Rapid Communications

# Substantially enhanced deuteron-triton fusion probabilities in intense low-frequency laser fields

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Deuteron-triton (DT) fusion is the primary fusion reaction used in controlled fusion research, mainly for its relatively high reaction cross sections compared to other fusion options. Even so, to attain appreciable reaction probabilities very high temperatures (on the order of 10–100 × $10^6$ K) are required, which are extremely challenging to achieve and maintain. Here, it is shown that intense low-frequency laser fields, such as those in the near-infrared regime for the majority of intense laser facilities around the world, are highly effective in transferring energy to the DT system and enhancing the fusion probabilities. The fusion probabilities are shown to be enhanced by, at least, an order of magnitude in 800-nm laser fields with intensities on the order of $10^{21}$ W/cm$^2$. The demanding temperature requirement of controlled nuclear fusion may be relaxed if intense low-frequency lasers are exploited.

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*Introduction.* Controlled nuclear fusion has the potential of supplying sustainable and clean energy solutions. In either magnetic confinement fusion [1–3] or inertial confinement fusion [4–6], which are two major schemes for controlled fusion research, the deuteron-triton (DT) fusion reaction ($\mathrm{D} + \mathrm{T} \rightarrow {}^4\mathrm{He} + n + 17.6$ MeV, where D and T denote $^2$H and $^3$H, respectively) is chosen for its relatively high reaction cross sections compared to other fusion options [7–9]. Even so, the required temperature is very high, usually on the order of 10–100 × $10^6$ K in order to attain appreciable fusion reaction probabilities. These temperatures are very challenging to achieve and maintain. New methods or tools that can further increase the DT reaction probabilities and relax the demanding temperature requirement are, therefore, particularly desirable.

The intense laser is a potential candidate. Rapid progress has been achieved on intense laser technologies since the invention of the chirped pulse amplification technique [10]. Light with intensities on the order of $10^{21}$–$10^{22}$ W/cm$^2$ can be generated nowadays, and a further increase for another one or two orders of magnitude can be expected in the near future, for example, with the extreme light infrastructure of Europe [11,12]. The possibility of using intense laser fields to influence nuclear processes, such as $\alpha$ decay, is intriguing and has attracted attention [13–17].

Whether intense laser fields can enhance the DT fusion probabilities remain unclear, yet this question is not only important for controlled fusion research, but also intriguing on its own. In the limit of very high laser frequencies as in the x-ray regime, qualitative estimations have been given by Queisser and Schützhold using a Floquet scattering method [18] and by Lv *et al.* using a Kramers-Henneberger approximation [19], indicating positive answers. However, these methods or approximations cannot be applied to lasers in the near-infrared regime for the majority of intense laser facilities. The difficulty originates from the large number of photons involved (e.g., exceeding 10 000) when high intensity combines with low frequency (photon energy). In the x-ray regime, by contrast, the involved number of photons is very limited, permitting simplifications.

The goal of the current Rapid Communication is to answer the questions whether how and by how much the DT fusion probabilities can be enhanced by intense laser fields. A physically intuitive analysis is presented that is capable of including the large numbers of involved photons. The results show that intense low-frequency laser fields are highly effective in transferring energy to the DT system and enhancing the fusion probabilities. This effectiveness is attributed to the energy properties of the Volkov state, the quantum state of a charge particle in an electromagnetic field. The DT fusion probabilities are shown to be enhanced by, at least, an order of magnitude in 800-nm laser fields with intensities on the order of $10^{21}$ W/cm$^2$. The results also show that low-frequency lasers are more efficient in enhancing DT fusion than high-frequency lasers. The demanding temperature requirement of controlled fusion may be relaxed if intense low-frequency laser fields are exploited.

