Particle representation for the kaonic $NN\bar{K}$ system

I. Filikhin* and B. Vlahovic
North Carolina Central University, Durham, North Carolina 27707, USA

(Received 15 January 2020; accepted 27 April 2020; published 18 May 2020)

The kaonic system $NN\bar{K}$ is studied based on the configuration space Faddeev equations. We consider two models associated with isospin "natural" basis and isospin "charge" basis. One basis is related to another by a unitary transformation. We show that the "particle representation" for $NN\bar{K}(s_{NN}=0)$ system motivated by the charge basis does not describe the system in terms of coupled particle channels $ppK^-/pn\bar{K}^0$. The coupling is associated with the nondiagonal elements of the matrix representation for the $N\bar{K}$ potential in the charge basis. The matrix can be diagonalized by a simple unitary transformation. With this relation, the Kyoto potential is discussed. The particle configurations of the $NN\bar{K}$ system may be classified by the presence or absence of the Coulomb interaction according to an analogy with the NNN system. The $NN\bar{K}(s_{NN}=0)$ system is represented by four particle configurations: ppK^-, npK^-, $np\bar{K}^0$, and $nn\bar{K}^0$.

DOI: 10.1103/PhysRevC.101.055203

I. INTRODUCTION

The quasibound states in the kaonic $NN\bar{K}(s_{NN}=0)$ system have been intensively studied in recent years. However, the theoretical predictions for the $NN\bar{K}(s_{NN}=0)$ binding energy are in disagreement with the values derived from existing experimental data [1]. The experimental data are above 100 MeV, while theoretical values are about 20 MeV, obtained with the chiral potentials, and about 50 MeV using the phenomenological potentials. The most recent J-PARC E15 experiment [2] supports the existence of the nuclear bound system ppK^- with an energy of $-47\pm3(\text{stat.})^{+3}_{-6}(\text{sys.})$ MeV that is in agreement with the theoretical prediction of the phenomenological approach of Ref. [3]. However, this does not mean that the agreement between theory and experiments is reached, because a two-peak structure of the ppK^- spectrum is proposed in Refs. [4,5] where the $ppK^- = (K^-p)^{I=0}p = \Lambda^* p$ system has the $J^\pi = 0^-$ ground state with energy about -100 MeV and a $J^\pi = 1^+$ excited state with energy about -50 MeV. Due to high momentum transfer found in the J-PARC E15 experiment, the measured peak of -50 MeV appears to belong to the excited state and not to the ground state.

Properties of the kaonic system are defined by the $N\bar{K}$ interaction having a significant difference for isospin singlet and triplet channels. The isospin singlet component of the $N\bar{K}$ potential generates a quasibound state corresponding to the $\Lambda(1405)$ resonance below the K^-p threshold. The resonance has a double state nature due to the coupling of the $N\bar{K}$ quasibound state and the $\pi\Sigma$ resonance [3,6,7]. Discussion about the experimental background and theoretical interpretations can be found in Refs. [1,5,8–10].

The theoretical consideration of the $NN\bar{K}$ system is based on the isotopic-spin formalism in which mesons $\bar{K}^0$ and K^- are two isospin states of the $\bar{K}$ particle with isospin $\frac{1}{2}$. The nucleon is also considered as an isospin $\frac{1}{2}$ particle having two states (proton and neutron) with different projections of the isospin. According to the isospin formalism, the isospins in the $NN\bar{K}$ system are summed as isospins of three identical particles. In the $NN\bar{K}(s_{NN}=0)$ system, the particle channels ppK^- and $pn\bar{K}^0$ can be defined due to the possible particle transition $n\bar{K}^0 \leftrightarrow pK^-$. The system can be found in the ppK^- or $pn\bar{K}^0$ state at the same time. The question is how these particle channels can be described within the isospin formalism.

A particle model was proposed in Ref. [11] within the "particle representation" for a dK^- atomic system. This system corresponds to the nuclear $NN\bar{K}(s_{NN}=1)$ state. The particle representation [11] is related to the coupled channels $nn\bar{K}^0/npK^-$, which are possible due to the particle transition mentioned above. The transformation of the isospin formalism to the particle representation can be obtained by unitary rotation [12] of the "natural" isospin basis for the $NN\bar{K}$ system, which includes the elements having fixed value of isospin of the pairs corresponding to the scheme $(i+j)+k$ of the isospin addition for particles i, j, k. We will call this rotation a transformation into the "given charge" isospin basis. The transformation leads to a modification of the matrix representation for the $N\bar{K}$ potential, which is diagonal in the natural isospin basis. In [11], the nondiagonal matrix elements of the new representation describe the coupling between $nn\bar{K}^0$ and npK^- channels. In that way, the particle picture for $NN\bar{K}(s_{NN}=1)$ is formally included in the isospin formalism.

As an example of such a transformation, one can point out the Kyoto $N\bar{K}$ potential proposed within a similar approach in Refs. [13,14]. In these papers, the given charge basis is named the "charge basis" and a particle model (with physical masses) is developed for the coupled $nn\bar{K}^0/npK^-$ channels. Note that

*ifilikhin@nccu.edu

 ©2020 American Physical Society

the motivation for the transformation of the isospin basis is the simplicity of the representation of the Coulomb potential, which becomes diagonal in the charge basis (see for example [15]).

