Suppression of hard two-photon-exchange contributions to $R_{e^+e^-}$ elastic scattering cross-section ratios: A phenomenological approach

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(Received 22 October 2019; accepted 15 April 2020; published 7 May 2020)

We present a new extraction of the positron-proton and electron-proton elastic-scattering cross sections ratio $R_{e^+e^-}$ based on a new phenomenological parametrization of the two-photon-exchange (TPE) corrections to electron-proton (ep) elastic-scattering cross section σ_R. We compare our results to several previous phenomenological extractions, TPE hadronic calculations, and world data on the ratio $R_{e^+e^-}$, including the recent direct measurements from the CLAS, VEPP-3, and OLYMPUS experiments. The ratio $R_{e^+e^-}$ as extracted from this work is consistent with unity, and shows little sensitivity to ε and essentially no Q^2 dependence, suggesting a smaller hard-TPE contribution to $R_{e^+e^-}$ than that predicted by several previous phenomenological extractions and TPE hadronic calculations. In this work, we show that, under certain assumptions and constraints imposed on the TPE amplitudes Y_E and Y_3, the proton magnetic form factor squared $(G_M^p)^2$, and the proton form factors ratio $\mu_p R_p = \mu_p G_E^p/G_M^p$, the TPE corrections to σ_R can be suppressed, yielding a small hard-TPE contribution to $R_{e^+e^-}$. Our results are in agreement with the recent analysis by Bytev and Tomasi-Gustafsson [Phys. Rev. C **99**, 025205 (2019)]. We believe that the assumption that hard-TPE corrections could account for the discrepancy in the ratio $\mu_p R_p$ is still an open question.

DOI: 10.1103/PhysRevC.101.055202

I. INTRODUCTION

The proton's electric, $G_E^p(Q^2)$, and magnetic, $G_M^p(Q^2)$, form factors (FFs) are fundamental quantities needed to parametrize the internal structure of the proton and many composite particles. These FFs are functions of Q^2, the four-momentum transferred squared of the virtual photon with longitudinal polarization parameter ε defined as $\varepsilon^{-1} = [1 + 2(1+\tau)\tan^2(\frac{\theta_e}{2})]$, where θ_e is the scattering angle of the electron, $\tau = Q^2/4M_p^2$ is a kinematics factor, and M_p is the mass of the proton. Utilizing electron scattering, the proton FFs ratio $\mu_p R_p = \mu_p G_E^p/G_M^p$ can be extracted by using two main techniques: The first is the Rosenbluth separation technique [1], where the unpolarized electron-proton (ep) cross section is measured, and the reduced cross section σ_R for ep elastic scattering in the one-photon-exchange (OPE) approximation or Born value is given by

$$\sigma_R(\varepsilon, Q^2) = \left[G_M^p(Q^2)\right]^2 + \frac{\varepsilon}{\tau}\left[G_E^p(Q^2)\right]^2. \quad (1)$$

The second technique is the polarization transfer or recoil polarization method [2], where measurement of the spin-dependent cross section is performed. The transverse P_t and longitudinal P_l polarization components of the recoil proton are measured simultaneously, and the ratio R_p in the OPE approximation [2–4] is determined as

$$R_p = \frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{(E+E')}{2M_p}\tan\left(\frac{\theta_e}{2}\right), \quad (2)$$

with E and E' being the initial and final energy of the incident electron, respectively. The ratio $\mu_p R_p$ as measured by the two techniques differs significantly for $Q^2 > 1.0$ (GeV/c)^2, and almost by a factor of three at high Q^2 [5–19]. While the Rosenbluth separation method predicts FF scaling $\mu_p R_p \approx 1$, the recoil polarization method predicts a linearly decreasing ratio with increasing Q^2 and flattening out of $\mu_p R_p$ for $Q^2 > 5.0$ (GeV/c)^2. Such a discrepancy in $\mu_p R_p$ suggested that missing higher-order radiative corrections to σ_R should be applied. In particular, inclusion of two-photon-exchange (TPE) correction diagrams [20–24] may explain the discrepancy. This is accomplished by adding the real function $F(\varepsilon, Q^2)$, which represents contributions coming from the interference of the OPE and TPE amplitudes, to the Born reduced cross section σ_{Born} or simply: $\sigma_R(\varepsilon, Q^2) = \sigma_{\text{Born}}(\varepsilon, Q^2) + F(\varepsilon, Q^2)$.