*DT fusion without laser fields.* Without the presence of laser fields, the cross section can be written in the following three-factor form [20]:

$$\sigma(E) = S(E)\frac{1}{E}\exp\left(-\frac{B_G}{\sqrt{E}}\right), \tag{1}$$

where $E$ is the energy in the center-of-mass frame. The term $1/E$ is called the geometrical factor. The exponential term is the probability of tunneling [21] through the repulsive DT Coulomb barrier, and $B_G = 34.38\sqrt{\mathrm{keV}}$ is called the Gamow constant. The slowly varying function $S(E)$ describes the nuclear physics when D and T are very close and nuclear

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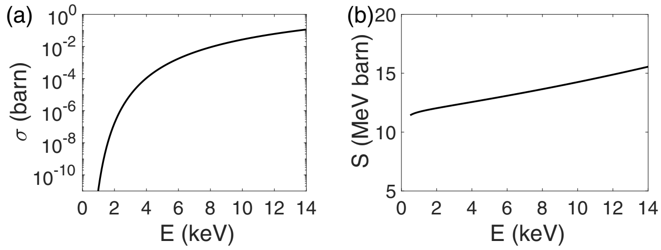


FIG. 1. (a) Cross section of DT fusion as a function of relative-motion energy $E$. (b) The corresponding $S$ function.

potentials are in effect. Here, the parametrization given by Bosch and Hale is used [22]

$$S(E) = \frac{A_1 + E[A_2 + E(A_3 + EA_4)]}{1 + E\{B_1 + E[B_2 + E(B_3 + EB_4)]\}}, \tag{2}$$

which yields accurate agreements to experimental data, especially for relatively low energies that are of relevance to controlled fusion research. The values of the parameters $A_i$'s and $B_i$'s can be found in Table IV of Ref. [22] and will not be repeated here. The cross section and the $S$ function are plotted in Fig. 1 for energies below 14 keV.

*The Volkov state and its energy distributions.* In the center-of-mass frame, the two-body DT system is described by a single particle with mass $\mu = m_1m_2/(m_1+m_2)$. Here, subscript 1 is for D, and 2 is for T. This relative-motion particle with energy $E$ can be described asymptotically by a plane wave,

$$\psi(\boldsymbol{r}, t) = \exp\{i\boldsymbol{p}\cdot\boldsymbol{r} - iEt\}, \tag{3}$$

where the momentum has magnitude $p = \sqrt{2\mu E}$.

In the presence of a laser field, the asymptotic state of the particle becomes a Volkov state [23],

$$\psi_V(\boldsymbol{r}, t) = \exp\left\{i\boldsymbol{p}\cdot\boldsymbol{r} - iEt - i\int_0^t H_I(t')dt'\right\}, \tag{4}$$

where $H_I$ is the interaction Hamiltonian with the laser field,

$$H_I(t) = -\frac{q}{\mu}\boldsymbol{p}\cdot\boldsymbol{A}(t) + \frac{q^2}{2\mu}A^2(t). \tag{5}$$

Note that $q = (q_1m_2 - q_2m_1)/(m_1+m_2) = 0.2e$ is an effective charge for relative motion, and $\boldsymbol{A}(t) = \hat{z}A_0 \sin\omega t$ is the vector potential of the laser field, assumed to be linearly polarized along the $z$ axis. The spatial variation of the vector potential is neglected, and further discussions on this point will be given later. Also, because the laser field is intense, the $A^2(t)$ term is kept.

The Volkov wave function with the above-given vector potential can be expanded in terms of photon numbers,

$$\psi_V(\boldsymbol{r}, t) = e^{i\boldsymbol{p}\cdot\boldsymbol{r}} \sum_{n=-\infty}^{\infty} e^{iu} F_n(u, v) e^{-i(E+U_p+n\omega)t}. \tag{6}$$

The coefficient $F_n(u, v)$ is calculated via

$$F_n(u, v) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-iu\cos\xi + iv\sin 2\xi + in\xi} d\xi. \tag{7}$$

For convenience, the following notations have been defined: $U_p = q^2A_0^2/4\mu$ (the ponderomotive energy), $u = u(\theta) = qpA_0\cos\theta/\mu\omega$, and $v = q^2A_0^2/8\mu\omega$. Here, $\theta$ is the angle between $\boldsymbol{p}$ and the $+z$ axis, and $\theta$ enters into the formalism through $u$. In a thermal environment, the direction between the particle momentum $\boldsymbol{p}$ and the laser polarization axis (the $z$ axis) is random.