In the present work, we discuss the channel interpretation of the particle representation based on the given charge isospin basis. The goal is to propose a description of the $NN\bar{K}$ system in terms of physical particles.

In our study, we used the Faddeev equations in configuration space [16] formulated for a three-body AAB system that includes two identical particles AA. The Faddeev equations enable the separation of components of the total wave function corresponding to the different particle arrangements $(AA)B$ and $A(AB)$.

II. FADDEEV EQUATION IN CONFIGURATION SPACE

The kaonic cluster $NN\bar{K}(s_{NN}=0)$ is represented by the three-body AAB system with two identical particles. The total wave function of the system is decomposed into the sum of the Faddeev components U and W corresponding to the $(NN)\bar{K}$ and $N(N\bar{K})$ types of rearrangements: $\Psi = U + W - PW$, where P is the permutation operator for two identical particles. In the expression for Ψ, the sign "$-$" corresponds to two identical fermions. Each component is expressed by corresponding Jacobi coordinates. The Faddeev equations are presented by two equations for the components U and W [17]:

$$\begin{aligned}\left(H_0^U + V_{NN} - E\right)U &= -V_{NN}(W - PW),\\ \left(H_0^W + V_{N\bar{K}} - E\right)W &= -V_{N\bar{K}}(U - PW),\end{aligned} \tag{1}$$

where H_0^U (H_0^W) is the kinetic energy operator presented in the Jacobi coordinates for corresponding rearrangement U (W). The wave function of the system AAB is automatically antisymmetrized with respect to two identical fermions. In the present work, we consider the s-wave approach for the kaonic systems. The total angular momentum $L=0$ and angular momenta in the subsystem $(NN)\bar{K}$ and $N(N\bar{K})$ are equal to zero.

III. ISOSPIN FORMALISM FOR KAONIC SYSTEM

The $NN\bar{K}(s_{NN}=0)$ system is a system with two identical particles described by Eq. (1). The Faddeev components U and W are expressed in terms of spin and isospin spaces:

$$U = \mathcal{U}\chi_{\rm spin}\eta^U_{\rm isospin}, \qquad W = \mathcal{W}\chi_{\rm spin}\eta^W_{\rm isospin},$$

We consider the $NN\bar{K}$ system with the triplet isospin state of the nucleon pair $I_{NN}=1$. The isospin basis for the $NN\bar{K}$ system in the state $I=\frac{1}{2}$ and $I_z=\frac{1}{2}$ can be written using the isospin functions: $\eta_{+-+}=\eta_+(1)\eta_-(2)\eta_+(3)$, $\eta_{-++}=\eta_-(1)\eta_+(2)\eta_+(3)$, $\eta_{++-}=\eta_+(1)\eta_+(2)\eta_-(3)$. Here, for example, $\eta_-(k)$ is the isospin eigenfunction of kth particle with a projection of $-\frac{1}{2}$. The three-body isospin basis η^U for the configuration $(1+2)+3$ includes two elements with different isospins (singlet or triplet) of the (1+2) pair and can be written as

$$\begin{aligned}\eta_1 &= \frac{1}{\sqrt{2}}(\eta_{+-+} - \eta_{-++}), \quad \text{singlet},\\ \eta_2 &= \sqrt{\frac{2}{3}}\left(\eta_{++-} - \frac{1}{2}\eta_{+-+} - \frac{1}{2}\eta_{-++}\right), \quad \text{triplet}.\end{aligned} \tag{2}$$

The basis η^W for the configurations $(3+1)+2$ [and $(2+3)+1$] can be obtained from Eq. (2) by cyclical permutations of the isospin projections [15,18]. The numerical coefficients in Eq. (2) are the Clebsch-Gordan coefficients defined by the algebraic combination of three isospin $\frac{1}{2}$ particles.

Note that in Ref. [13] the element η_2 of Eq. (2) was decomposed into the sum of two terms: $\eta_2 = \eta_2^1 + \eta_2^2 = \sqrt{\frac{2}{3}}\eta_{++-} - \frac{1}{\sqrt{6}}(\eta_{+-} + \eta_{-+})\eta_+$. The first (second) term can formally correspond to the $(pp)K^-$ $[(np)\bar{K}^0]$ configurations within the condition that the isospin functions of nucleons and kaons are different. Such clear correspondence is not possible for the configuration $N(N\bar{K})$. While conclusions about the existence of the coupled fractions $(pp)K^-/(np)\bar{K}^0$ have been made, further analysis of the system $NN\bar{K}$ within this isospin basis using the proposed notation was not performed in [13], and the correspondence of the term η_2^2 to the $(np)\bar{K}^0$ fraction has not been used anywhere else.

The spin states of the $NN\bar{K}$ system can be described by spin states of the nucleon pair, which can be spin singlet or spin triplet. The singlet spin function $\chi^{s=0}(NN)$ is an asymmetrical function regarding the permutation of nucleons, $\chi^{s=0}(NN) = \frac{1}{\sqrt{2}}(\chi_{+-} - \chi_{-+})$, which provides the sign "+" in Eq. (1). The triplet spin function $\chi^{s=1}(NN)$ is symmetric, which gives the sign "$-$" in Eq. (1).

