α-decay half-lives of lead isotopes within a modified generalized liquid drop model

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Theoretical half-lives of α decay for lead (Pb) isotopes ($Z = 82$) in the range $178 \leqslant A \leqslant 220$ systematically have been investigated within the modified generalized liquid drop model (MGLDM). Akrawy, Royer, AKRE, and Modified RenB formulas are used to evaluate α-decay half-lives. The experimental Q values are used. The results are compared with the experimental data and with Coulomb and proximity potential model. The standard deviations are evaluated; the results indicate that the MGLDM and Akrawy are the best model and formula for evaluating α decay for Pb isotopes.

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I. INTRODUCTION

One of the physical processes that can be easily described by quantum mechanics is the α decay which is one of the most important decay modes for heavy and superheavy nuclei [1–12], and also important tools to study the nuclear force and nuclear structure [13–17].

In 1928 the α decay was first explained by Gamow [18] and Condon and Gurney [19] based on the quantum tunneling effect which can be considered as the first successful quantum description of nuclear phenomena.

The different theoretical models have been used to describe the α-decay half-lives, like the preformed cluster model (PCM) [20,21], the generalized liquid drop model (GLDM) [22–25], the fissionlike model (FLM) [26], the effective liquid drop model (ELDM) [27], the Coulomb and proximity potential model [28–30], and the modified generalized liquid drop mode (MGLDM) [31–33].

The isotopes of Pb have been particularly well studied and in the environmental media find a wide range of applications [34]. They provide valuable insights into the origin of Pb within a sample, and allow for reliable fingerprinting of their source, which is useful for a variety of applications, from tracing sources of pollution-related Pb to the origins of Pb in archaeological artefacts [35].

The naturally occurring stable isotopes of lead are ^{204}Pb, ^{206}Pb, ^{207}Pb, and ^{208}Pb with half-lives as 1.4×10^{20} yr, 2.5×10^{21} yr, 1.9×10^{21} yr, and 2.6×10^{21} yr [36], and abundances of 1.5%, 24%, 22%, and 52.5%, respectively [37]. A total of 43 lead isotopes are now known, including very unstable synthetic species.

The generalized liquid drop model (GLDM) presented by Royer [38] is one of the theoretical models that is widely used for α-decay study [39–42]. Based on it, Santhosh *et al.* [31] modified the GLDM by commingling the nuclear proximity potential suggested by Blocki and co-workers [43,44], that used to evaluate α-decay half-lives for Pb isotopes.

Four empirical formulas are used in this study: the Akrawy formula [45], the Royer formula [46], the AKRE formula [47], and the modified RenB formula [48], to describe the α-decay half-lives.

We have organized the present article as follows. In Sec. II, we introduce a modified generalized liquid drop model used for this study. In Sec. III, the formalism of α-decay half-lives is described using four empirical formulas. The results and discussions are included in Sec. IV. The conclusion is given in Sec. V.

II. GENERALIZED LIQUID DROP MODEL WITH NEW PROXIMITY POTENTIAL

In GLDM, the potential barrier for α decay is given as [39]

$$E = E_V + E_S + E_C + E_R + E_P, \tag{1}$$

where the terms E_V, E_S, E_C, E_R, and E_P represent the barrier terms of volume, surface, Coulomb, rotational, and proximity energy respectively.

For one body shape, the barrier terms in MeV are given by

$$E_V = -15.494(1 - 1.8I^2)A, \tag{2}$$

$$E_S = 17.9439(1 - 2.6I^2)A^{2/3}\left(S/4\pi R_0^2\right), \tag{3}$$

$$E_C = 0.6e^2(Z^2/R_0) \times 0.5 \int [V(\theta)/V_0][R(\theta)/R_0]^3 \sin\theta d\theta. \tag{4}$$

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Here I is the relative neutron excess and S is the surface of the deformed nucleus, $V(\theta)$ is the electrostatic potential at the surface, and V_0 is the surface potential of the sphere.