The impact of TPE effects on ep scattering observables was studied in great details theoretically [20–22,25–67], phenomenologically [13,68–87], and experimentally [11,14,15,88]. See Refs. [20,21,89] for detailed reviews. For example, experimentally, and while some studies focused on verifying the discrepancy by measuring or constraining the TPE contributions to σ_R and their effect on the ratio $\mu_p R_p$ [12–15], other studies focused on examining the ε dependence and nonlinearity of σ_R [68,69,71]. Some studies also examined the ε dependence of the ratio $\mu_p R_p$ [11,88] and the possibility of any deviation from the OPE prediction.

However, no deviation from the OPE predictions was observed because the ratio $\mu_p R_p$ showed essentially no ε dependence. Phenomenologically, studies focused mainly on extracting the TPE contributions based on the experimentally observed discrepancy on $\mu_p R_p$ using combined elastic ep cross-section and polarization measurements [13,22,71,73,76–78,80,85–87]. Other studies aimed at extracting the TPE amplitudes with fewer assumptions and constraints [72,74,81,82,84]. However, measuring the positron-proton to electron-proton cross-section ratio $R_{e^+e^-}(\varepsilon, Q^2)$ is by far the most direct technique used to measure the TPE effect. Depending on the charge of the lepton (electron or positron) involved, the function $F(\varepsilon, Q^2)$ changes sign accordingly, yielding an amplified signal when the ratio $R_{e^+e^-}(\varepsilon, Q^2)$ is constructed.

It should be noted here that σ_R in the Born approximation is the measured elastic cross section after applying radiative corrections that account for photon radiation from the charged particle $\delta^{\pm}$, or $\sigma_R = \sigma_{\text{elastic}}(1+\delta^{\pm})$. The radiative correction $\delta^{\pm}$, + (−) for positron (electron) includes, in addition to the charge-even terms δ_{even} (vertex-type corrections), the charge-odd terms δ_{odd} which change sign depending on the sign of the lepton, or $\delta^{\pm} = \mp\delta_{\text{odd}} + \delta_{\text{even}}$. The δ_{odd} term can also be broken down into the hard-TPE and soft-TPE contributions, or $\delta_{\text{odd}} = \delta_{2\gamma} + \delta_{\text{soft}}$. The measured ratio $R^{\text{meas}}_{e^+e^-}(\varepsilon, Q^2)$ is then defined as

$$R^{\text{meas}}_{e^+e^-}(\varepsilon, Q^2) = \frac{\sigma(e^+p \to e^+p)}{\sigma(e^-p \to e^-p)} = \frac{1+\delta_{\text{even}}-\delta_{\text{odd}}}{1+\delta_{\text{even}}+\delta_{\text{odd}}}, \quad (3)$$

and any deviation of $R^{\text{meas}}_{e^+e^-}$ from unity is a clear signature of δ_{odd} or charge-odd contributions to the cross section.

After correcting $R^{\text{meas}}_{e^+e^-}$ for δ_{even} and δ_{soft}, the ratio can now be written as

$$R_{e^+e^-}(\varepsilon, Q^2) = R_{2\gamma} = \frac{1-\delta_{2\gamma}}{1+\delta_{2\gamma}} \approx 1 - 2\delta_{2\gamma}, \quad (4)$$

with $\delta_{2\gamma}$ being the fractional TPE correction for *ep* scattering or $\delta_{2\gamma} = F(\varepsilon, Q^2)/\sigma_{\text{Born}}$, and any deviation of $R_{2\gamma}$ from unity is a clear signature of the hard-TPE effect.

II. PHENOMENOLOGICAL TWO-PHOTON-EXCHANGE CONTRIBUTION

In this section, we summarize several previous and recent phenomenological studies of the ratio $R_{e^+e^-}$ which we compare with our extracted ratio.