We assumed that s-wave spin/isospin dependent V_{NN} and $V_{N\bar{K}}$ potentials are used. The Faddeev components U and W are represented by the form

$$U = \sum_{i=1}^{2}\mathcal{U}_i\eta_i^U\chi^{s=0}, \qquad W = \sum_{i=1}^{2}\mathcal{W}_i\eta_i^W\chi^{s=0}. \tag{3}$$

This form is used for the separation of the spin-isospin variables in Eq. (1). The corresponding matrices of representation of the operators I, P and the potentials V_{NN} and $V_{N\bar{K}}$ in the bases η^U and η^W were calculated by the projection procedure [19]:

$$\langle\eta^U|I|\eta^W\rangle = I^{(U,W)} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix},$$

$$\langle\eta^U|P|\eta^W\rangle = P^{(U,W)} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix},$$

$$\langle\eta^W|I|\eta^U\rangle = I^{(W,U)} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix},$$

$$\langle\eta^W|P|\eta^W\rangle = P^{(W,W)} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}.$$

The matrices of the potentials have diagonal forms according to the fixed value of the isospin or spin of a pair of particles in Eq. (2). For the $NN\bar{K}$ system, this separation leads to the following form of the Faddeev equations:

$$\begin{aligned}
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U} \\
&\quad = -v_{NN}^t\left(-\frac{1}{2}\mathcal{W}^t + \frac{\sqrt{3}}{2}\mathcal{W}^s - \frac{1}{2}p\mathcal{W}^t + \frac{\sqrt{3}}{2}p\mathcal{W}^s\right), \\
&\left(H_0^W + v_{\bar{K}N}^s - E\right)\mathcal{W}^s \\
&\quad = -v_{\bar{K}N}^s\left(\frac{\sqrt{3}}{2}\mathcal{U} + \frac{1}{2}p\mathcal{W}^s - \frac{\sqrt{3}}{2}p\mathcal{W}^t\right), \\
&\left(H_0^W + v_{\bar{K}N}^t - E\right)\mathcal{W}^t \\
&\quad = -v_{\bar{K}N}^t\left(-\frac{1}{2}\mathcal{U} - \frac{\sqrt{3}}{2}p\mathcal{W}^s - \frac{1}{2}p\mathcal{W}^t\right).
\end{aligned} \tag{4}$$

The isospin singlet and triplet $\mathcal{W}$ components of the wave function are denoted by indexes s and t, and the exchange operator p acts on the particles' coordinates only. Here, the Faddeev component U is restricted by a single element corresponding to the singlet spin state of the nucleons. The isospin singlet component of U is equal to zero due to the right-hand side of the corresponding equation in Eq. (1) being equal to zero within the applied s-wave approximation. The corresponding equation is therefore not shown in the set (4).

To describe the pair interactions, one can use s-wave Akaishi-Yamazaki (AY) [3] or simulating Hyodo-Weise (sHW) potentials [20]. The potentials effectively reduce the coupled-channel dynamics into a single channel $N\bar{K}$ interaction. The effective NN potential may be chosen to be the same as in Ref. [3]. Our calculation, performed for the AY model, gives a $NN\bar{K}(s_{NN} = 0)$ binding energy of 46.3 MeV and width of 50 MeV, which are close to the values of 47.7 MeV and 60 MeV, respectively, obtained in Ref. [21].

IV. CHARGE BASIS AND PARTICLE REPRESENTATION

The definition of the charge basis for a three-body kaonic system and the Kyoto $N\bar{K}$ potential can be found in Refs. [13,14]. The components of the Kyoto $N\bar{K}$ potential were presented by two terms in the form

$$V^{\pm} = \tfrac{1}{2}v_{\bar{K}N}^s \pm \tfrac{1}{2}v_{\bar{K}N}^t. \tag{5}$$

This form can be obtained by a unitary transformation of the natural isospin basis given by Eq. (2). The new isospin basis was chosen by the particles' isospin projections, which are motivated by the "isospin charge" set of particles: ppK^- or pnK^-, $(++-)$ or $(+--)$. The isospin functions η_{+-+}, η_{-++}, and η_{++-} are a new isospin basis τ which is not related to definite values of pair isospins. Let us call the new basis the "given charge basis." To obtain the transformation matrix, one has to add an additional element to Eq. (2). This element describes the isospin state of the $NN\bar{K}(I_{NN} = 1)$ $[NN\bar{K}(s_{NN} = 0)]$ system with isospin $\frac{3}{2}$ and the projection $\frac{1}{2}$ (or $-\frac{1}{2}$). For the configuration $(1 + 2) + 3$, this basis element is written as

$$\eta_3 = \frac{1}{\sqrt{3}}(\eta_{+-+} + \eta_{-++} + \eta_{++-}), \quad \text{triplet}, \tag{6}$$

where the pair $(1 + 2)$ is in the triplet isospin state. The pair potentials NN and $\bar{K}N$ have diagonal representation in the natural isospin basis defined by Eqs. (2) and (6):

$$V = \mathrm{diag}\{v^s, v^t, v^t\}. \tag{7}$$

The transformation matrix of the bases is given as

$$\eta = S\tau, \tag{8}$$

where $\tau = (\tau_1, \tau_2, \tau_3)^T$, $\tau_1 = \eta_{+-+}$, $\tau_2 = \eta_{-++}$, $\tau_3 = \eta_{++-}$, and

$$S = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{9}$$

The matrix S is unitary: $S^T S = I$. In the given charge basis (8), the matrix representation for potentials has nondiagonal elements:

$$\begin{aligned}
S^T V S &= \begin{pmatrix} \frac{1}{2}(v^t + v^s) & \frac{1}{2}(v^t - v^s) & 0 \\ \frac{1}{2}(v^t - v^s) & \frac{1}{2}(v^t + v^s) & 0 \\ 0 & 0 & v^t \end{pmatrix} \\
&= \begin{pmatrix} V^+ & V^- & 0 \\ V^- & V^+ & 0 \\ 0 & 0 & v^t \end{pmatrix},
\end{aligned} \tag{10}$$

where $V^+ = \frac{1}{2}(v^t + v^s)$ and $V^- = \frac{1}{2}(v^t - v^s)$. These matrix elements are interpreted in Refs. [11,13,14] as a coupling of the channels ppK^- and npK^0.

One can find a similar channel interpretation for the two-body system $N\bar{K}$. The coupled channel Schrödinger equation was written in Refs. [13,14] as

$$\left[H_0 - E + \begin{pmatrix} V_+ & V_- \\ V_- & V_+ \end{pmatrix}\right]\phi = 0, \tag{11}$$

where $\phi = (\phi_1, \phi_2)^T$, ϕ_1 (ϕ_2) corresponds to the K^-p $(\bar{K}^0 n)$ state of $N\bar{K}$, $V_+ = V^+ + V^-$, and $V_- = V^+ - V^-$.

The unitary transformation $t = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} -1 & 1 \\ 1 & 1 \end{smallmatrix}\right)$, $\tilde{\phi} = t^T\phi$, separates the channels as follows

$$\left[H_0 - E + 2\begin{pmatrix} V^- & 0 \\ 0 & V^+ \end{pmatrix}\right]\tilde{\phi} = 0. \tag{12}$$

The last equations imply a redefinition for singlet and triplet components of the $N\bar{K}$ potential. The potentials v^s and v^t are not clearly defined in this model because the components ϕ_1 and ϕ_2 are represented as $(-+)$ and $(+-)$ and the components $\tilde{\phi}_1$ and $\tilde{\phi}_1$ are antisymmetric and symmetric combinations of ϕ_1 and ϕ_2. In another words, the potentials in Eq. (12) have to be defined as triplet and singlet components of the $N\bar{K}$ potential. However, it is not possible due to the definition of Eqs. (11) and (10) for V^+ and V^- in which v^s and v^t are already used. The interpretation of the components ϕ_1 and ϕ_2 as pK^- and $n\bar{K}^0$ components is arbitrary. In the isospin

space, we have a system of identical particles. For the $NN\bar{K}$ system, the number of "channels" and the number of basis elements is equal. The problem is when these numbers are not equal as in the $NN\bar{K}$ system: two physical channels ppK^- and $np\bar{K}^0$ and three basis elements η_{+-+}, η_{-++}, and η_{++-}. One element of the basis is extra for the "channel interpretation." In the work [11], the problem was artificiality solved by the introduction of an additional channel $pn\bar{K}^0$. Such a solution follows the method used to construct the isospin basis as cyclical permutations of isospins of particles: $\{++-, +-+, -++\}$. The descriptions of the $NN\bar{K}$ system as a system of particles can be written in the following form: $\{ppK^-, pK^-p, K^-pp\}$ or $\{p\bar{K}^0n, pn\bar{K}^0, n\bar{K}^0p\}$ or $\{ppK^-, pn\bar{K}^0, npK^0\}$ (mixed variant) or $\{ppK^-, pn\bar{K}^0, K^-pp\}$. These four variants reproduce the form $\{++-, +-+, -++\}$ formally. The set of physical particles may be arbitrarily chosen as in Ref. [11]. In Ref. [14], the isospin basis τ was restricted by two elements $-+-$ and $+--$ described as K^-pn and $\bar{K}^0nn$, respectively. The Kyoto potential simulates the particle transition on this restricted basis using isospin exchange operators, resulting in the permutation of pairs of isospin projections $-+$ and $+-$ (K^-p and $\bar{K}^0n$), which is a transformation of the basis elements into each other. Thus, the coupling of the channels has appeared. Notice, however, that the isospin basis is formed as a basis of identical particles using cyclical permutations.

The unitary transformation S for the matrix representation of Eq. (1) in the basis η leads to the matrices

$$S^T V I^{(U,W)} S = \begin{pmatrix} 0 & V^+ & V^- \\ 0 & V^- & V^+ \\ v^t & 0 & 0 \end{pmatrix},$$

$$S^T V P^{(U,W)} S = \begin{pmatrix} 0 & V^- & V^+ \\ 0 & V^+ & V^- \\ v^t & 0 & 0 \end{pmatrix}, \qquad (13)$$

$$S^T V I^{(W,U)} S = \begin{pmatrix} V^- & 0 & V^+ \\ V^+ & 0 & V^- \\ 0 & v^t & 0 \end{pmatrix},$$

$$S^T V P^{(W,W)} S = \begin{pmatrix} V^+ & 0 & V^- \\ V^- & 0 & V^+ \\ 0 & v^t & 0 \end{pmatrix}. \qquad (14)$$

