Structure of ^{16}C and the $B(E2)$ problem

S. Karataglidis[1,2] and K. Murulane[1]

[1]*Department of Physics, University of Johannesburg, P.O. Box 524, Auckland Park, 2006, South Africa*

[2]*School of Physics, University of Melbourne, Victoria 3010, Australia*

(Received 25 February 2020; accepted 2 June 2020; published 17 June 2020)

The $B(E2)$ value for the decay of the 2_1^+ state of ^{16}C to ground has been the subject of much discussion. The existing measurements extend over a large range, almost an order of magnitude, from 0.63 to 4.15 e^2 fm^4. Analyses assuming a simple model of two neutrons coupled to a ^{14}C core give reasonable agreement but require the inclusion of a large effective charge for the neutrons. To assess this situation, a large-scale $(2+4)\hbar\omega$ shell-model calculation of ^{16}C has been made from which the wave functions have been used to obtain the $B(E2)$ value. As a check, comparison is made with available data on the spectrum of ^{16}C and intermediate-energy elastic proton scattering. It is found that with the new model a much smaller effective charge is needed, if at all, to explain the accepted $B(E2)$ value.

DOI: 10.1103/PhysRevC.101.064316

I. INTRODUCTION

The structure of the heavy carbon isotopes, above ^{14}C, are of interest given the closed $0p$ neutron shell at ^{14}C. One then has a minimum configuration of $2\hbar\omega$ character in the states of these nuclei, because the $0p$ proton shell is open (the minimal configuration is a closed $0p_{3/2}$ orbit, with the $0p_{1/2}$ orbit empty). The open $0p$ shell for the protons suggests significant mixing of additional $\hbar\omega$ components. All the carbon isotopes exhibit neutron skins, given that the neutron separation energies are relatively large (energies in MeV): 1.218, 4.251, 0.73, 4.188, 0.53, and 3.3, for ^{15}C, ^{16}C, ^{17}C, ^{18}C, ^{19}C, and ^{20}C, respectively [1–4]. There are indications that ^{17}C and ^{19}C may exhibit neutron halos given that their separation energies are quite low compared with the other isotopes. However, from analyses of intermediate-energy inelastic proton-scattering data [5], the ground states of both ^{17}C and ^{19}C appear to have a neutron density distribution consistent more with a skin than with a halo.

Given that ^{17}C may be described within a collective model as a neutron coupled to ^{16}C, assuming a simple rotor for ^{16}C [5], it is important to understand the structure of ^{16}C. Little is known of its spectrum: the ground state is $0^+;2$, the first-excited state at 1.77 MeV is a 2^+ state, and the second-excited state is at 3.03 MeV [2]. The second-excited state is tentatively assigned 0^+. At higher energies there are only a cluster of three states at $\approx$4 MeV and a state at 6.11 MeV. There is some evidence for negative-parity states above 7 MeV [6], but these assignments are tentative at best.

Recent shell-model calculations [7–9] considered the structure of ^{16}C by using effective interactions derived from free nucleon-nucleon (NN) interactions. Fujii *et al.* [7] calculated the spectrum of ^{16}C in a no-core shell model, incorporating all shells from the $0s$ to the $0f1p$ shell. They sought to explain the $B(E2)$ value for the decay from the first-excited state. They described the low $B(E2)$ value by the inclusion of both an effective operator and an effective interaction. However, as the authors state, the shell model used is within an incomplete space in energy, so the removal of center-of-mass spuriosity is not exact. They also conclude in their analysis that the $B(E2)$ value is sensitive to the value of the effective neutron charge. With both corrections, they obtain a $B(E2)$ value of 0.82 e^2 fm^4, which agrees reasonably well with the experimental value of 0.63 e^2 fm^4 [10]. On the basis of that experimental value along with analyses of the $B(E2)$ values in other carbon isotopes, using antisymmetric molecular dynamics, Kanada-En'yo [11] concluded that the proton structure in carbon influences the neutron skins in those systems. As the minimum configurations admitted in the even-mass carbon isotopes heavier than ^{14}C is $2\hbar\omega$, the space used is incomplete, because $2\hbar\omega$ components in ^{16}C must necessarily include the $1p1h$ excitations from the $0d1s$ to the $0g1d2s$ shell, and effective charges must be used to calculate electromagnetic observables to account for the limitations of the assumed model.