When the nuclei are separated,

$$E_V = -15.494\left[\left(1-1.8I_1^2\right)A_1 + \left(1-1.8I_2^2\right)A_2\right], \quad (5)$$

$$E_S = 17.9439\left[\left(1-2.6I_1^2\right)A_1^{2/3} + \left(1-2.6I_2^2\right)A_2^{2/3}\right], \quad (6)$$

$$E_C = \frac{0.6e^2Z_1^2}{R_1} + \frac{0.6e^2Z_2^2}{R_2} + \frac{e^2Z_1Z_2}{r}. \quad (7)$$

Here A_i, Z_i, R_i, and I_i are the mass number, charge number, radii, and relative neutron excess of the parent and daughter nuclei, r is the distance between the centers of the parent and daughter nuclei.

The nuclear proximity potential term E_P given by Blocki *et al.* [43] is given as

$$E_p(z) = 4\pi\gamma b\left[\frac{C_1C_2}{(C_1+C_2)}\right]\Phi\left(\frac{z}{b}\right), \quad (8)$$

TABLE I. Calculation of α-decay half-lives for Pb isotopes using different theoretical formulas with experimental values.

			$T_{1/2}$ (s)			
A	l_{min}	Q value (MeV)	Expt.	CPPM [51]	GLDM [51]	MGLDM
178	0	7.824	1.20×10^{-04}	4.03×10^{-04}	1.66×10^{-04}	3.186×10^{-04}
179	2	7.629	3.50×10^{-03}	2.59×10^{-03}	1.14×10^{-03}	1.617×10^{-03}
180	0	7.452	4.20×10^{-03}	6.01×10^{-03}	2.09×10^{-03}	4.301×10^{-03}
181	2	7.269	3.60×10^{-02}	3.91×10^{-02}	1.46×10^{-02}	2.191×10^{-02}
182	0	7.099	5.61×10^{-02}	9.55×10^{-02}	2.79×10^{-02}	6.159×10^{-02}
183	2	6.962	5.94×10^{-01}	4.63×10^{-01}	1.49×10^{-01}	2.358×10^{-01}
184	0	6.807	6.13×10^{-01}	$1.09 \times 10^{+00}$	2.73×10^{-01}	6.445×10^{-01}
185	2	6.729	$1.85 \times 10^{+01}$	$3.31 \times 10^{+00}$	9.43×10^{-01}	$1.568 \times 10^{+00}$
186	0	6.504	$1.21 \times 10^{+01}$	$1.65 \times 10^{+01}$	$3.45 \times 10^{+00}$	$8.832 \times 10^{+00}$
187	7	6.427	$1.53 \times 10^{+02}$	$1.65 \times 10^{+03}$	$1.25 \times 10^{+01}$	$1.321 \times 10^{+02}$
188	0	6.143	$2.70 \times 10^{+02}$	$5.62 \times 10^{+02}$	$9.35 \times 10^{+01}$	$2.657 \times 10^{+02}$
189	2	5.905	$3.90 \times 10^{+03}$	$1.09 \times 10^{+04}$	$1.87 \times 10^{+03}$	$3.875 \times 10^{+03}$
190	0	5.732	$1.78 \times 10^{+04}$	$4.78 \times 10^{+04}$	$5.96 \times 10^{+03}$	$1.95 \times 10^{+04}$
191	0	5.487	$7.98 \times 10^{+05}$	$8.66 \times 10^{+05}$	$1.70 \times 10^{+05}$	$3.22 \times 10^{+05}$
192	0	5.255	$3.56 \times 10^{+06}$	$1.62 \times 10^{+07}$	$1.39 \times 10^{+06}$	$5.52 \times 10^{+06}$
193	0	5.049		$2.57 \times 10^{+08}$	$3.51 \times 10^{+07}$	$8.08 \times 10^{+07}$
194	0	4.772	$8.80 \times 10^{+09}$	$1.42 \times 10^{+10}$	$7.99 \times 10^{+08}$	$4.02 \times 10^{+09}$
195	0	4.489		$1.25 \times 10^{+12}$	$1.02 \times 10^{+11}$	$3.19 \times 10^{+11}$
196	0	4.259	$7.40 \times 10^{+09}$	$6.57 \times 10^{+13}$	$2.21 \times 10^{+12}$	$1.53 \times 10^{+13}$
197	0	3.923		$4.04 \times 10^{+16}$	$1.75 \times 10^{+15}$	$8.23 \times 10^{+15}$
198	0	3.743		$1.75 \times 10^{+18}$	$3.20 \times 10^{+16}$	$3.32 \times 10^{+17}$
