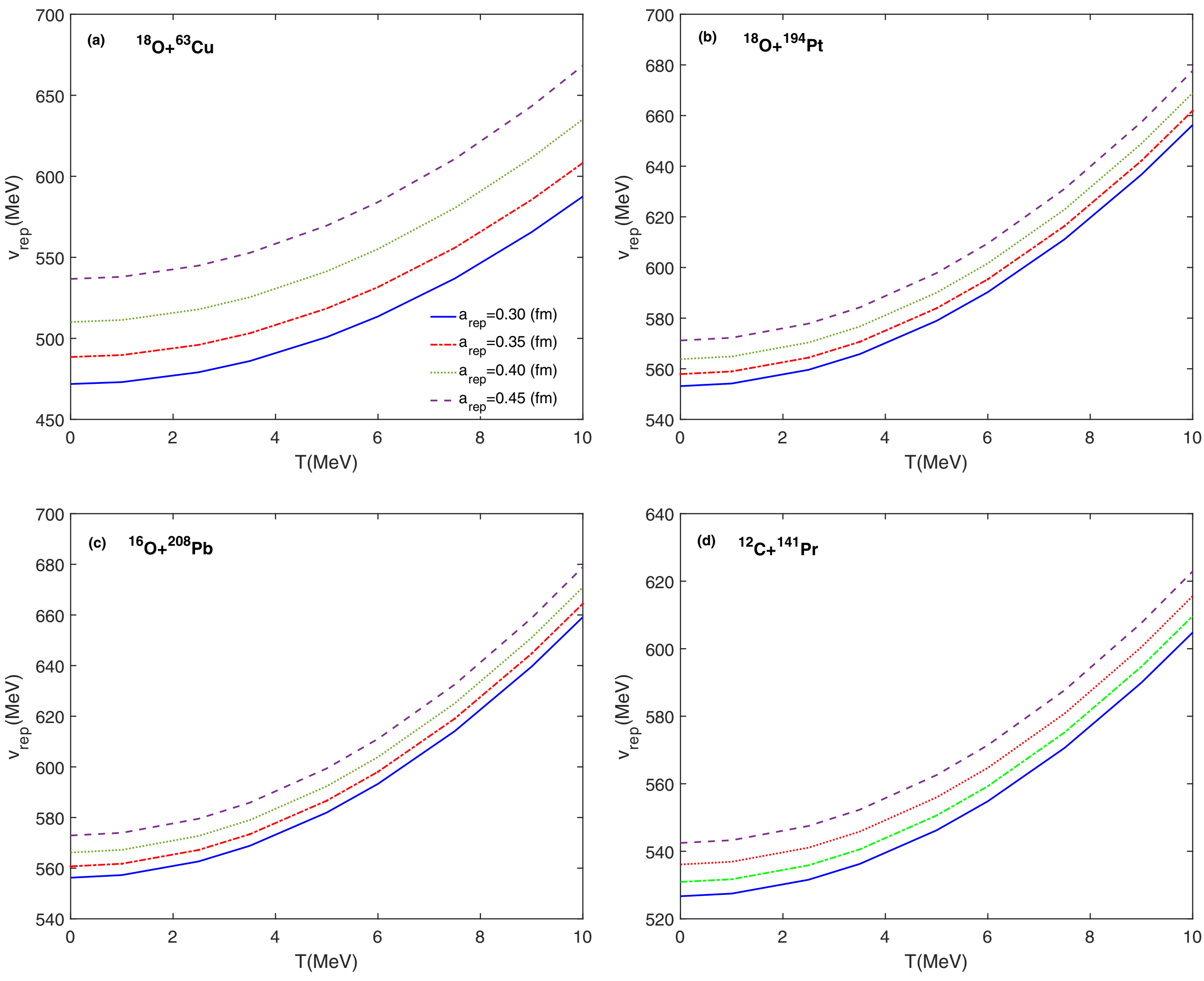


FIG. 2. Variation of repulsive strength with temperature.