Measurements of the lifetime of the 2_1^+ state [12,13] in ^{16}C suggest larger values of the $B(E2)$. Wiedeking *et al.* report a value of 4.15 ± 0.73 e^2 fm^4 [12] obtained from a lifetime measurement of the 2_1^+ state in ^{16}C from the ^{9}Be(^{9}Be, $2p$) fusion-evaporation reaction. The subsequent measurements of the lifetime by Ong *et al.* [13,14] report values for the $B(E2)$ from $1.4 \pm 0.6 \pm 0.4$ to $2.7 \pm 0.2 \pm 0.7$ e^2 fm^4, a large variation, but all a factor of two below that reported by Wiedeking *et al.*, but a factor of two above that measured by Imai *et al.* [10]. Ong *et al.* attribute the discrepancy the inclusion of the γ-ray angular distribution into their previous measurement [10], which leads to a reduction in the observed lifetime by a factor of four.

Guiding the analyses of the $B(E2)$ value in ^{16}C has been the assumption that the ground state of ^{16}C may be described by a dominant configuration of $\nu(sd)^2$ coupled to a ^{14}C core. This has been assumed by Wiedeking *et al.* [12] and in the

shell-model calculation of Corragio *et al.* [8]. Extensions to that model suggest the inclusion of proton configurations would influence the $B(E2)$ [15], while the inclusion of more complicated neutron *sd* shell configurations may also explain it [16]. The inclusion of proton configurations is consistent with the conclusions of Kanada-En'yo [11]. In all such analyses using the simple two-neutron model, an effective neutron charge of $\approx 0.4e$ has been required in order to fit the measured or adopted value.

II. SHELL-MODEL CONSIDERATIONS

As an extension beyond these simple models, we performed a no-core $(2+4)\hbar\omega$ shell-model calculation for the positive-parity states of ^{16}C, using a single-particle basis encompassing the six major shells from the $0s_{1/2}$ to the $0h1f2p$ shells. The model space is complete in $2\hbar\omega$ while the only limitation in $4\hbar\omega$ components is the exclusion of the (neutron) $1p1h$ components from the $0d1s$ to the $0i1g2d3s$ shell. The shell-model interaction of Zheng *et al.* [17] was used and the calculations performed using the code OXBASH [18]. We have also performed a complete, no-core, $(0+2+4)\hbar\omega$ calculation of the ground state of ^{14}C, using the same single-particle basis and shell-model interaction, to test the assumption of the simple $\nu(sd)^2$ model for ^{16}C. The wave functions obtained for the ground states of both nuclei are

$$|^{14}\mathrm{C}_{\mathrm{gs}}\rangle = 62.54\%|0\hbar\omega\rangle + 21.03\%|2\hbar\omega\rangle + 16.43\%|4\hbar\omega\rangle,$$
$$|^{16}\mathrm{C}_{\mathrm{gs}}\rangle = 72.82\%|2\hbar\omega\rangle + 27.18\%|4\hbar\omega\rangle. \quad (1)$$

While there are dominant components corresponding to the configuration $\nu(sd)^2$ coupled to the ground state of ^{14}C, the significant admixing of $4\hbar\omega$ components in the ground state of ^{16}C suggests a more complicated wave function. Those components that include the $\nu(sd)^2$ constitute $\approx 60\%$ of the total wave function: 22.63% comes from $\nu(0d_{\frac{5}{2}})^2$ while 22.37% is $\nu(1s_{\frac{1}{2}})^2$. The other 40% of the total wave function comes from more complicated configurations, including those involving proton admixing.