199	2	3.377		$1.46 \times 10^{+22}$	$2.03 \times 10^{+20}$	$4.47 \times 10^{+21}$
200	0	3.185		$1.59 \times 10^{+24}$	$1.30 \times 10^{+22}$	$2.41 \times 10^{+23}$
201	2	2.891		$1.52 \times 10^{+28}$	$9.57 \times 10^{+25}$	$5.22 \times 10^{+27}$
202	0	2.624		$1.04 \times 10^{+32}$	$3.07 \times 10^{+29}$	$1.26 \times 10^{+31}$
203	2	2.37		$4.06 \times 10^{+36}$	$8.42 \times 10^{+33}$	$3.66 \times 10^{+35}$
204	0	2.004		$1.75 \times 10^{+44}$	$1.08 \times 10^{+41}$	$1.63 \times 10^{+43}$
205	2	1.502		$2.56 \times 10^{+59}$	$3.23 \times 10^{+55}$	$1.74 \times 10^{+58}$
206	0	1.170		$1.40 \times 10^{+74}$	$2.30 \times 10^{+69}$	$9.51 \times 10^{+72}$
207	2	0.427		$1.90 \times 10^{+157}$	$2.02 \times 10^{+148}$	$7.40 \times 10^{+155}$
208	0	0.552		$8.05 \times 10^{+131}$	$1.40 \times 10^{+124}$	$3.73 \times 10^{+130}$
209	5	2.282		$6.81 \times 10^{+38}$	$2.64 \times 10^{+35}$	$2.87 \times 10^{+37}$
210	0	3.826	$3.69 \times 10^{+16}$	$1.83 \times 10^{+17}$	$3.32 \times 10^{+15}$	$3.34 \times 10^{+16}$
211	2	3.605		$2.41 \times 10^{+19}$	$6.33 \times 10^{+17}$	$3.98 \times 10^{+18}$
212	0	3.332		$1.92 \times 10^{+22}$	$1.74 \times 10^{+20}$	$2.84 \times 10^{+21}$
213	5	3.047		$5.25 \times 10^{+25}$	$5.86 \times 10^{+23}$	$6.93 \times 10^{+24}$
214	0	2.799		$1.38 \times 10^{+29}$	$4.90 \times 10^{+26}$	$1.60 \times 10^{+28}$
215	0	2.649		$2.59 \times 10^{+31}$	$1.34 \times 10^{+29}$	$2.88 \times 10^{+30}$
216	0	2.329		$1.05 \times 10^{+37}$	$1.37 \times 10^{+34}$	$1.03 \times 10^{+36}$
217	0	2.179		$1.14 \times 10^{+40}$	$1.91 \times 10^{+37}$	$1.05 \times 10^{+39}$
218	0	1.879		$1.59 \times 10^{+47}$	$5.69 \times 10^{+43}$	$1.30 \times 10^{+46}$
219	0	1.679		$9.77 \times 10^{+52}$	$3.16 \times 10^{+49}$	$7.36 \times 10^{+51}$
220	0	1.419		$1.89 \times 10^{+62}$	$1.05 \times 10^{+58}$	$1.29 \times 10^{+61}$

with the nuclear surface tension coefficient

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2/A^2]\ \mathrm{MeV/fm^2}, \quad (9)$$

where N, Z, and A represent the neutron number, proton, and mass numbers of parent nucleus respectively,Φ represents the universal proximity potential [44], given as

$$\Phi(\varepsilon) = -4.41e^{-\varepsilon/0.7176}, \quad \text{for } \varepsilon > 1.9475, \quad (10)$$

$$\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.016\,96\varepsilon^2 - 0.051\,48\varepsilon^3, \quad \text{for } 0 \leqslant \varepsilon \leqslant 1.9475, \quad (11)$$

with $\varepsilon = z/b$, where the width (diffuseness) of the nuclear surface $b \approx 1$ fm and Süsmann central radii C_i of fragments related to sharp radii R_i as

$$C_i = R_i - \left(\frac{b^2}{R_i}\right). \quad (12)$$

For R_i we use semiempirical formula in terms of mass number A_i as [43]