First, we start by summarizing the theoretical work of Guichon and Vanderhaeghen [22], which is the framework used by several phenomenological extractions below. The hadronic vertex function was expressed in terms of three independent complex amplitudes or generalized FFs, which are functions of Q^2 and ε: $\tilde{G}^p_E(\varepsilon, Q^2)$, $\tilde{G}^p_M(\varepsilon, Q^2)$, and $\tilde{F}_3(\varepsilon, Q^2)$. These generalized FFs can be broken into the usual OPE and the TPE contributions as $\tilde{G}^p_{E,M}(\varepsilon, Q^2) = G^p_{E,M}(Q^2) + \Delta G^p_{E,M}(\varepsilon, Q^2)$, with $Y_{2\gamma}(\nu, Q^2)$ defined as $\text{Re}(\frac{\nu \tilde{F}_3}{M_p^2|G^p_M|})$, and $\nu = M_p^2\sqrt{(1+\varepsilon)/(1-\varepsilon)}\sqrt{\tau(1+\tau)}$. The reduced cross section σ_R is expressed in terms of these amplitudes as

$$\sigma_R = \left|\tilde{G}^p_M\right|^2\left[1 + \frac{\varepsilon}{\tau}\frac{\left|\tilde{G}^p_E\right|^2}{\left|\tilde{G}^p_M\right|^2} + 2\varepsilon\left(1 + \frac{\left|\tilde{G}^p_E\right|}{\tau\left|\tilde{G}^p_M\right|}\right)Y_{2\gamma}\right]. \quad (5)$$

Their results suggested that, while small TPE contributions would significantly modify the ratio $\mu_p R_p$ as extracted by the Rosenbluth separation method, they had negligible impact on the polarization transfer data. We assume that the TPE contributions come mainly from the amplitude $Y_{2\gamma}(\nu, Q^2)$, which was parametrized to yield a correction to σ_R that is proportional to ε. While the applied TPE corrections reduced the values of G^p_E extracted by polarization transfer measurement slightly, they have significantly reduced G^p_E as extracted by the Rosenbluth separation method, making the ratio $\mu_p R_p$ from both techniques almost comparable. On the other hand, the G^p_M value is unaffected by TPE contributions.

Based on the framework of Ref. [22], Arrington [73] extracted the TPE amplitudes $\Delta G^p_{E,M}$ and $Y_{2\gamma}$. The amplitudes were assumed to be ε independent and ΔG^p_E was set to zero. The amplitude $Y_{2\gamma}(Q^2)$ was extracted by taking the difference between polarization and Rosenbluth measurements. The amplitude ΔG^p_M was extracted by using the high-ε data on $R_{e^+e^-}$ as constraints [90] and requiring the contribution of ΔG^p_M to σ_R cancels that of $Y_{2\gamma}$ in the limit $\varepsilon \to 1$. The TPE amplitudes were then parametrized as a function of Q^2.

Based on the TPE parametrization of Chen *et al.* [71] given below:

$$\sigma_R = \left(G^p_M\right)^2\left(1 + \frac{\varepsilon}{\tau}R_p^2\right) + A(Q^2)y + B(Q^2)y^3, \quad (6)$$

where $y = \sqrt{(1-\varepsilon)/(1+\varepsilon)}$, $A(Q^2) = \alpha G_D^2(Q^2)$, $B(Q^2) = \beta G_D^2(Q^2)$, and $G_D(Q^2)$ is the dipole parametrization $G_D(Q^2) = \{1 + Q^2/[0.71(\text{GeV}/c)^2]\}^{-2}$, Alberico *et al.* [91] extracted the proton FFs and the TPE parameters α and β by using two different fits. In the first fit, they fixed the ratio $\mu_p R_p$, and fit the world data on σ_R to extract the TPE parameters. In the second fit, cross-section and polarization transfer data were fit simultaneously, with G^p_E, G^p_M, α, and β being the parameters of the fit. The ratio $R_{e^+e^-}$ was extracted by using these two fits. Note that the TPE correction applied was taken as the dipole parametrization $G_D(Q^2)$, which is a nearly constant fractional correction to σ_R at large Q^2 which is dominated by G^p_M. The two fits are almost identical, with $R_{e^+e^-}$ showing essentially no Q^2 dependence because the Q^2 dependence of the TPE correction applied is taken as the dipole parametrization $G_D(Q^2)$.