The full low-energy spectrum for ^{16}C is shown in Fig. 1. Therein, there is excellent agreement of the results of the shell-model calculation with the observed 2_1^+ and 0_2^+ states, while the cluster at 4 MeV is reproduced reasonably well. However, there is an indication of a larger energy separation between the calculated energies of the 2_2^+, the 3^+, and 4^+ states. The state observed at 6.11 MeV [2] is indicated as a third 2^+ state in the model.

Note that only the positive-parity states of ^{16}C have been calculated within this model. The requisite model for the negative-parity states requires a $(1+3+5)\hbar\omega$ space calculated within the same single-particle basis, which leads to matrices with dimensions much larger than can be handled by the shell-model code. Most of the states observed above 7 MeV are tentatively assigned as negative parity [6], while there is one (4^+) state given at 9.1 MeV. The present model predicts two additional 4^+ states at 7.85 and 9.37 MeV.

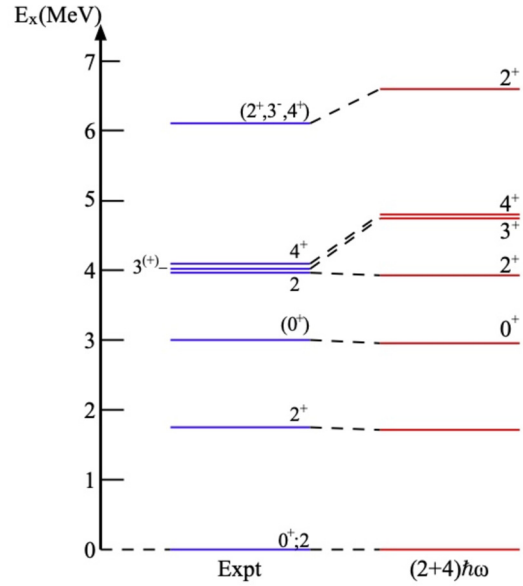

FIG. 1. Low-energy spectrum of ^{16}C. The experimental spectrum [2] is compared with the result obtained from the $(2+4)\hbar\omega$ model described in text.

III. ELASTIC PROTON SCATTERING AND $B(E2)$

To assess the wave functions obtained from the shell model, we have analyzed the available data [19] of the elastic scattering of 300 MeV protons from ^{16}C. The microscopic, nonlocal, Melbourne *g*-folding model for intermediate energy nucleon-nucleus scattering [20] was used, wherein the one-body density-matrix elements obtained from the ground-state wave function were folded with the Bonn B *NN* interaction [21] to obtain the complex and fully nonlocal optical potential. The Bonn B interaction has been used successfully for the descriptions of elastic scattering across the mass range, as well as inelastic scattering for select examples [20]. It should be noted that the upper energy limit for the applicability of the Melbourne *g*-folding model is 300 MeV [20,22]. Harmonic oscillators were assumed for the single-particle states in the nucleus, with oscillator parameter $b = 1.7$ fm. That value corresponds to an oscillator energy of 14 MeV, that which was used in obtaining the shell-model interaction [17], and which is also appropriate for mass-16 nuclei [23].

The optical potential from the folding model may be described, in coordinate space, in terms of a direct and exchange potential, viz. [20]

$$U_{\mathrm{OM}}(\mathbf{r},\mathbf{r}';E) = \delta(\mathbf{r}-\mathbf{r}')\sum_i n_i \int \varphi_i^*(\mathbf{s}) g_{\mathrm{D}}(\mathbf{r},\mathbf{s};E)\varphi_i(\mathbf{s})\, d\mathbf{s} + \sum_i n_i \varphi_i^*(\mathbf{r}) g_{\mathrm{E}}(\mathbf{r},\mathbf{r}';E)\varphi_i(\mathbf{r}') = U_{\mathrm{D}}\delta(\mathbf{r}-\mathbf{r}') + U_{\mathrm{E}}, \quad (2)$$

where n_i is the occupation number for orbit i, $\varphi_i(\mathbf{r})$ is the single-particle wave function, and g_{D} and g_{E} are the direct and exchange parts of the g matrix generated from the *NN* interaction. The code DWBA98 [24] was used to calculate the