TABLE II. Fusion parameters of the Wong formula. The barrier height $V_b$, its position $R_b$, and $\hbar\omega$ are in MeV, femtometers, MeV, respectively. The upper and lower data, respectively, are for $a_{\rm rep} = 0.35$ and 0.45 fm.

| | M3Y | | | M3Y + rep ($T = 0$) | | | M3Y + rep ($T = 5$ MeV) | | | M3Y + rep ($T = 10$ MeV) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Fusion reaction | $R_b$ | $V_b$ | $\hbar\omega$ | $R_b$ | $V_b$ | $\hbar\omega$ | $R_b$ | $V_b$ | $\hbar\omega$ | $R_b$ | $V_b$ | $\hbar\omega$ |
| $^{18}O + ^{63}Cu$ | 10.13 | 30.59 | 3.23 | 10.10 | 30.63 | 3.30 | 10.09 | 30.64 | 3.30 | 10.09 | 30.65 | 3.32 |
| | | | | 9.82 | 31.11 | 3.91 | 9.79 | 31.16 | 3.99 | 9.69 | 31.31 | 4.23 |
| $^{18}O + ^{194}Pt$ | 11.67 | 71.98 | 4.17 | 11.60 | 72.15 | 4.35 | 11.59 | 72.16 | 4.36 | 11.58 | 72.17 | 4.40 |
| | | | | 11.16 | 73.52 | 5.56 | 11.15 | 73.64 | 5.60 | 10.88 | 74.10 | 6.38 |
| $^{16}O + ^{208}Pb$ | 11.62 | 75.85 | 4.51 | 11.56 | 76.05 | 4.70 | 11.55 | 76.06 | 4.71 | 11.54 | 76.09 | 4.73 |
| | | | | 11.08 | 77.63 | 6.08 | 10.98 | 77.79 | 6.37 | 10.84 | 78.40 | 6.80 |
| $^{12}C + ^{141}Pr$ | 10.87 | 43.66 | 4.13 | 10.82 | 43.75 | 4.26 | 10.81 | 43.76 | 4.27 | 10.81 | 43.77 | 4.29 |
| | | | | 10.44 | 44.56 | 5.31 | 10.40 | 44.63 | 5.44 | 10.25 | 44.87 | 5.88 |

Therefore, using Eqs. (10), (11), and (15), the temperature-dependent repulsive factor is determined as

$$v_{\rm rep}(T) = \frac{2A_p\left(\frac{K}{18} + a'_{LD}T^2\right) - V_N^{(\rm attr.)}(0)}{V_N^{(\rm rep)}(0)}. \quad (17)$$

Using the temperature-dependent effective potential, three essential parameters of the Wong formula are derived as a function of temperature. The introduced temperature is the temperature of the compound nucleus and can be determined through the excitation energy of the compound nucleus as

$$T = \sqrt{\frac{E^*}{a_{LD}}}, \quad (18)$$

$$E^* = E_{\rm c.m.} + Q_{\rm in}, \quad (19)$$

where $Q_{\rm in}$ is the incident $Q$ value. Therefore, the corresponding temperature is a function of $E_{\rm c.m.}$.

## III. RESULTS

The described model in the previous section is now applied to find out the effect of nuclear temperature and repulsive core on the fusion cross section of four heavy ion fusion reactions. The spherical or deformed nuclei with very small deformation parameters have been selected [24]. The wide range of nuclear temperatures, up to very high-temperature $T = 10$ MeV, have been considered to analyze the temperature dependence of the repulsive factor, the effective potential, and the parameters of the Wong formula. Nevertheless, the evaluation of the fusion cross section has been performed based on the temperature of the compound nucleus at each incident energy [Eq. (18)]. The calculated level-density parameter of the compound nucleus of four reactions $^{18}O + ^{63}Cu$, $^{18}O + ^{194}Pt$, $^{16}O + ^{208}Pb$, and $^{12}C + ^{141}Pr$ have been listed in Table I. The values of nuclear incompressibility $K$ have been adopted from Ref. [25].