In the laser field, the charge particle does not have a definite energy. Instead, it has a series of possible energies $E_n \equiv E + U_p + n\omega$ for all integers $n$. That is, the particle can absorb or emit integer numbers of photons in the laser field. The probability with energy $E_n$ is $P_n(u, v) = |F_n(u, v)|^2$, and the total probability is equal to unity $\sum_{n=-\infty}^{\infty} P_n = 1$.

Some example $P_n$ distributions are given in Fig. 2 for laser wavelengths 800, 400, and 100 nm. The same intensity of $1 \times 10^{20}$ W/cm$^2$ is used. The bare energy (energy without laser fields) $E = 5$ keV, corresponding to a temperature of about $55 \times 10^6$ K, a typical temperature in controlled fusion research. For each wavelength, three $\theta$ angles, namely, 0°, 45°, and 90°, are shown as labeled in each figure.

The most important feature is that higher $E_n$ components are easier to be populated with longer wavelengths. For 800 nm and $\theta = 0°$, $E_n$ with $-1400 < n < 1400$ are populated as shown in Fig. 2(a). The photon energy for this wavelength is 1.55 eV, and the populated range of energy is 3.0 keV $< E_n <$ 7.3 keV ($U_p$ is about 0.1 keV). For 400 nm, although the photon energy doubles, the populated number of photons is $-350 < n < 350$, and the populated range of energy is 4.0 keV $< E_n <$ 6.2 keV as shown in Fig. 2(b). For 100 nm, only $-25 < n < 25$ or 4.7 keV $< E_n <$ 5.3 keV are populated as shown in Fig. 2(c). As $\theta$ increases from 0° to 90°, the populated range of $E_n$ decreases as can be seen by comparing each column of Fig. 2. $P_n$ for $\theta > 90°$ is the same as that for $(180° - \theta)$.

The range of populated $n$ (or $E_n$) can be understood from classical mechanics by analyzing the kinetic energy of the particle over a laser cycle. Or one may impose a stationary phase condition to the integrant in Eq. (7) and solve for the allowed range of $n$. These two analyses lead to identical results. The upper limit of $n$ is found to be $|u| + 2v$, and the lower limit is $-|u| + 2v$ (if $|u|/8v > 1$) or $-u^2/16v - 2v$ (if $|u|/8v \leqslant 1$). These analyses are valid when $n$ is large (the correspondence principle).

Another important feature is that $P_n$ does not decrease with increasing $|n|$. Instead, the general trend is that $P_n$ increases with $|n|$ (while fluctuating), reaches two high peaks near the maximally populated $|n|$, then terminates. This means that the particle can have substantial probabilities being with energies that are considerably higher (or lower) than its bare energy.

In short, intense low-frequency laser fields are highly effective in transferring energy to the DT system. This will lead to substantially enhanced fusion probabilities as will be shown as follows.

*Enhanced DT fusion probabilities in intense laser fields.* A component $E_n$ with $n > 0$ will lead to a higher fusion probability than the bare energy $E$ does. The corresponding component $E_{-n}$ will lead to a lower fusion probability than the bare energy $E$ does. A net gain, however, can be obtained due to the exponential dependency of the DT fusion cross section on the energy, especially at relatively low energies as can be seen from Fig. 1(a) and from Fig. 2 on the linear scale (red dashed