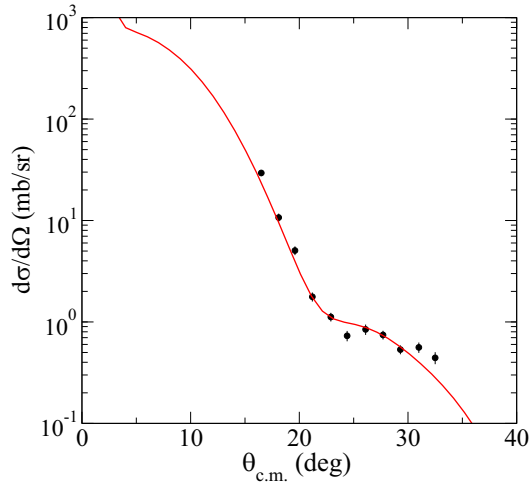

FIG. 2. Differential cross section for the elastic scattering of 300 MeV protons from ^{16}C. The data [19], obtained in the inverse kinematics, are compared with the result of the microscopic optical model calculation described in text.

optical potential and from it obtain the scattering observables. (See Ref. [20] for full details, because it has a description of how the scattering amplitudes are obtained directly from the optical potential found by the folding of the effective NN interaction with the structure of the target. Inelastic scattering is described by a distorted-wave approximation.)

The result of the calculation for the differential cross section is compared with the data [19] in Fig. 2. As shown in Fig. 2, the result of the g-folding calculation agrees very well with the data, with no fitting required.

Given the large variation in the quoted $B(E2)$ values for the transition in ^{16}C, as listed in Table I, we adopt the value given by Ong *et al.*, 2.6 e^2 fm^4, as the benchmark, and as that lying in the middle of the range of values. From our shell-model calculation, we find a $B(E2)$ value of 1.35 e^2 fm^4, using bare operators and the same oscillator parameter as used in the prediction of the proton elastic scattering shown in Fig. 2, well within the range of values indicated in Table I. The inclusion of an effective charge of $0.08e$, for both protons and neutrons, gives a value of 2.61 e^2 fm^4, while one of $0.12e$ gives a value of 3.39 e^2 fm^4. These are much smaller values of effective charge than the value of $0.4e$ used in the models assuming the $\nu(sd)^2$ model for ^{16}C. Together with the agreements found between experiment and model results for the spectrum and the scattering, this indicates that the large-scale shell model adopted gives a far more reliable indication of the structure of ^{16}C. It is clear that the assumption of $\nu(sd)^2$ for the structure of ^{16}C is too simplistic.

TABLE I. Evaluated $B(E2)$ values for the transition $2_1^+ \rightarrow 0_1^+$ in ^{16}C, in units of e^2 fm^4.

Author	$B(E2)$ value
Imai *et al.* [10]	$0.63 \pm 0.11 \pm 0.16$
Wiedeking *et al.* [12]	4.15 ± 0.73
Ong *et al.* [13,14]	$2.7 \pm 0.2 \pm 0.7$[a]
	$2.4 \pm 0.4 \pm 0.6$[b]
	$1.4 \pm 0.6 \pm 0.4$[c]
Fortune [16]	3.5 ± 0.3[d]

[a]Inelastic channel at 72A MeV.
[b]Breakup channel at 79A MeV.
[c]Inelastic channel at 40A MeV.
[d]Simple average of available data.

IV. CONCLUSION

A large scale, no-core, shell-model calculation, in $(2+4)\hbar\omega$ model space, which is complete save for the $1p$-$1h$ transitions to the $0i1g2d3s$ shell, has been used to obtain the spectrum and wave functions of ^{16}C. There is very good agreement found between the results of the calculation for the spectrum, 300 MeV elastic proton scattering, and the $B(E2)$ value, with experiment. This is especially so given that there has been no fitting to the data being described, except in the case of the $B(E2)$ value. It should be noted that the descriptions of the spectrum, proton elastic-scattering differential cross section, and $B(E2)$ value are entirely self-consistent, because there has been no general adjustment to any of the observables. For the $B(E2)$ value, the bare operators give an acceptable value, well within the range of the quoted experimental values. The inclusion of a much smaller effective charge than previously reported gives a value close to the somewhat larger experimental values now accepted. Overall, the agreements in the spectrum, proton elastic scattering, and the $B(E2)$ value, suggest that the shell-model calculations presented provide a far more reliable description of the structures of ^{16}C. The assumption of a simple $\nu(sd)^2$ structure, while indicated as a dominant component of the total ground-state wave function of ^{16}C, is not entirely valid.