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}. \quad (13)$$

The interaction potential for parent nucleus and α decay can be expressed as

$$E(r) = E_V + E_S + E_C + E_R + E_P. \quad (14)$$

The barrier penetrability P is calculated by the following integral:

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \sqrt{2B(r)(E(r) - E_{sph})}dr\right], \quad (15)$$

where $R_{\mathrm{in}} = R_1 + R_2, B(r) = \mu$, and $R_{\mathrm{out}} = e^2Z_1Z_2/Q. R_1, R_2$ are the radius of the daughter and α-particle nuclei respectively, μ and E_{sph} are the reduced mass and the released energy respectively.

The decay half-lives for the α particle by the decay constant λ can be calculated as

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right), \quad (16)$$

where the assault frequency υ is taken as $10^{20}\ \mathrm{s^{-1}}$.

III. FORMALISM OF α-DECAY HALF-LIVES

A. Akrawy formula

Akrawy and Ahmed [45] predicted a new empirical formula for α-decay half-lives of 356 nuclei from $106 \geqslant Z \geqslant 120$ from ground state to ground state transitions, the formula given as

$$\log_{10}\left(T_{1/2}^{\mathrm{Akrawy}}\right) = a + b\sqrt[6]{A}\sqrt{Z} + c\frac{Z}{\sqrt{Q}} + d\frac{\sqrt{\ell(\ell+1)}}{Q} + eI + f\mu I^2[\ell(\ell+1)]^{1/4}, \quad (17)$$

where the parameters Z, A, Q, and I represent the atomic number, mass number, decay energy (MeV), and nuclear asymmetry term ($I = \frac{N-Z}{A}$), respectively; μ is reduced mass and ℓ is the orbital angular momentum quantum number. The coefficients a, b, c, d, e, and f are determined by the least square fitting method.

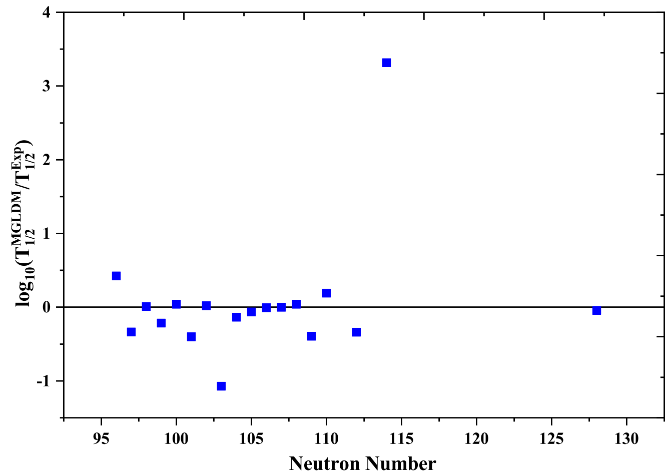

FIG. 1. The ratio of $\log_{10}(T_{1/2}^{\mathrm{MGLDM}}/T_{1/2}^{\mathrm{exp}})$ vs neutron number of the parent nuclei.

B. Royer formula

Royer [46] presented an empirical formula for α-decay half-lives; the formula depends on mass number (A), charge number (Z) of parent nuclei, and the decay energy (Q) released during the decay. The formula is given as

$$\log_{10}\left(T_{1/2}^{\mathrm{Royer}}\right) = a + bA^{1/6}\sqrt{Z} + c\frac{Z}{\sqrt{Q}}. \quad (18)$$

The free parameters a, b, and c are determined by fitting experimental data given in Ref. [49].

C. AKRE formula

Based on the Royer formula, Akrawy and Poenaru [47] set a modified formula of α-decay half-lives by adding the nuclear asymmetry term I, given as

$$\log_{10}\left(T_{1/2}^{\mathrm{AKRE}}\right) = a + bA^{1/6}\sqrt{Z} + c\frac{Z}{\sqrt{Q}} + dI + eI^2. \quad (19)$$

The free parameters a, b, c, d, and e are determined by fitting experimental data given in Ref. [49].

