Properties of excited 0^+ states in ^{14}C and ^{14}O

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

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I have computed expected energies and widths of excited 0^+ states in ^{14}O, using experimental information from ^{14}C and a variety of wave functions. Agreement for the 0_2 state is fair, but my calculations do not support a recent suggestion that a previously unknown state unresolved from the second 2^+ state might be the 0_3 state.

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I. INTRODUCTION

In ^{16}C [1,2], all the low-lying states have positive parity and are of the dominant configuration $^{14}\mathrm{C}_{1p} \otimes (sd)^2$, where the subscript $1p$ denotes structure totally within the $1p$ shell. If the $1d_{3/2}$ orbital is neglected, this space contains six states—two each with $J^\pi = 0^+$ and 2^+, one 3^+, and one 4^+. These $(sd)^2$ states also exist in ^{14}C [3], as $^{12}\mathrm{C}_{1p} \otimes (sd)^2$, along with one p-shell state with $J^\pi = 0^+$ and one 2^+ [4]. (The second p-shell 0^+ and 2^+ states and a p-shell 1^+ state exist at much higher excitation energy.) Long ago, energies and wave functions of the ^{14,16}C states were obtained in a simple $(sd)^2$ shell-model (sm) calculation [2] (hereinafter referred to as Fo78), where they were found to give good agreement with results of the $^{12,14}\mathrm{C}(t, p)$ reactions. This sm calculation used local single-particle energies and global two-body matrix elements. Because of the near equality of the $5/2^+ - 1/2^+$ energy splitting in ^{13}C and ^{15}C [3], the $(sd)^2$ wave-function admixtures in ^{14}C and ^{16}C are virtually identical [2].

Of course, in ^{14}C, the p-shell and $(sd)^2$ states can mix. The first two 2^+ states, at 7.01 and 8.32 MeV, have been long held to be approximately equal mixtures of the two configurations [5]. The mixing of 0^+ states is significantly weaker, but not negligible. An analysis of the (t, p) data suggested a mixing intensity of 12(3)% for the first two 0^+ states [6]. There is strong evidence that the second $(sd)^2$ 0^+ state does not mix with the p-shell state. In the (t, p) reactions, the angular distributions for the second 0^+ state in ^{16}C and the third one in ^{14}C are virtually identical—in both shape and magnitude. These cross sections are small because of destructive interference between s^2 and d^2 configurations. Because there is no p-shell state in ^{16}C to mix with, equality of the results for ^{14}C and ^{16}C strongly suggests the total absence of such mixing in ^{14}C.

This is a simple model, but it contains the physics that is important for present purposes. I return to this point below. I have used previous and modified wave functions of 0^+ states in ^{14}C to calculate expected energies and widths of their mirrors in ^{14}O.

II. ANALYSIS AND RESULTS

Wave-function intensities of the second 0^+ state in ^{14}C are listed in Table I. The first row contains the original results [2], while the next two rows list modifications obtained by allowing participation of the $1d_{3/2}$ orbital [7] and of the $1p$ shell [6]. The p-shell mixing was obtained [6] by fitting the $^{12}\mathrm{C}(t, p)$ angular distributions for this state and the ground state (g.s.). Mirror energy calculations have been performed with a Woods-Saxon nuclear potential having r_0, $a = 1.26$, 0.60 fm, plus the Coulomb potential of a uniform sphere with $r_{0c} = 1.40$ fm. Such calculations have been amazingly successful in reproducing energy shifts in many mirror pairs, including ^{8}He / ^{8}C [8], ^{9}Be / ^{9}B [9,10], ^{10}Be / ^{10}C [10], ^{11}Be / ^{11}N [11], ^{11}Li / ^{11}O [12–14], ^{12}Be / ^{12}O [15,16], ^{14}B / ^{14}F [17], ^{15}C / ^{15}F [18–20], ^{16}C / ^{16}Ne [8], ^{17}N / ^{17}Ne [21,22], ^{17}C / ^{17}Na [23], ^{18}O / ^{18}Ne [24–26], ^{18}N / ^{18}Na [22,27], ^{19}F / ^{19}Ne [28], ^{19}N / ^{19}Mg [29–31], ^{20}F / ^{20}Na [32], ^{20}O / ^{20}Mg [33], ^{22}Ne / ^{22}Mg [34], and ^{40}K / ^{40}Sc [35]. Whenever wave functions are reliably known, this procedure reproduces energy shifts to within 30–40 keV. Such was the case for the nine lowest positive-parity states in ^{18}O / ^{18}Ne [25]. A calculation provides reasonable agreement for the four lowest negative-parity states in ^{14}C / ^{14}O [36].