Based on the theoretical framework of Borisyuk and Kobushkin [72], Qattan, Alsaad, and Arrington [78] fit the world data on σ_R for $Q^2 \geqslant 0.39\ (\text{GeV}/c)^2$ to the form

$$\sigma_R = \left(G^p_M\right)^2\left(1 + \frac{\varepsilon}{\tau}R_p^2\right) + 2a(Q^2)\left(G^p_M\right)^2(1-\varepsilon). \quad (7)$$

In this parametrization, the TPE contribution is linear in ε and vanishes in the limit $\varepsilon \to 1$ (Regge limit). In this analysis, R_p was fixed to its OPE value with G^p_M and $a(Q^2)$ being the parameters of the fit. The TPE parameter $a(Q^2)$ was best parametrized as

$$a(Q^2) = -0.0191\sqrt{Q^2} \pm 0.0014\sqrt{Q^2} \pm 0.003. \quad (8)$$

Following the same procedure of Ref. [78], and based on Eq. (7), Qattan, Arrington, and Alsaad [80] performed an improved extraction of the proton FFs, and the TPE parameter $a(Q^2)$ including data sets used in Ref. [78] and the low-Q^2 data from Refs. [13,92]. They used their recent improved parametrization of the ratio $\mu_p R_p$ given by

$$\mu_p R_p = \frac{1}{1 + 0.1430Q^2 - 0.0086Q^4 + 0.0072Q^6}, \quad (9)$$

with an absolute uncertainty in the fit $\delta^2_{R_p}(Q^2) = \mu_p^{-2}\{(0.006)^2 + [0.015\ln(1+Q^2)]^2\}$. The TPE parameter $a(Q^2)$ was best parametrized as $a(Q^2) = 0.016 - 0.030(Q^2)^{1/2}$, with Q^2 in $(\mathrm{GeV}/c)^2$. The ratio $R_{e^+e^-}$ was extracted by using the form

$$R_{e^+e^-}(\varepsilon, Q^2) = \frac{1-\delta_{2\gamma}}{1+\delta_{2\gamma}} \approx 1 - \frac{4a(Q^2)(1-\varepsilon)}{\left(1+\frac{\varepsilon}{\tau}R_p^2\right)}. \quad (10)$$

Guttmann *et al.* [74] expressed σ_R/G^2_{Mp}, the ratio $-\mu_p\sqrt{\tau(1+\varepsilon)/(2\varepsilon)}P_t/P_l$, and the ratio P_l/P_l^{Born} in terms of the ratio R_p, and the real parts of the TPE amplitudes, relative to G_M^p, or $Y_M(\varepsilon, Q^2) = \mathrm{Re}(\delta\tilde{G}_M^p/G_M^p)$, $Y_E(\varepsilon, Q^2) = \mathrm{Re}(\delta\tilde{G}_E^p/G_M^p)$, and $Y_3(\varepsilon, Q^2) = (\nu/M_p^2)\mathrm{Re}(\tilde{F}_3/G_M^p)$ as

$$\frac{\sigma_R}{\left(G_M^p\right)^2} = 1 + \frac{\varepsilon}{\tau}\left(\frac{G_E^p}{G_M^p}\right)^2 + 2Y_M + \frac{2\varepsilon}{\tau}\frac{G_E^p}{G_M^p}Y_E + 2\varepsilon\left(1 + \frac{G_E^p}{\tau G_M^p}\right)Y_3 + O(e^4), \quad (11a)$$

$$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} = \frac{G_E^p}{G_M^p} + Y_E - \frac{G_E^p}{G_M^p}Y_M + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E^p}{G_M^p}\right)Y_3 + O(e^4), \quad (11b)$$

$$\frac{P_l}{P_l^{\mathrm{Born}}} = 1 - 2\varepsilon\left[1 + \frac{\varepsilon}{\tau}\left(\frac{G_E^p}{G_M^p}\right)^2\right]^{-1} \times \left\{\left[\frac{\varepsilon}{1+\varepsilon}\left[1 - \frac{1}{\tau}\left(\frac{G_E^p}{G_M^p}\right)^2\right] + \frac{G_E^p}{\tau G_M^p}\right]Y_3 + \frac{G_E^p}{\tau G_M^p}\left[Y_E - \frac{G_E^p}{G_M^p}Y_M\right]\right\} + O(e^4), \quad (11c)$$

and the ratio $R_{e^+e^-}$ was extracted at $Q^2 = 2.64$, 3.20, and 4.10 $(\mathrm{GeV}/c)^2$ using σ_R data from Ref. [15].