The variations of the repulsive factor with temperature in the wide range of $0 \leqslant T \leqslant 10$ MeV have been plotted in Figs. 2(a)–2(d) for four reactions with different values of repulsive diffuseness $a_{\rm rep} = 0.30,\ 0.35,\ 0.40,$ and 0.45 fm. An increasing behavior is observed in this range. The moderate gradient is seen in $0 < T < 3$ MeV. Larger values of $a_{\rm rep}$ give larger repulsive strength coefficients. Also, a similar manner is obtained for variation of $\Delta U$ with temperature.

Figure 3 shows the variations of the effective potential with relative distance for the $^{18}O + ^{63}Cu$ reaction

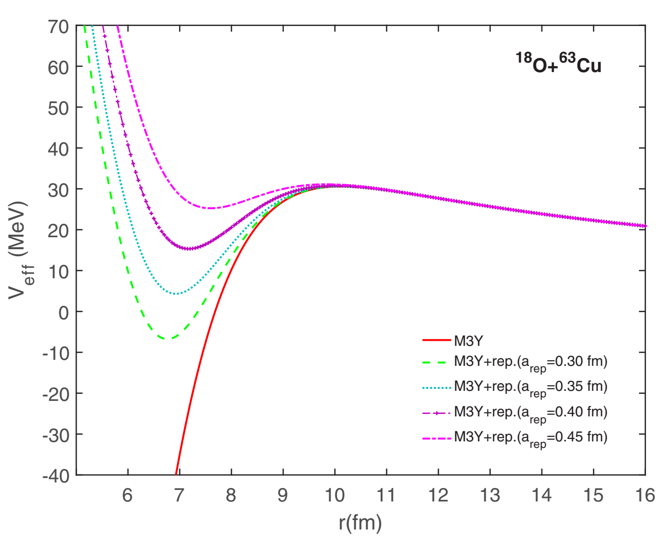


FIG. 3. Variation of effective potential with distance at $T = 0$.

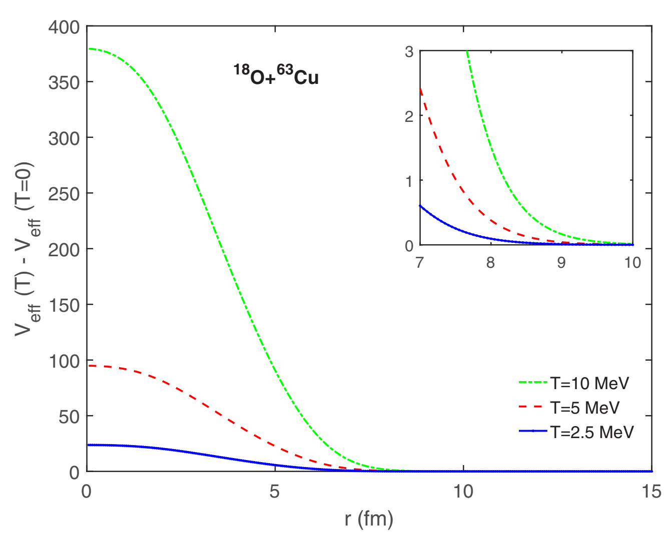


FIG. 4. Temperature dependence of the effective potential for $a_{\rm rep} = 0.35$ fm.