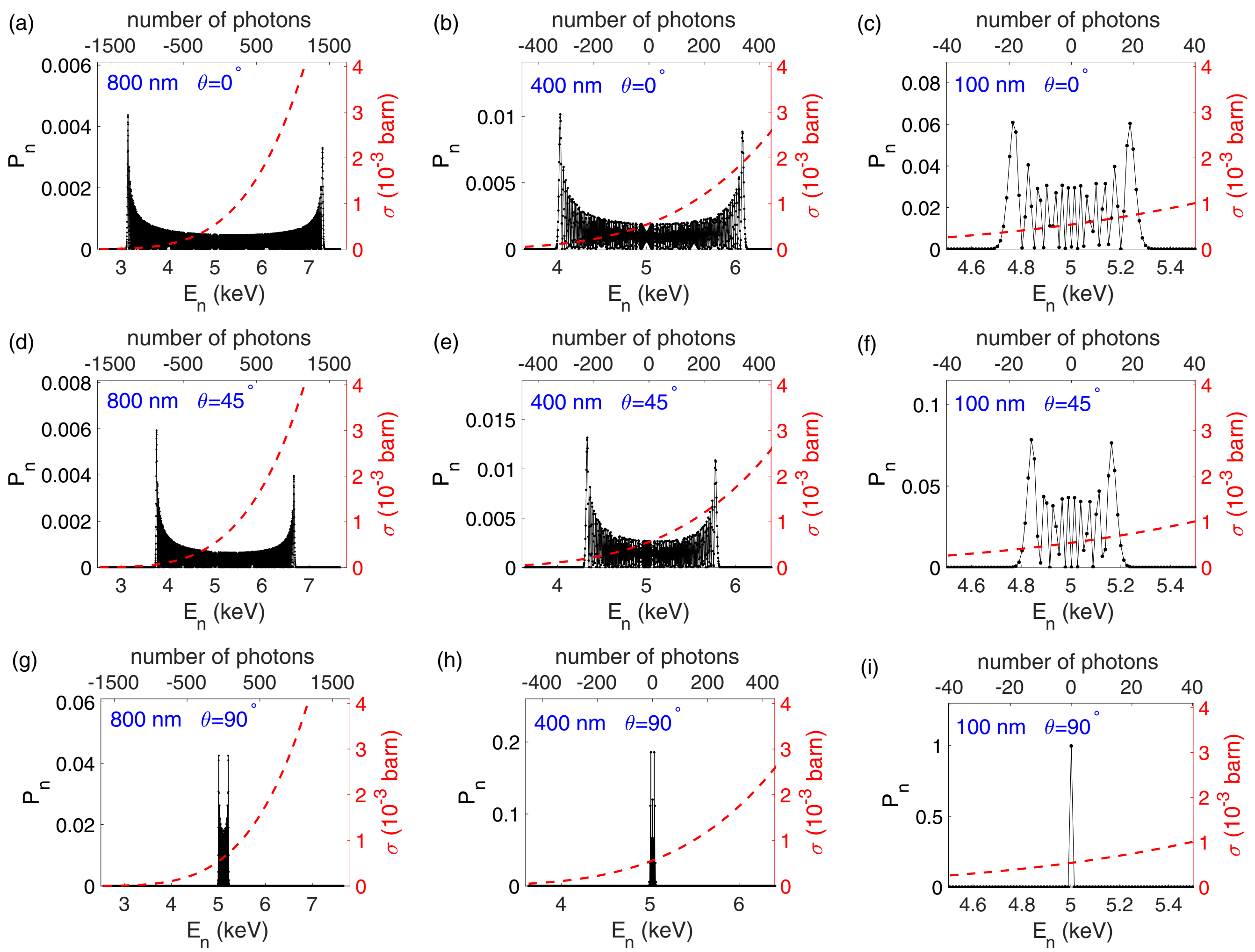


FIG. 2. Distributions of $P_n$ for laser wavelength 800 nm (left column), 400 nm (middle column), and 100 nm (right column). The laser intensity is $1 \times 10^{20}$ W/cm$^2$ for all the three wavelengths. For each wavelength, three $\theta$ angles are shown as labeled in the figure. $\sum_{n=-\infty}^{\infty} P_n = 1$ holds for each figure. The fusion cross section $\sigma(E)$ has also been shown in each panel (red dashed curve) for the corresponding energy range in a linear scale (right axis).

line in each panel). This is the mechanism of enhanced DT fusion probabilities in intense laser fields.