Recently, and based on the formalism of Ref. [74], Eq. (11a), Qattan [81] extracted the TPE amplitudes Y_M, Y_E, and Y_3 as a function of ε at fixed Q^2 up to $Q^2 \sim 4.0$ $(\mathrm{GeV}/c)^2$. As the ratio $\mu_p R_p$ was experimentally confirmed to be independent of ε [11,88], the ratio $-\sqrt{\tau(1+\varepsilon)/(2\varepsilon)}P_t/P_l$ was constrained to its OPE value, $R_p = G_E^p/G_M^p$, by setting the TPE terms in Eq. (11b) to zero. That allowed for the amplitude $Y_M(\varepsilon, Q^2)$ to be expressed in terms of the remaining amplitudes $Y_E(\varepsilon, Q^2)$ and $Y_3(\varepsilon, Q^2)$. Because of the experimentally observed linearity of σ_R, and to reserve as possible the linearity of σ_R, as well as to account for any possible nonlinearities in the TPE amplitudes, all TPE amplitudes were expanded as a second-order polynomial as

$$Y_E(\varepsilon, Q^2) = (\alpha_0 + \alpha_1\varepsilon + \alpha_2\varepsilon^2),$$
$$Y_3(\varepsilon, Q^2) = (\beta_0 + \beta_1\varepsilon + \beta_2\varepsilon^2), \quad (12)$$

where α_i and β_i $(i = 0, 1, 2)$ are functions of Q^2 only.

Imposing the Regge limit where TPE corrections and TPE amplitudes must vanish as $\varepsilon \to 1$ yields the following constraints: $\alpha_0 = -(\alpha_1 + \alpha_2)$ and $\beta_0 = -(\beta_1 + \beta_2)$, and $\sigma_R/(G_M^p)^2$ can be expressed as

$$\frac{\sigma_R}{\left(G_M^p\right)^2} = 1 + \frac{\varepsilon}{\tau}R_p^2 + \left[\frac{2}{R_p} + \frac{2\varepsilon R_p}{\tau}\right] \times [\alpha_1(\varepsilon - 1) + \alpha_2(\varepsilon^2 - 1)] + \left[\frac{2}{R_p}\left(1 - \frac{2\varepsilon R_p}{1+\varepsilon}\right) + 2\varepsilon\left(1 + \frac{R_p}{\tau}\right)\right] \times [\beta_1(\varepsilon - 1) + \beta_2(\varepsilon^2 - 1)], \quad (13)$$

where $(G_M^p)^2$, α_1, α_2, β_1, and β_2 are the parameters of the fit. The value of R_p was fixed to that of Eq. (9), and $(G_M^p)^2$ was determined by fitting world data on σ_R to the following form:

$$\left[G_M^p(Q^2)\right]^2 = \frac{a(Q^2) + b(Q^2)}{\left(1 + \frac{R_p^2}{\tau}\right)}, \quad (14)$$

which assumed linearity of the experimentally observed σ_R, and the vanishing of σ_R at $\varepsilon = 1.0$. World data on σ_R covering Q^2 points up to $Q^2 = 4.0$ $(\mathrm{GeV}/c)^2$ were used in the analysis, with σ_R measured at a minimum of 5ε points. The TPE coefficients $\alpha_k(Q^2)$ and $\beta_k(Q^2)$ $(k = 0, 1, 2)$ were best parametrized as a second-order polynomial in Q^2 of the form $\alpha(\beta)_{(0,1,2)}(Q^2) = (a_0 + a_1Q^2 + a_2Q^4)$. The TPE amplitudes are found to be on the few-percentage-points level and behave roughly linearly with ε as Q^2 increases, where they become nonlinear at high-Q^2 values. While both Y_E and Y_3 amplitudes differ in magnitude as Q^2 increases, they have opposite sign to each other and tend to partially cancel each other. The TPE correction to σ_R seems to be driven mainly by Y_M and to lesser extent by Y_3 which is in agreement with previously reported phenomenological extractions at $Q^2 = 2.50$ $(\mathrm{GeV}/c)^2$ [74].

Borisyuk and Kobushkin [87] proposed new parametrizations of the three TPE amplitudes $\delta\mathcal{G}_E$, $\delta\mathcal{G}_M$, and $\delta\mathcal{G}_3$ normalized to G_M and valid for $0 < Q^2 < 8.0$ $(\mathrm{GeV}/c)^2$. The TPE correction to the unpolarized cross section is given by

$$\frac{\delta\sigma}{\sigma} = \frac{F(\varepsilon, Q^2)}{\sigma_{\mathrm{Born}}} = \frac{2}{\varepsilon R_p^2 + \tau}\mathrm{Re}\left[\varepsilon R_p^2\frac{\delta\mathcal{G}_E}{G_E} + \tau\frac{\delta\mathcal{G}_M}{G_M}\right], \quad (15)$$

where R_p is the recoil polarization ratio. Equation (15) can be expressed in terms of the TPE amplitudes normalized to G_M as

$$\frac{\delta\sigma}{\sigma} = \frac{F(\varepsilon, Q^2)}{\sigma_{\mathrm{Born}}} = \frac{2}{1 + \frac{\varepsilon}{\tau}R_p^2}\mathrm{Re}\left[\frac{\varepsilon R_p}{\tau}\frac{\delta\mathcal{G}_E}{G_M} + \frac{\delta\mathcal{G}_M}{G_M}\right]. \quad (16)$$