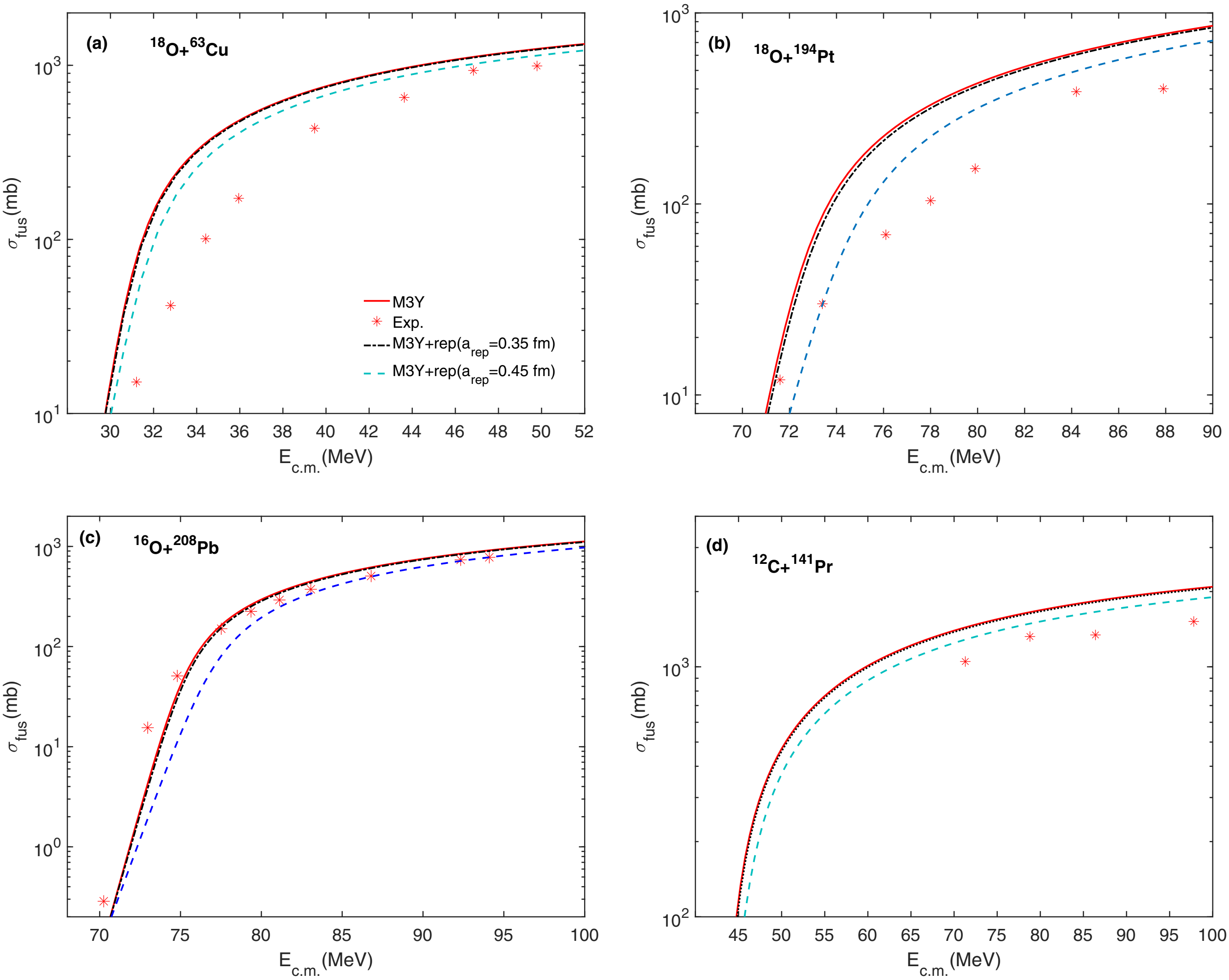


FIG. 5. Variation of the fusion cross section with the center-of-mass energy. The experimental data have been extracted from Ref. [26].

at zero temperature. The effective potentials have been calculated by considering the attractive double-folding nuclear potential (M3Y) and attractive plus repulsive potentials (M3Y + rep) with different values of repulsive diffuseness $a_{\rm rep} = 0.30,\ 0.35,\ 0.40$, and 0.45 fm. By increasing $a_{\rm rep}$, the depth of the potential well is decreased noticeably whereas the fusion barrier is increased slightly. Moreover, the curvature is increased significantly. So, three parameters of the Wong formula $(R_b, V_b, \hbar\omega)$ change with $a_{\rm rep}$, and $\hbar\omega$ varies much larger than two other parameters.