The new set of the Faddeev equations has to include additional equations with isospin triplet potential corresponding to the isospin element $I = 3/2$ in the η basis. The transformation of the isospin basis η does not result in isospinless representation. The total wave function is represented by the form

$$\Psi = U + W - PW = \sum_{i=1}^{3} \left(\mathcal{U}_i \tau_i^U + (1+P)\mathcal{W}_i \tau_i^W\right).$$

Here, the spin basis χ_{spin} is not shown due to the restriction by a single element corresponding to the singlet spin state of nucleons. In the isospin basis τ, the Faddeev equations (1) read

$$\begin{aligned}
&\left(H_0^U + V^+ - E\right)\mathcal{U}_1 + V^-\mathcal{U}_2 \\
&\quad = -V^+(\mathcal{W}_2 + p\mathcal{W}_3) - V^-(\mathcal{W}_3 + p\mathcal{W}_2), \\
&\left(H_0^U + V^+ - E\right)\mathcal{U}_2 + V^-\mathcal{U}_1 \\
&\quad = -V^-(\mathcal{W}_2 + p\mathcal{W}_3) - V^+(\mathcal{W}_3 + p\mathcal{W}_2), \\
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U}_3 = -v_{NN}^t(1+p)\mathcal{W}_1, \\
&\left(H_0^W + V^+ - E\right)\mathcal{W}_1 + V^-\mathcal{W}_2 \\
&\quad = -V^-(\mathcal{U}_1 + p\mathcal{W}_3) - V^+(\mathcal{U}_3 + p\mathcal{W}_1), \\
&\left(H_0^W + V^+ - E\right)\mathcal{W}_2 + V^-\mathcal{W}_1 \\
&\quad = -V^+(\mathcal{U}_1 + p\mathcal{W}_3) - V^-(\mathcal{U}_3 + p\mathcal{W}_1), \\
&\left(H_0^W + v_{N\bar{K}}^t - E\right)\mathcal{W}_3 = -v_{N\bar{K}}^t(\mathcal{U}_3 + p\mathcal{W}_3). \qquad (15)
\end{aligned}$$

The numerical coefficients coming from the isospin addition in Eq. (4) are reduced in Eq. (15) to be equal to 1. The form (15) has to keep the two-body $N + (N\bar{K})$ threshold. However, this is hidden by the nondiagonal elements of matrix of potentials. Diagonalization can be done by the matrix

$$t_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} t_0 & 0 \\ 0 & 1 \end{pmatrix},$$

$\mathcal{W}' = t_1\mathcal{W}$, and $\mathcal{U}' = t_1\mathcal{U}$. After diagonalization, the left-hand side of the new equations for $\mathcal{W}_1$ ($\mathcal{U}_1$) and $\mathcal{W}_2$ ($\mathcal{U}_1$) does not include the potentials $V^\pm$, which are replaced by v^t and v^s. The new matrices of potentials are diagonal. Equatio (15) is rewritten as

$$\begin{aligned}
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U}_1 = -\frac{1}{\sqrt{2}} v_{NN}^t(\mathcal{W}_2 + p\mathcal{W}_2 + \mathcal{W}_3 + p\mathcal{W}_3), \\
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U}_3 = -v_{NN}^t(1+p)\mathcal{W}_1, \\
&\left(H_0^W + V^+ - E\right)\mathcal{W}_1 + V^-\mathcal{W}_2 \\
&\quad = -V^-\left(\frac{1}{\sqrt{2}}\mathcal{U}_1 + p\mathcal{W}_3\right) - V^+(\mathcal{U}_3 + p\mathcal{W}_1), \\
&\left(H_0^W + V^+ - E\right)\mathcal{W}_2 + V^-\mathcal{W}_1 \\
&\quad = -V^+\left(\frac{1}{\sqrt{2}}\mathcal{U}_1 + p\mathcal{W}_3\right) - V^-(\mathcal{U}_3 + p\mathcal{W}_1), \\
&\left(H_0^W + v_{N\bar{K}}^t - E\right)\mathcal{W}_3 = -v_{N\bar{K}}^t(\mathcal{U}_3 + p\mathcal{W}_3) \qquad (16)
\end{aligned}$$

after the transformation t_1 for the components $\mathcal{U}_1$ and $\mathcal{U}_2$. The component $\mathcal{U}_2'$ is equal to zero within the s-wave approach and is not shown. We keep the notation $\mathcal{U}_1$ for the component $\mathcal{U}_1'$. To perform the transformation, we take into account that

$$t_0 \begin{pmatrix} V^+ & V^- \\ V^- & V^+ \end{pmatrix} t_0^T = \begin{pmatrix} v^t & 0 \\ 0 & v^s \end{pmatrix},$$

$$t_0 \begin{pmatrix} V^- & V^+ \\ V^+ & V^- \end{pmatrix} t_0^T = \begin{pmatrix} v^t & 0 \\ 0 & -v^s \end{pmatrix}.$$