For a fixed Q^2 value, the three TPE amplitudes, normalized to G_M, were parametrized as a function of ε as

$$\frac{\delta\mathcal{G}}{G_M} = a_1\sqrt{1-\varepsilon} + a_2\sqrt{\varepsilon(1-\varepsilon)} + a_3(1-\varepsilon) + a_4(1-\epsilon)^2, \quad (17)$$

where the coefficients a_i $(i = 1, \ldots, 4)$ are functions of Q^2 only, and are given by

$$a_i = a_{i1} + a_{i2}Q^2 + (a_{i3} + a_{i4}Q^2)\ln(Q^2 + D) + \frac{a_{i5}}{1 + \frac{Q^2}{m^2}} + \frac{a_{i6}}{1 + \frac{Q^2}{\mu^2}}, \quad (18)$$

TABLE I. Fit results for the TPE coefficients $\alpha_{(1,2)}(Q^2)$ and $\gamma(Q^2)$.

Coefficient	$a_0 \times 10^{-3}$	$a_1 \times 10^{-3}$	$a_2 \times 10^{-3}$	χ^2_ν
$\alpha_1(Q^2)$	-4.38 ± 18.46	$+21.38 \pm 38.34$	-7.47 ± 11.36	4.05
$\alpha_2(Q^2)$	$+13.19 \pm 6.70$	-27.33 ± 15.08	$+6.90 \pm 4.14$	3.39
$\gamma(Q^2)$	$+999.54 \pm 1.43$	$+107.84 \pm 8.03$	-32.60 ± 5.41	4.47

where the coefficients $a_{i1...i6}$ $(i = 1, \ldots, 4)$ are constants and listed in Table I of Ref. [87] for each of the three TPE amplitudes. Here, $D = 0.046$ (GeV)^2, $m = 0.359\,898$ (GeV), and $\mu = 0.0954$ (GeV).

III. RESULTS AND DISCUSSION

Recently [84], we improved on and extended to low- and high-Q^2 values, up to $Q^2 = 5.20$ (GeV/c)^2, the previous phenomenological extractions [74,81] of the three TPE amplitudes Y_M, Y_E, and Y_3 as a function of ε. In our work, we followed the same procedure of Ref. [81], but constrained the two amplitudes Y_E and Y_3 in Eq. (13) to $Y_3(\varepsilon, Q^2) = -\gamma(Q^2)Y_E(\varepsilon, Q^2)$, or effectively $\beta_k(Q^2) = -\gamma(Q^2)\alpha_k(Q^2)$ $(k = 1, 2)$, as these two amplitudes were reported to have opposite signs, and with the tendency to partially cancel each other [74,81]. Here, the TPE coefficient $\gamma(Q^2)$ is a function of Q^2 only. With the new constraint, the number of fitting parameters is reduced to only three: $\alpha_1(Q^2)$, $\alpha_2(Q^2)$, and $\gamma(Q^2)$, which allows for σ_R measurements taken at Q^2 points with a minimum of three ε points to be included in the analysis. World data on σ_R used in the analysis of Refs. [80,81] as well as new data from Refs. [13–17,92–95] were used, and fit to Eq. (13) for a total of 142Q^2 points extending over both the low- and high-Q^2 regions up to $Q^2 = 5.2$ (GeV/c)^2. Note that both R_p and $(G_M^p)^2$ were constrained in Eq. (13) to their values as given by Eqs. (9) and (14), respectively. The fitting procedure was based on the Levenberg-Marquardt nonlinear least squares fitting method with the reduced χ^2 or χ^2_ν defined as

$$\chi^2_\nu = \chi^2/\nu = \frac{1}{\nu}\sum_{i=1}^{n_p=142} = \left(\sigma_{R_i}^{\text{meas.}} - \sigma_{R_i}^{\text{comp.}}\right)^2 \Big/ \delta_i^2, \qquad (19)$$

where $\nu = (n_\text{p} - n_\text{parameters})$ is the number of degrees of freedom. In general, the χ^2_ν values of the σ_R fits are reasonable and ranged from $0.03 < \chi^2_\nu < 1.80$, except for a handful of low-Q^2 points from Refs. [13,92] where $\chi^2_\nu > 1.80$. In addition, we observed nonlinearity in our σ_R fits at high-Q^2 points. Such nonlinearity was also reported in several previous hadronic- and perturbative-QCD-based calculations in that range [29,34,46,47].