For the second 0^+ state, it can be noted from Table I that the mirror energy calculation with the original wave function misses the experimental energy by about 300 keV, considerably worse agreement than is usually encountered in such calculations. The addition of the other two components improves the situation somewhat, but the disagreement is still about 150 keV. The last line lists the configuration intensities that reproduce the mirror energy exactly. Note that these indicate a significantly reduced s^2 component.

The proton decay of the 0_2 state can proceed only by p-wave emission to the g.s. of ^{13}N, for which the single-particle (sp) width is 156 keV. The p-shell spectroscopic factor for ^{13}C to ^{14}C (g.s.) [and hence for ^{13}N to ^{14}O (g.s.)] is 1.73 [4], so that a 12% admixture of this configuration into the 0_2 state provides $S = 0.21$, so that we have $\Gamma_{\mathrm{calc}} = S\Gamma_{\mathrm{sp}} = 32$ keV. This is consistent with the experimental limit of <50 keV [3] and approximately compatible with the computed value of 22 keV in [37]. Note, however, a width limit of < 12 keV in an investigation of the $^{14}\mathrm{N}(^3\mathrm{He}, t)$ reaction [38] (which quotes an experimental resolution width of 33 keV).

Before discussing the third 0^+ state, I briefly examine the first 1^- state. In ^{14}C (^{14}O), its dominant configuration is an s neutron (proton) coupled to the g.s. of ^{13}C (^{13}N). Given the

TABLE I. Wave-function intensities of second 0^+ state in ^{14}C and the calculated energy of its mirror in ^{14}O.

Source	Intensities				$E_x(^{14}\mathrm{O})$ (MeV)	
	p shell	$(1d_{5/2})^2$	$(2s_{1/2})^2$	$(1d_{3/2})^2$	Calc.	Expt.[a]
Fo78[b]		0.458	0.542		5.63	5.92
Fo78 + $d_{3/2}$[c]		0.412	0.488	0.10	5.73	5.92
$+p$ shell[d]	0.12	0.363	0.429	0.088	5.77	5.92
Varied s^2/d^2[e]	0.12	0.561	0.231	0.088	5.92	5.92

[a]Reference [3].
[b]Reference [2].
[c]Reference [7].
[d]Reference [6].
[e]Varied to reproduce ^{14}O experimental energy.

excitation energy of 6.094 MeV in ^{14}C, a calculation for a pure s configuration provides an energy of 5.05 MeV in ^{14}O [36], 120 keV below the experimental excitation energy of 5.17 MeV. However, it is clear from the width of this state that it has an s spectroscopic factor S less than unity. The experimental width is 38.1(18) keV [3], and the sp width for s-wave decay to ^{13}N (g.s.) is 54 keV [36], leading to $S = \Gamma_{\mathrm{exp}}/\Gamma_{\mathrm{sp}} = 0.71(3)$, in remarkable agreement with $S = 0.75$ for the ^{14}C mirror state from the ^{13}C(d, p) reaction [39]. An energy calculation with $S = 0.75$ gives $E_x = 5.15$ MeV, only 20 keV from the experimental value.

I turn now to the third 0^+ state. An independent estimate of any possible p-shell component in it can be made by examining its width, which is reported to be 18 keV in the ^{12}C(t, p) reaction [3,40]. The only energetically allowed neutron decay of this state is via $p_{1/2}$ emission to the ^{13}C g.s., with $E_n = 1.57$ MeV. The width to be expected is $\Gamma_{\mathrm{calc}} = S\Gamma_{\mathrm{sp}}$, where Γ_{sp} is the single-particle width and S is a spectroscopic factor $S = \varepsilon^2 S_{1\mathrm{p}}$. The quantity ε^2 is the intensity of a possible p-shell component, and $S_{1\mathrm{p}}$ is the spectroscopic factor connecting the pure p-shell state to ^{13}C (g.s.) $- S_{1\mathrm{p}} = 1.6$ to 2.0. The sp width is large enough that it is difficult to calculate, but it is in the range of 2.5 MeV, with some uncertainty. Thus, an experimental width of 18 keV corresponds to $\varepsilon^2 \sim 0.004$, quite a small number.

For the third 0^+ state, the mirror energy has been computed for a variety of wave functions (Table II). The original wave function produces an energy of 8.46 MeV in ^{14}O. Recall from the Introduction that the (t, p) reaction [2] provides strong evidence that this state contains very little p-shell strength. In any case, the presence of p-shell, fp-shell, or $(d_{3/2})^2$ components in the third 0^+ state will produce a smaller energy shift from ^{14}C to ^{14}O, and hence a higher predicted excitation energy in ^{14}O. The alternate wave function in Table II is that containing only s^2 and d^2 components, and orthogonal to the wave function of the 0_2 state in the last line of Table I. Finally, I show results for a 0_3 state that is pure s^2. Of course, a dominant s^2 component is unrealistic, because it would produce a large ^{12}C(t, p) cross section, whereas this cross section is found to be small [2,40].