Transforming the components $\mathcal{W}_1$ and $\mathcal{W}_2$ in Eq. (16), we obtain

$$\begin{aligned}
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U}_1 = -\frac{1}{\sqrt{2}} v_{NN}^t(1+p)(\mathcal{W}_2 + \mathcal{W}_3), \\
&\left(H_0^U + v_{NN}^t - E\right)\mathcal{U}_3 = -v_{NN}^t(1+p)\mathcal{W}_1,
\end{aligned}$$

$$
\begin{aligned}
&\big(H_0^W + v^t_{N\bar K} - E\big)\mathcal{W}'_1 \\
&\quad = -\frac{1}{\sqrt{2}}\, v^t_{N\bar K}\bigg(\frac{1}{\sqrt{2}}\,\mathcal{U}_1 + \mathcal{U}_3 + p\mathcal{W}_1 + p\mathcal{W}_3\bigg), \\
&\big(H_0^W + v^s_{N\bar K} - E\big)\mathcal{W}'_2 \\
&\quad = -\frac{1}{\sqrt{2}}\, v^s_{N\bar K}\bigg(\frac{1}{\sqrt{2}}\,\mathcal{U}_1 - \mathcal{U}_3 + p\mathcal{W}_3 - p\mathcal{W}_1\bigg), \\
&\big(H_0^W + v^t_{N\bar K} - E\big)\mathcal{W}_3 = -v^t_{N\bar K}(\mathcal{U}_3 + p\mathcal{W}_3),
\end{aligned}
$$

where $\mathcal{W}_1 = \frac{1}{\sqrt{2}}\,(\mathcal{W}'_1 - \mathcal{W}'_2)$ and $\mathcal{W}_2 = \frac{1}{\sqrt{2}}\,(\mathcal{W}'_1 + \mathcal{W}'_2)$. After substitution, we have

$$
\begin{aligned}
&\big(H_0^U + v^t_{NN} - E\big)\mathcal{U}_1 \\
&\quad = -\frac{1}{\sqrt{2}} v^t_{NN}(1+p)\bigg(\frac{1}{\sqrt{2}}\,(\mathcal{W}'_1 + \mathcal{W}'_2) + \mathcal{W}_3\bigg), \\
&\big(H_0^U + v^t_{NN} - E\big)\mathcal{U}_3 = -\frac{1}{\sqrt{2}}\, v^t_{NN}(1+p)(\mathcal{W}'_1 - \mathcal{W}'_2), \\
&\big(H_0^W + v^t_{N\bar K} - E\big)\mathcal{W}'_1 \\
&\quad = -\frac{1}{\sqrt{2}} v^t_{N\bar K}\bigg[\frac{1}{\sqrt{2}}\,\mathcal{U}_1 + \mathcal{U}_3 + p\mathcal{W}_3 + \frac{1}{\sqrt{2}}\,p(\mathcal{W}'_1 - \mathcal{W}'_2)\bigg], \\
&\big(H_0^W + v^s_{N\bar K} - E\big)\,\mathcal{W}'_2 \\
&\quad = -\frac{1}{\sqrt{2}} v^s_{N\bar K}\bigg[\frac{1}{\sqrt{2}}\,\mathcal{U}_1 - \mathcal{U}_3 + p\mathcal{W}_3 - \frac{1}{\sqrt{2}}\,p(\mathcal{W}'_1 - \mathcal{W}'_2)\bigg], \\
&\big(H_0^W + v^t_{N\bar K} - E\big)\mathcal{W}_3 = -v^t_{N\bar K}(\mathcal{U}_3 + p\mathcal{W}_3). \qquad (17)
\end{aligned}
$$

We have thus obtained an analog for Eq. (4) where the equations including isospin singlet or triplet potentials are not associated with physical channels. There is no a reason to interpret Eq. (17) as equations which are describing the channel coupling. The channel interpretation requires at least two isospin singlet potentials in the equations presenting the $ppK^-/np\bar K^0$ system (v_{pK^-} and $v_{n\bar K^0}$). Equation (17) includes only a single equation with this potential. The obvious reason for the difference is that the set of $N\bar K$ potentials of the isospin model is defined by the set of pair isospin states, which are possible when the total isospin is $I = \frac{1}{2}$ or $I = \frac{3}{2}$. The set is composed of the singlet, triplet, and triplet potentials. Therefore, the particle interpretation [11,13,14] proposed within the given charge formalism does not correspond to the equations which describe the $NN\bar K$ system.

We can restrict Eq. (17) to keep two components $\mathcal{U}_1$, $\mathcal{W}'_1$ and $\mathcal{W}'_2$ as in Ref. [14] and ignore the components $\mathcal{U}_3$ and $\mathcal{W}_3$ corresponding to the $I = \frac{3}{2}$ state. In the notations of Eq. (4), the restricted set takes the form

$$
\begin{aligned}
&\big(H_0^U + v^t_{NN} - E\big)\mathcal{U} = -v^t_{NN}(1+p)\bigg(\frac{1}{2\sqrt{3}}\,\mathcal{W}^s - \frac{1}{2}\mathcal{W}^t\bigg), \\
&\big(H_0^W + v^s_{N\bar K} - E\big)\mathcal{W}^s = -v^s_{N\bar K}\bigg(\frac{\sqrt{3}}{2}\,\mathcal{U} + \frac{1}{2}p\mathcal{W}^s + \frac{\sqrt{3}}{2}\,p\mathcal{W}^t\bigg), \\
&\big(H_0^W + v^t_{N\bar K} - E\big)\mathcal{W}^t = -v^t_{N\bar K}\bigg(-\frac{1}{2}\,\mathcal{U} + \frac{1}{2}p\mathcal{W}^t + \frac{1}{2\sqrt{3}}\,p\mathcal{W}^s\bigg). \qquad (18)
\end{aligned}
$$

TABLE I. An analogy between the $NN\bar K$ and NNN systems, where particle configuration is defined by using the Coulomb interaction.