D. Modified RenB formula

Depending on the (Ren) formula [50], Akrawy *et al* [48] modified the α-decay half-lives formula by introducing the nuclear asymmetry I factor and orbital angular momentum quantum number ℓ, given as

$$\log_{10}\left(T_{1/2}^{M\mathrm{RenB}}\right) = a\sqrt{\mu}Z_1Z_2Q^{-1/2} + b\sqrt{\mu Z_1Z_2} + c + dI + eI^2 + f[\ell(\ell+1)], \quad (20)$$

where Z_1, Z_2 are the atomic number of parent and daughter, respectively.

IV. RESULTS AND DISCUSSION

The α-decay half-lives of lead isotopes (Pb) in the range $178 \leqslant A \leqslant 220$ have been calculated using different theoretical formulas of CPPM, GLDM, and MGLDM and empirical formulas of Akrawy, Royer, AKRE, and MRenB. The α-decay energy Q is calculated from [51]

$$Q = \Delta M_p - (\Delta M_\alpha + \Delta M_d) + k\big(Z_p^\varepsilon - Z_d^\varepsilon\big),$$

where ΔM_p, ΔM_α, ΔM_d are the mass excesses of parent, α particle, and daughter, respectively taken from [52]. The term $k\ ({Z_p}^\varepsilon - {Z_d}^\varepsilon)$ represents the screening effect of atomic electrons [53], in which the different values of k and ε are taken from [54].

During α decay, the transfer of angular momentum transition are considered from the spin-parity selection rule given as

$$|J_i - J_j| \leqslant \ell \leqslant J_i + J_j \text{ and } \frac{\pi_i}{\pi_j} = (-1)^\ell, \tag{21}$$

where J_i, J_j, π_i, and π_j are the spin and parity of the parent and daughter nucleus respectively.

The GLDM analytical formula of Royer [51] has been tested to calculate the α-decay half-lives of $^{178-220}$Pb isotopes

TABLE II. The comparison of α-decay half-lives for Pb isotopes using Akrawy, Royer, AKRE, and MRenB formulas with available experimental half-lives.

		$\log_{10}T_{1/2}$				
A	Q value (MeV)	Expt.	Akrawy	Royer	AKRE	MRenB
178	7.824	−3.9208	−3.7461	−3.7522	−3.8638	−3.9486
179	7.629	−2.4559	−2.7566	−2.9856	−3.3087	−2.7634
180	7.452	−2.3768	−2.6483	−2.6534	−2.7073	−2.7692
181	7.269	−1.4437	−1.5390	−1.7729	−2.1910	−1.6396
182	7.099	−1.2510	−1.5263	−1.5305	−1.5362	−1.5789
183	6.962	−0.2262	−0.4178	−0.6685	−1.1445	−0.6019
184	6.807	−0.2125	−0.5380	−0.5411	−0.5083	−0.5349
185	6.729	1.2672	0.4963	0.2121	−0.2821	0.2341
186	6.504	1.0828	0.5627	0.5605	0.6239	0.6098
187	6.427	2.1847	2.2591	1.4395	0.9413	5.4041
188	6.143	2.4314	1.9928	1.9913	2.0790	2.0733
189	5.905	3.5911	4.1020	3.8159	3.2926	3.7783
190	5.732	4.2504	3.7950	3.7937	3.9001	3.8988
191	5.487	5.9020	5.3712	5.9549	5.4415	5.4312
192	5.255	6.5515	6.1607	6.1591	6.2800	6.2788
193	5.049		7.7690	8.4829	7.9867	7.9602
194	4.772	9.9400	8.9168	8.9146	9.0453	9.0409
195	4.489		11.3580	12.2588	11.7498	11.7358
196	4.259	9.8700	12.3548	12.3515	12.4892	12.4777
197	3.923		15.7525	16.8784	16.3422	16.3587
198	3.743		16.5131	16.5079	16.6505	16.6281
199	3.377		22.3500	22.4025	21.8207	22.3647
200	3.185		22.1189	22.1107	22.2598	22.2214
201	2.891		28.3707	28.5890	27.9521	28.5609
202	2.624		29.4866	29.4739	29.6332	29.5733
203	2.37		36.7584	37.2495	36.4717	37.2132
204	2.004		41.0248	41.0041	41.1894	41.0972
205	1.502		59.3466	60.7685	59.2488	60.5573
206	1.170		69.3408	69.2982	69.5885	69.4246
207	0.427		156.1281	161.8722	156.4061	160.5763
208	0.552		124.0989	124.0124	124.5348	124.2431
209	2.282		39.7220	38.8954	38.7944	41.0965
210	3.826	16.6000	15.5213	15.5259	15.4648	15.4086
211	3.605		20.4875	18.5999	19.3820	20.3571
212	3.332		20.2370	21.4533	22.2833	20.0757
213	3.047		27.9815	24.8407	25.7202	28.8336
214	2.799		26.6824	28.2041	29.1399	26.4720
215	2.649		30.1173	30.4594	31.4133	32.3741
216	2.329		34.1232	36.0128	37.0212	33.8625
217	2.179		38.5876	39.0212	40.0569	41.3156
218	1.879		43.7377	46.1139	47.2014	43.4287
219	1.679		51.2841	51.8669	52.9899	54.6429
220	1.419		57.9947	61.1106	62.2765	57.6461