To be more quantitative, given the laser parameters (intensity and wavelength), one may define an effective DT fusion cross section for each angle $\theta$ as

$$\sigma_L(E,\theta) = \sum_{n=-\infty}^{\infty} P_n[u(\theta), v]\sigma(E + U_p + n\omega), \tag{8}$$

and an angle-averaged effective fusion cross section as

$$\sigma_L(E) = \frac{1}{2}\int_0^{\pi} \sigma_L(E,\theta)\sin\theta\, d\theta. \tag{9}$$

It is to be emphasized that the laser field does not change the cross-section function $\sigma(E)$ *per se*, it just changes the energy of the particle before tunneling.

Figure 3 shows angle-averaged $\sigma_L$ under different laser intensities and wavelengths for $E = 1$, 5, and 10 keV. These energies cover a typical range of temperatures in controlled fusion research. One can see from all the three energies that $\sigma_L$'s are substantially higher than the corresponding laser-free cross sections. The stronger the laser intensity, the larger the $\sigma_L$. For $E = 1$ keV, a wavelength 800 nm, and an intensity of $5 \times 10^{21}$ W/cm$^2$, the enhancement is over nine orders of magnitude. For $E = 5$ keV and the same laser parameters, the enhancement is over two orders of magnitude. For $E = 10$ keV and the same laser parameters, the enhancement is over one order of magnitude. The factor of enhancement drops as $E$ increases because the cross-section function $\sigma(E)$ [Fig. 1(a)] increases more slowly as $E$ increases.

One also sees from Fig. 3 that longer wavelengths are more efficient in enhancing the fusion probability. The factor of enhancement drops as the wavelength changes from 800 to 400 to 100 nm. This is a direct consequence of the probability distribution $P_n$ explained above in Fig. 2: It is easier to populate high-$E_n$ components using longer wavelengths.

Without laser fields, the DT fusion cross section for 1 keV ($1.37 \times 10^{-11}$ b) is over nine orders of magnitude smaller than that for 10 keV (0.027 b). This gap can be filled, to a large extent, by intense laser fields. For example, the angle-

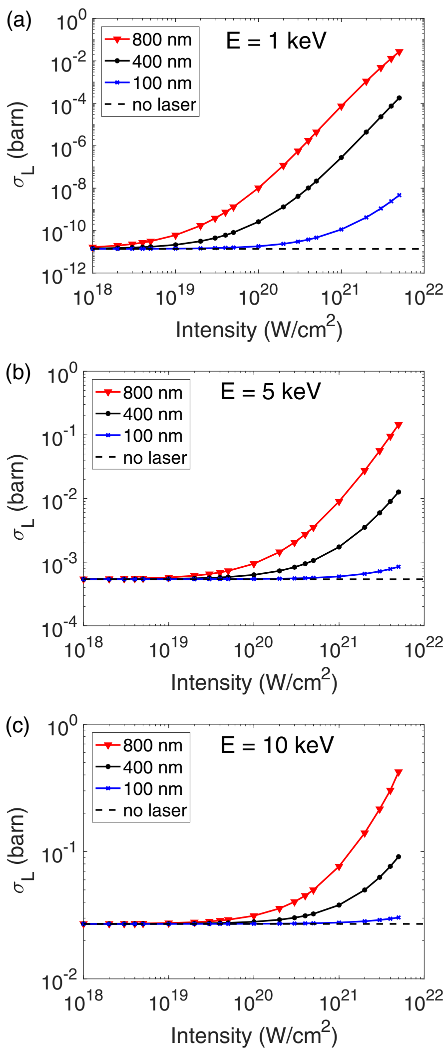


FIG. 3. Angle-averaged effective DT fusion cross-section $\sigma_L$ under different laser intensities and wavelengths for $E =$ (a) 1 keV, (b) 5 keV, and (c) 10 keV. The horizontal dashed line in each figure marks the corresponding laser-free cross section.

averaged $\sigma_L$ with laser wavelength 800 nm and intensity $5 \times 10^{21}$ W/cm$^2$ is 0.0272 b for 1 keV, and 0.423 b for 10 keV. The gap shrinks to about one order of magnitude. Therefore, it should be possible to relax the DT-fusion temperature requirement, which is known to be difficult to achieve and maintain, by using intense low-frequency laser fields.