[1] F. Ajzenberg-Selove, Nucl. Phys. A **523**, 1 (1991).
[2] D. R. Tilley, H. R. Weller, and C. M. Cheeves, Nucl. Phys. A **564**, 1 (1993).
[3] D. R. Tilley, H. R. Weller, C. Cheves, and R. Chasteler, Nucl. Phys. A **595**, 1 (1995).
[4] D. R. Tilley, C. M. Cheves, J. H. Kelley, S. Raman, and H. R. Weller, Nucl. Phys. A **636**, 249 (1998).
[5] S. Karataglidis, K. Amos, P. Fraser, L. Canton, and J. P. Svenne, Nucl. Phys. A **813**, 235 (2008).
[6] H. G. Bohlen, R. Kalpakchieva, B. Gebauer, S. M. Grimes, H. Lenske *et al.*, Phys. Rev. C **68**, 054606 (2003).
[7] S. Fujii, T. Mizusaki, T. Otsuka, T. Sebe, and A. Arima, Phys. Lett. B **650**, 9 (2007).
[8] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **81**, 064303 (2010).
[9] C. Yuan, T. Suzuki, T. Otsuka, F. Xu, and N. Tsunoda, Phys. Rev. C **85**, 064324 (2012).

[10] N. Imai, H. J. Ong, N. Aoi, H. Sakurai, K. Demichi, H. Kawasaki *et al.*, Phys. Rev. Lett. **92**, 062501 (2004).
[11] Y. Kanada-En'yo, Phys. Rev. C **71**, 014310 (2005).
[12] M. Wiedeking, P. Fallon, A. O. Macchiavelli, J. Gibelin, M. S. Basunia, R. M. Clark, M. Cromaz, M. A. Deleplanque, S. Gros *et al.*, Phys. Rev. Lett. **100**, 152501 (2008).
[13] H. J. Ong *et al.*, Eur. Phys. J. A **42**, 393 (2009).
[14] H. J. Ong, N. Imai, D. Suzuki, H. Iwasaki, H. Sakurai, T. K. Onishi *et al.*, Phys. Rev. C **78**, 014308 (2008).
[15] A. O. Macchiavelli, M. Petri, P. Fallon, S. Paschalis, R. M. Clark, M. Cromaz, and I. Y. Lee, Phys. Rev. C **90**, 067305 (2014).
[16] H. T. Fortune, Phys. Rev. C **93**, 044322 (2016).
[17] D. C. Zheng, B. R. Barrett, J. P. Vary, W. C. Haxton, and C.-L. Song, Phys. Rev. C **52**, 2488 (1995).
[18] OXBASH-MSU (the Oxford-Buenos-Aries-Michigan State University shell model code). A. Etchegoyen, W. D. M. Rae, and N. S. Godwin (MSU version by B. A. Brown, 1986); B. A. Brown, A. Etchegoyen, and W. D. M. Rae, MSUCL Report Number 524 (1986).
[19] S. Terashima *et al.*, RIKEN Accel. Prog. Rep. **47**, xviii (2014).
[20] K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis, and J. Raynal, Adv. Nucl. Phys. **25**, 275 (2000).
[21] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[22] P. K. Deb, B. C. Clark, S. Hama, K. Amos, S. Karataglidis, and E. D. Cooper, Phys. Rev. C **72**, 014608 (2005).
[23] S. Karataglidis, P. J. Dortmans, K. Amos, and R. de Swiniarski, Phys. Rev. C **53**, 838 (1996).
[24] J. Raynal, computer code DWBA70 (NEA 1209/02), 1998 version.