In order to analyze the influence of nuclear temperature on the effective potential, the variations of $V_{\rm eff}(T) - V_{\rm eff}(0)$ with distance have been plotted in Fig. 4 for the $^{18}{\rm O} + ^{63}{\rm Cu}$ reaction with repulsive diffuseness $a_{\rm rep} = 0.35$ fm at different temperatures $T = 2.5,\ 5$, and 10 MeV. These temperatures cover the entire incident energies up to high energies. As expected, $V_{\rm eff}(T)$ has larger values, and a difference between $V_{\rm eff}(T)$ and $V_{\rm eff}(0)$ is noticeable in the interior region whereas, in the barrier region, it decreases and finally disappears at far distances. Obviously, this discrepancy is increased with temperature but in the barrier region is very small.

The calculated parameters of the Wong formula $(R_b, V_b, \hbar\omega)$ for mentioned reactions have been listed in Table II. The second to fifth columns give calculated parameters without repulsion and with a repulsive term by $T = 0,\ 5$, and 10 MeV, respectively. The data of upper and lower cases in columns three to five are corresponded to $a_{\rm rep} =$ 0.35 and 0.45 fm, respectively. By including the repulsive term, $R_b$ is decreased whereas $V_b$ and $\hbar\omega$ are increased a little. A similar behavior is observed by an increase in temperature and repulsive diffuseness. However, noticeable changes are obtained by an increase in repulsive diffuseness.

Figures 5(a)–5(d) show the variations of the fusion cross sections of $^{18}{\rm O} + ^{63}{\rm Cu}$, $^{18}{\rm O} + ^{194}{\rm Pt}$, $^{16}{\rm O} + ^{208}{\rm Pb}$, and $^{12}{\rm C} + ^{141}{\rm Pr}$ reactions with $E_{\rm c.m.}$ in the near and above barrier regions. The experimental data have been extracted from the Nuclear Reactions Video website [26]. The calculated cross section of three forms of the potential M3Y, M3Y + rep

($a_{rep} = 0.35$ fm), and M3Y + rep ($a_{rep} = 0.45$ fm) have been shown in these figures. The temperature of the compound nucleus has been determined through Eqs. (18) and (19). Obtained temperatures for these reactions do not exceed the value of $T = 4$ MeV. Hence, based on the results of Table II, it is not expected to observe a noticeable effect from the nuclear temperature. Two potentials M3Y and M3Y + rep ($a_{rep} = 0.35$ fm) give close results but smaller cross sections are obtained by M3Y + rep ($a_{rep} = 0.45$ fm). As expected, in the above barrier energies fusion cross section is decreased by an increase in the barrier height, and better agreement between theory and experiment is observed by the inclusion of the repulsive nuclear potential.

## IV. SUMMARY AND CONCLUSION

The main results of the present paper illustrated the role of inclusion of the repulsive nuclear potential and its temperature dependence on calculations of the fusion cross section of four heavy ion reactions through the Wong formula. The double-folding potential was considered for the calculation of both attractive and repulsive nuclear potentials. For relatively larger values of repulsive diffuseness, the barrier height and curvature were increased whereas the barrier position was back-shifted. As a result of varying these parameters, the fusion cross section was decreased in the above barrier energies. In the below barrier energies because of the increase in tunneling probability and frequency, the cross section was increased by an increase in the repulsive diffuseness. Here, the corrected form of the temperature dependence of the repulsive nuclear potential was introduced. By including this potential and considering the temperature up to $T = 10$ MeV, the enhancement was observed in the repulsive strength and effective potential. However, this noticeable effect was observed far from the barrier position. Consequently, negligible influence of temperature on near-barrier distances could not notably change the parameters of the Wong formula, barrier position, barrier height, and frequency. The fusion cross sections of these reactions were determined by considering the temperature of the hot compound nucleus that was not exceeded more than $T = 4$ MeV. Hence, the temperature dependence of the repulsive nuclear potential could not affect the fusion cross section. The effect of the temperature dependence of the surface diffuseness parameter of attractive and repulsive nuclear potentials can be evaluated in future studies. Although, better agreement between calculated cross sections and experiment were obtained by considering the repulsive term. Attending to the results of Ref. [8], this effect may be justified as a correction term which amend the negative exchange term of the nucleon-nucleon potential instead of the Pauli repulsion assumption.


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