Coulomb interaction	$NN\bar K(s_{NN} = 0)$	NNN
No	$nn\bar K^0$	nnp, ^{3}H, n-d
Yes	ppK^-	ppn, ^{3}He, p-d

Obviously, the solutions of Eqs. (17) and (4) have to be different due to difference of the right-hand sides of the equations. To obtain the same solution, we have to keep the components $\mathcal{U}_3$ and $\mathcal{W}_3$ in Eq. (17).

To continue the analysis for the problem of separation particle channels, one can consider an analogy using the three-body nucleon system NNN. The isospin variables of wave functions of both systems $NN\bar K$ and NNN are the same. The particle picture for the NNN system can be defined when the Coulomb interaction is taken (or not taken) into account. We present this analogy in Table I. The NNN system is taken to be the npn system or ^{3}H nucleus when the Coulomb potential is not included in the physical model. The same model with the Coulomb potential describes the ppn system or ^{3}He nucleus. Note, that the known low-energy p-d scattering problem is more complicated than the problem of n-d scattering due to different asymptotic boundary conditions. According to the analogy, one has to separate two particle configurations of the kaonic cluster $NN\bar K(s_{NN} = 0)$. The first one is the $nn\bar K^0$ system and the second one is the ppK^- system. The systems are described by the same equations (4) [or (17)]. There is a small energy gap between ground state levels corresponding systems that are induced by the Coulomb potential.

In Ref. [19], the given charge isospin basis was used to formulate a three-nucleon problem for the system AAB where the proton and neutron are nonidentical particles. The three-body problems for the systems npn and ppn were separately considered; however, they were based on the same Faddeev equations, taking into account the Coulomb potential and the difference of the nn and pp interactions. The approach [19] allows us to study the charge symmetry breaking effect of the nucleon-nucleon interaction in a phenomenological way. Also, one can define an analog of the charge symmetry breaking effect of the nucleon-nucleon interaction for the systems $nn\bar K^0$ and ppK^-.

The particle configurations npK^- and $np\bar K^0$ can be formally described by the Faddeev equation (4) for the AAB system. Comparing with the ppK^- configuration, one has to exchange the np and pp potentials and the corresponding sets of the Coulomb potentials. However, the physical particle systems npK^- and $np\bar K^0$ require the use of Faddeev equations formulated for the system ABC in which all particles are nonidentical, which violates the isospin equivalency of nucleons. An attempt to use the issospinless model for the coupled systems $ppK^-/np\bar K^0$ ($s_{NN} = 0$) and $npK^-/nn\bar K^0$ ($s_{NN} = 1$) is presented in Ref. [22].

Thus, we can separate four particle configurations related to the $NN\bar K(s_{NN} = 0)$ system. The configurations can be sorted by two sets, ppK^-, $pn\bar K^0$ and npK^-, $nn\bar K^0$, having the opposite projections of total isospin. The different inputs for

the systems are defined by the difference between the pair nuclear and Coulomb potentials. For example, the systems ppK^- and $nn\bar{K}^0$ differ by addition of the Coulomb potential in the system ppK^- and the distinction of pp and nn spin-singlet potentials. In addition, according to the charge conservation principle, these configurations can be classified as neutral and positively charged.

In Ref. [13], the authors consider two different particle systems for the $NN\bar{K}(s_{NN}=0)$ state with the notation ppK^--$np\bar{K}^0$ and npK^--$nn\bar{K}^0$. This classification is dictated by charge conservation (the projections of the total isospin) and the particle transition. The transition generates the coupling between the channels. It means that the $NN\bar{K}(s_{NN}=0)$ system can be found as the ppK^- system and the $np\bar{K}^0$ system, simultaneously, with equal probability [13]. In our interpretation, within the framework of the simple models for the $N\bar{K}$ interaction known from literature, the notation ppK^--$np\bar{K}^0$ is not possible. The systems ppK^- and $pn\bar{K}^0$ are noncoupled different systems or are equivalent systems when the Coulomb potential is not taken into account.

V. SUMMARY

The kaonic system $NN\bar{K}$ is an example of a three-body system with an isospin-dependent $N\bar{K}$ interaction. The "natural" isospin basis includes two elements to describe the rearrangement channel $N(N\bar{K})$. These elements correspond to the singlet and triplet states of the $N\bar{K}$ pair when the total isospin of the system is $I=\frac{1}{2}$. Without the Coulomb interaction, the three-body isospin states with $I=\frac{1}{2}$ and $I=\frac{3}{2}$ are not coupled.

The "particle representation" [11,13,14] for kaonic clusters is formally related to the "given charge basis," which was obtained by a unitary transformation of the natural isospin basis. The charge basis is complete and includes the three-body isospin state $I=\frac{3}{2}$ with a triplet $N\bar{K}$ pair state. The $V^{\pm}$ potentials obtained in the given charge basis may be replaced by v^t and v^s potentials using a simple transformation. The matrix representation related to the $V^{\pm}$ potentials can be diagonalized to include the singlet v^s and triplet v^t component of the $N\bar{K}$ potential. Therefore, the Faddeev equations written in the new isospin basis have to include, as an input, the singlet, triplet, and again triplet components of the $N\bar{K}$ potential. The resulting set of Faddeev equations does not have a structure that can describe the particle channel coupling $ppK^-/np\bar{K}^0$ for the $NN\bar{K}(s_{NN}=0)$ system with the input of the pK^-, $n\bar{K}^0$, $p\bar{K}^0$ potentials. A possibility for a channel interpretation of the $NN\bar{K}(s_{NN}=0)$ system may be the particle model for coupled isospinless $ppK^-$and $np\bar{K}^0$ systems [22]. A similar analysis for the $NN\bar{K}(s_{NN}=1)$ system leads to the same result. The coupled channel particle interpretation $nn\bar{K}^0/npK^-$ has to include the $n\bar{K}^0$, pK^-, and $p\bar{K}^0$ potentials, in disagreement with the input of the isospin model for the $NN\bar{K}(s_{NN}=1)$ system.