*Further remarks.* In the above analyses, the laser field does not alter the DT-fusion process from the fundamental level, such as affecting the $S$ function. This is justified by the fact that what now regarded as very intense laser fields, such as those with intensities on the order of $10^{21}$ W/cm$^2$, are still negligible compared to nuclear potentials. The laser field has little effect on processes inside a nucleus or when the D and T are very close to each other. The role of the laser field is to change the particle energy before tunneling, and this energy change has a substantial effect on the fusion probabilities.

For simplicity, the plane-wave Volkov state has been used, which does not include the DT Coulomb potential. If one replaces, in Eq. (4), the spatial part $e^{i\boldsymbol{p}\cdot\boldsymbol{r}}$ with a Coulomb wave-function $\phi_{\boldsymbol{p}}(\boldsymbol{r})$, then the resulting wave function is called a Coulomb-Volkov state [24,25], which is an approximate solution to the full Coulomb-plus-laser system. The discussions above will not be affected because the Coulomb-Volkov state has exactly the same temporal part, hence, the energy distributions as the plane-wave Volkov state.

For simplicity, the spatial variation of the laser field is not taken into account. This approximation holds when the spatial range of motion of the particle is much smaller than the spatial range across which the laser field amplitude changes appreciably. The former range can be estimated by the quiver motion amplitude of the particle $qA_0/\mu\omega$, and the latter range can be estimated by the radius of the laser focal spot $R_c$. The required condition is $qA_0/\mu\omega \ll R_c$. For example, for $R_c \approx 1$ $\mu$m and intensity $5 \times 10^{21}$ W/cm$^2$, the frequency needs to satisfy $\omega \gg 0.0013$ a.u. or the wavelength $\lambda \ll 3.5 \times 10^4$ nm. The wavelengths used in this Rapid Communication obviously satisfy this condition.

The timescale of forming the Volkov state or of laser acceleration is on the order of an optical cycle (2.6 fs for 800 nm). For comparison, in inertial confinement fusion, the typical confinement time is on the order of 10 ps [6], which is much longer than the time needed for laser acceleration.

It is worth mentioning that the Volkov state has been widely used and very successful in strong-field atomic physics, the research discipline studying the interaction between atoms and intense laser fields to describe the state of an ionized electron [26–29].

Extension to elliptically or circularly polarized laser fields is straightforward. It is found that elliptical or circular polarization does not lead to higher efficiencies in enhancing the DT fusion probabilities, mainly due to the reduction of the laser amplitude from $A_0$ to $A_0/\sqrt{1+\varepsilon^2}$ with $\varepsilon$ as the degree of ellipticity.

*Conclusion.* In conclusion, the physics of using intense laser fields to enhance the DT fusion probabilities has been considered. The questions whether, how, and by how much the DT fusion probabilities can be enhanced by intense laser fields, especially those with frequencies in the near-infrared regime for the majority of intense laser facilities, have been answered. The combination of high intensity and low frequency leads to highly effective energy transfer from the laser field to the DT system due to energy properties of the quantum Volkov state. The results show that the probabilities of DT fusion can be substantially enhanced by, at least, an order of magnitude in 800-nm lasers with intensities on the order of $10^{21}$ W/cm$^2$. The results also show that low-frequency lasers are more efficient in enhancing DT fusion than high-frequency lasers. The results indicate that intense low-frequency laser fields can be very helpful to controlled fusion research, and the demanding temperature requirement may be relaxed if intense low-frequency laser fields are fully exploited.

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