In a detailed investigation of $1p$ and $2p$ decays of states of ^{14}O, Charity *et al.* [37] reported that their data for the second 2^+ state at 7.768 MeV had a larger width and slightly different energy than the values listed in the compilation [3]. They suggested the presence of an additional state, which they identified as the third 0^+ state. Their extracted energy for the additional state was 7.669(53) MeV, with a width of <128 keV. Note that this energy is smaller than any of the computed values in Table II, even the one for a pure s^2 state. The calculated 0_3 energy in ^{14}O in [37] was larger than 8 MeV, and relatively constant, for all values of the continuum coupling potential V_0. The calculated energy spacing between the second and third 0^+ states varied from about 1.5 to 3.5 MeV [37] over the displayed range of V_0.

Table II also contains the computed widths for the 0_3 state. These are for s-wave decay to the $1/2^+$ first-excited state of ^{13}N. The next-to-last column lists the widths calculated from the given wave functions if the energy of the 0_3 state is 7.669 MeV. The last column lists the calculated widths if the 0_3 state is at the calculated energy. Note that this second set of widths are sufficiently large that the state might be difficult to observe.

If the 0_3 state is indeed at 7.669 MeV, the original wave function provides an excellent value for the width, but misses

TABLE II. Results for third 0^+ state in ^{14}C and ^{14}O.

Source	Intensities		E_x (MeV)			Γ_{calc} (^{14}O) (keV)[a]	
	d^2	s^2	^{14}C (expt.)	^{14}O (calc.)	^{14}O (expt.)	$E_x = 7.669$	$E_x = E_{\mathrm{calc}}$
Fo78	0.542	0.458	9.746(7)	8.459	7.669(53)?	110	1273
Alternate[b]	0.292	0.708	9.746(7)	8.184		170	1155
Pure s^2	0.0	1.0	9.746(7)	7.853		240	586

[a]$\Gamma_{\mathrm{expt}} < 128$ keV [37].
[b]Orthogonal to s^2, d^2 component of 0_2.

TABLE III. Percentage of s^2 component in first three 0^+ states of ^{14}C and ^{14}O.

State	Percentage	
	Previous	Ref. [37]
g.s.	9	<1
0_2	43	~59
0_3	46	~10

the energy by almost 800 keV. In my calculations, the decrease in energy as one moves from ^{14}C to ^{14}O is a consequence of the so-called Thomas-Ehrman (TE) effect, in which the energy in the proton-rich member of a mirror pair is lower for the s orbital. Charity *et al.* state that their reduced energy is not a TE shift. In fact, the s^2 components of their 0^+ wave functions differ considerably from mine, as can be noted in Table III. Reference [37] states that coupling of the shell model configurations to the continuum can "provide strong energy shifts." However, they did not calculate any energies in ^{14}C, nor any energy shifts from ^{14}C to ^{14}O.

It would have appeared extremely unlikely that the excitation energy shift from ^{14}C to ^{14}O could be as large as 2 MeV. This expectation is supported by the calculations presented above. The shift for 0_2 is only 0.67 MeV. The TE shift increases with decreasing binding of the neutron state and with increasing s occupancy, but a great deal of experience has demonstrated that a potential model calculation reproduces these effects quite well.

Charity *et al.* observed a state at an excitation energy of 8.787(13) MeV, which they suggest might be 1^-. It decays preferentially to the $1/2^+$ state of ^{13}N. Its energy is close to that computed from the original wave function for the 0_3 state, but their reported width is 182(32) keV—considerably smaller than would be expected for the 0_3 state if it is at this energy.

III. SUMMARY

I have computed expected energies and widths of excited 0^+ states in ^{14}O, using experimental information from ^{14}C and a variety of wave functions. Agreement for the 0_2 state is fair, but there is no agreement for 0_3. Charity *et al.* suggest the presence of a previously unknown state unresolved from the second 2^+ state, and suggest it is the 0_3 state. My calculations do not support this hypothesis. Reference [37] did not prove that their possible new state is the third 0^+, and I have not proven that it is not. However, if it is, the present calculations demonstrate that its mirror energy shift is considerably larger than expected, and larger than ones previously encountered. Perhaps the energy and width for the second 2^+ state in the compilation should be adjusted slightly.

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