The Kyoto $N\bar{K}$ potential, including isospin exchange operators, simulates the particle transition in the framework of the isospin formalism. However, the isospin part of the Kyoto $N\bar{K}$ potential, defined for the restricted given charge basis, contradicts the definition of the singlet (v^s) and triplet (v^t) components of the $N\bar{K}$ potentials in the natural isospin basis.

The particle interpretation for the $NN\bar{K}(s_{NN}=0)$ kaonic system is formally possible with the presence or absence of the Coulomb interaction. The $NN\bar{K}(s_{NN}=0)$ system with the Coulomb potential may be described as the ppK^- system. The $np\bar{K}^0$ system may be related to the $NN\bar{K}$ $(s_{NN}=0)$ system where the Coulomb potential is ignored. For the charged $NN\bar{K}(s_{NN}=0)$ kaonic cluster, there exist two quasibound states ppK^- and $np\bar{K}^0$ separated by the energy gap induced by the Coulomb potential. Generally, one can determine four particle configurations related to the $NN\bar{K}(s_{NN}=0)$ bound state, using the difference of pair potentials and masses of kaons to define each configuration: ppK^-, npK^-, $np\bar{K}^0$, and $nn\bar{K}^0$.

ACKNOWLEDGMENTS

We thank Prof. D. Markoff and Dr. A. Karoui for useful discussions. This work is supported by the National Science Foundation Grant No. HRD-1345219 and NASA Grant No. NNX09AV07A.

[1] A. Gal, EPJ Web Conf. **130**, 01030 (2016).
[2] S. Ajimura *et al.* (J-PARC E15 Collaboration), Phys. Lett. B **789**, 620 (2019).
[3] T. Yamazaki and Y. Akaishi, Phys. Rev. C **76**, 045201 (2007).
[4] W. Mu Mu Phyo, K. S. Myint, Y. Akaishi, and T. Yamazaki, JPS Conf. Proc. **17**, 082005 (2017).
[5] Y. Akaishi and T. Yamazaki, arXiv:1903.10687.
[6] R. H. Dalitz and S. F. Tuan, Ann. Phys. (NY) **10**, 307 (1960).
[7] D. Jido, T. Sekihara, Y. Ikeda, T. Hyodo, Y. Kanada-En'yo, and E. Oset, Nucl. Phys. A **835**, 59 (2010).
[8] A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. **88**, 035004 (2016).
[9] T. Nagae, Nucl. Phys. A **954**, 94 (2016).
[10] T. Sekihara, E. Oset, and A. Ramos, PoS (Hadron2017) 199 (2018); JPS Conf. Proc. **26**, 023009 (2019).
[11] J. Revai, Phys. Rev. C **94**, 054001 (2016).
[12] P. Doleschall, J. Revai, and N. V. Shevchenko, Phys. Lett. B **744**, 105 (2015).
[13] Sh. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara, and T. Hyodo, Phys. Rev. C **95**, 065202 (2017).
[14] T. Hoshino, Sh. Ohnishi, W. Horiuchi, T. Hyodo, and W. Weise, Phys. Rev. C **96**, 045204 (2017).
[15] A. A. Kvitsinskii, Yu. A. Kuperin, S. P. Merkuriev, A. K. Motovilov, and S. L. Yakovlev, Sov. J. Part. Nuclei **17**, 113 (1986).
[16] L. D. Faddeev and S. P. Merkuriev, *Quantum Scattering Theory for Several Particle Systems* (Kluwer Academic, Dordrecht, 1993), pp. 398.
[17] Yu. A. Kuperin, D. M. Latypov, S. P. Merkuriev, M. Bruno, and F. Cannata, Sov. J. Nucl. Phys. **53**, 582 (1991).

[18] S. P. Merkur'ev, Yu. A. Kuperin, and A. A. Kvitsinskij, preprint IFVE-OMVT–82-161, Inst. Fiziki Vysokikh Ehnergij, Serpukhov, 1982 (in Russian), http://www.iaea.org/inis/collection/NCLCollectionStore/_Public/14/798/14798017.pdf

[19] I. Filikhin, V. M. Suslov, and B. Vlahovic, J. Phys. G: Nucl. Part. Phys. **46**, 105103 (2019).

[20] D. Jido and Y. Kanada-En'yo, Phys. Rev. C **78**, 035203 (2008).

[21] S. Maeda, A. Akaishi, and T. Yamazaki, Proc. Jpn. Acad., Ser. B **89**, 418 (2013).

[22] B. Vlahovic, I. Filikhin, and R. Ya. Kezerashvili, SciPost Phys. Proc. **3**, 044 (2020).