TABLE III. Standard deviation (σ) for the proposed models and semiempirical formulas.

Model	σ
MGLDM	0.849
GLDM [51]	0.897
CPPM [51]	1.061
Akrawy	0.774
Royer	0.809
AKRE	0.947
MRenB	1.103

in the specified mass number range; the results are listed in column 6 of Table I. The calculated α-decay half-lives using the modified GLDM (MGLDM) proposed by Santhosh *et al.* [31] is presented in column 5 of Table I. Another theoretical formula of CPPM has been tried for evaluating the α-decay half-lives of the same Pb isotopes as shown in column 7 of Table I. Through the comparison made between results obtained from these theoretical formulas with the experimental half-lives listed in column 4 of Table I, best agreements were found for the MGLDM formula. This good prediction may refer to the improvement done in the proximity potential of the GLDM model, in which the physical terms of asymmetry parameter I and surface tension S are included. Furthermore, the factor $\log_{10}(T_{1/2}^{\mathrm{MGLDM}}/T_{1/2}^{\mathrm{exp}})$ has been evaluated and plotted in Fig. 1 for the available experimental half-lives, where most of the points are near zero and within ± 0.5, which agrees with the assumption of constant preformation factor in our MGLDM model.

On the other hand, Different empirical formulas, Akrawy, Royer, AKRE, and MRenB, have been tried to calculate the logarithm of α-decay half-lives for the $^{178-220}$Pb isotopes. The results are presented in Table II in which the available experimental $\ln T_{1/2}$ are also listed in column 3 for comparison. Among these empirical formulas, Akrawy reveals better prediction of the experimental half-lives of α decay, where the involvement of asymmetry factor and angular momentum dependence terms contributes to this improvement.

Later on, the standard deviation of the present approach has been used to distinguish the agreement of each formula with the experimental results. The following formula can be used to evaluate the standard deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(\log_{10}T_{1/2}^{\mathrm{theor}} - \log_{10}T_{1/2}^{\mathrm{exp}}\right)^2\right]} \tag{22}$$

Table III presents the calculated standard deviation values for the selected seven formulas. For the theoretical formulas, the minimum standard deviation was 0.849 for the MGLDM formula, and the maximum was 1.061 for the CPPM formula, while within the empirical formulas, the minimum and maximum values were 0.774 and 1.103 for Akrawy and MRenB formulas; the Royer formula shows a deviation of 0.809.

These values again certify the good agreement of the MGLDM, Royer, and Akrawy formulas over the other tested

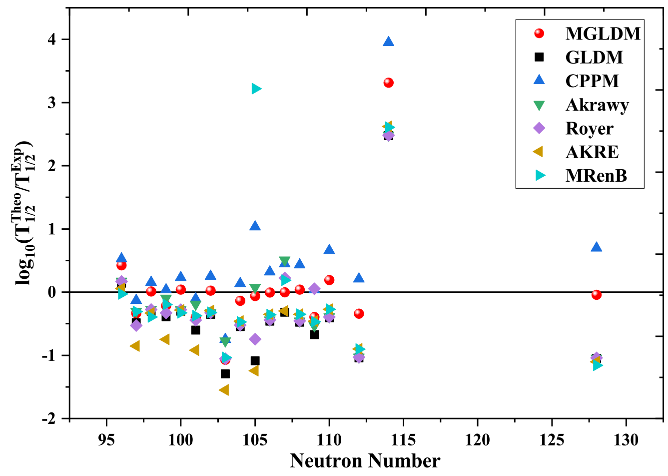

FIG. 2. Comparison of the calculated $\log_{10}(T_{1/2}^{\mathrm{theor}}/T_{1/2}^{\mathrm{exp}})$ of various theoretical approaches.

formulas. Thus, these formulas can be used to reproduce the α-decay half-lives of $^{178-220}$Pb isotopes, respectively. In order to assess the quality of "residual" preformation factor, a plot (Fig. 2) with several panels for $\log_{10}(T_{1/2}^{\mathrm{theor}}/T_{1/2}^{\mathrm{exp}})$ corresponding to various theoretical approaches are also studied. From Fig. 2 it is clear that none of the analyzed approaches were able to properly describe the residual α-core correlations in the preformation factor for nuclei with Z greater than the $Z = 82$ magic number, and further investigations are necessary.

Figure 3 shows the plot of $\log_{10}T_{1/2}$ of alpha decay from the studied Pb isotopes against neutron number of the daughter nuclei. In this figure, the maximum $\ln T_{1/2}$ for ^{207}Pb ($Z = 82$, $N = 125$) and the minimum for ^{178}Pb ($Z = 82$, $N = 96$) have been observed, which reflects the role of closure shell effects relative to the magicity of the neutron or proton numbers and the doubly magic nuclei (proton and neutron numbers).

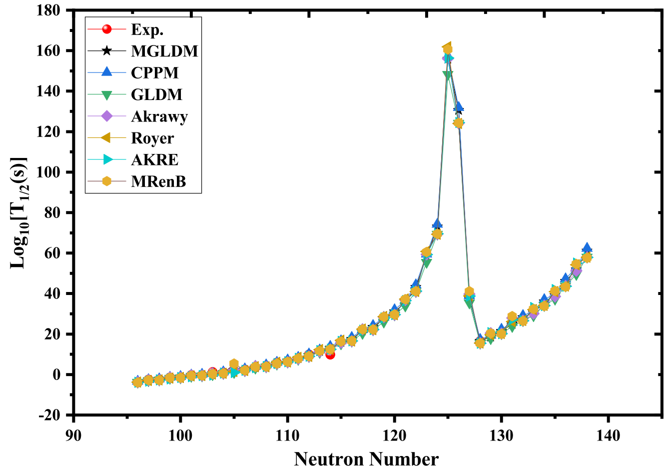

FIG. 3. The comparison of α-decay half-lives for Pb isotopes using Akrawy, Royer, AKRE, and modified RenB formulas with MGLDM and CPPM models.

V. CONCLUSIONS

In this study, the theoretical modified generalized liquid drop model MGLDM has been used to calculate the α-decay half-lives of $^{178-220}$Pb isotopes besides the GLDM and CPPM formulas. Four other empirical formulas, Akrawy, Royer, AKRE, and MRenB, have also been tried in the calculations. Through half-life calculations the shell closure effect and nuclei magicity have been observed. The best agreement with the available experimental α-decay half-lives were obtained from Akrawy, Royer and MGLDM for $^{178-220}$Pb isotopes, indicated by their corresponding standard deviations. Thus, the formulas can be depended on in the reproduction of the existing α-decay half-lives and the unknown ones. From our study it is clear that none of the analyzed approaches were able to properly describe the residual α-core correlations in the preformation factor for nuclei with Z greater than the $Z = 82$ magic number, and further investigations are necessary.

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