The extracted TPE coefficients are on the few-percentage-points level, and they are best parametrized as a second-order-polynomial of the form $\alpha_{1,2}(\gamma)(Q^2) = (a_0 + a_1Q^2 + a_2Q^4)$, with Q^2 in (GeV/c)^2. The results of the fits are listed in Table I. The χ^2_ν values of the fits are clearly large. In our fits, we do not exclude any Q^2 data points that yielded large TPE amplitudes, exceeding the 10% level, as was done in the analysis of Ref. [81]. In an attempt to improve and lower the high χ^2_ν values obtained, we fit the TPE coefficients, excluding Q^2 points, which yielded TPE amplitudes exceeding the 10%, 20%, and 30% level. However, the χ^2_ν values did not improve significantly after these new fits as the central fits were essentially unchanged. Therefore, we concluded that the large χ^2_ν values obtained are driven mainly by the tension between the different data sets, the scatter of the data points, and the reduced number of fitting parameters, rather than a limitation of the fit function used. The extracted TPE coefficients along with their fits are shown in Fig. 2 of Ref. [84]. Finally, our final fits of the TPE coefficients do not exclude any of the Q^2 points used. The extracted TPE amplitudes show strong nonlinearity in ε at low Q^2, and behave roughly linearly with ε as Q^2 increases. Again, the amplitudes Y_E, and Y_3 have opposite signs with the tendency to partially cancel each other.

In this work, we use our parametrization of the TPE amplitudes and that of Borisyuk and Kobushkin [87], referred to as "BK18" throughout the text, to construct the TPE contributions to ep elastic scattering $F(\varepsilon, Q^2)$, and the ratio $R_{e^+e^-}$. Note that the ratio $R_{e^+e^-}$ as extracted based on the BK18 parametrization was not shown or discussed before. We then compare our results to several previous TPE phenomenological extractions: "qattan17" [81] (dotted black line), "qattan12" [76] (dashed magenta line), "qattan15" [80] (solid magenta line), "Bernauer" [13] (dashed-dotted black line), "ABGG" [91] (solid red line), "BK18" [87] (dashed-dotted magenta line), and "Arrington" [73] (dashed blue line).

The ε dependence of the function $F(\varepsilon, Q^2)$ normalized to $(G_M^p)^2$ as extracted from this work (solid black line) for a range of Q^2 values is shown in Fig. 1. The ratio $F(\varepsilon, Q^2)/(G_M^p)^2$ as extracted from this work is very much consistent with zero showing little sensitivity to ε and essentially no Q^2 dependence. This suggests that the imposed constraint on the amplitudes Y_E and Y_3 in our TPE parametrization is suppressing TPE contributions to σ_R provided that the "true" magnetic form factor of the proton squared $(G_M^p)^2$, and the ratio R_p are also constrained to their values as given by Eqs. (9) and (14), respectively. Here, the constants $a(Q^2) = (\tilde{G}_M^p)^2$ and $b(Q^2) = (\tilde{G}_E^p)^2/\tau$, with $\tilde{G}_E^p$ and $\tilde{G}_M^p$ being the electric and magnetic FFs of the proton as extracted using the Rosenbluth separation method.

Contrary to our results, all previous phenomenological extractions shown predict a ratio that is different from zero. At low Q^2, and while the Qattan15, Qattan17, Bernauer, and BK18 extractions predict a positive ratio, the Qattan12, ABGG, and Arrington parametrizations predict a negative ratio. As Q^2 increases, the ratio starts to decrease and changes sign as predicted by several low-Q^2 calculations [26,29,40,80,96,97], and then it starts to increase in magnitude slowly where it becomes linear, yielding effectively similar slopes except for the Arrington parametrization which shows larger slope as the recoil polarization ratio was corrected for

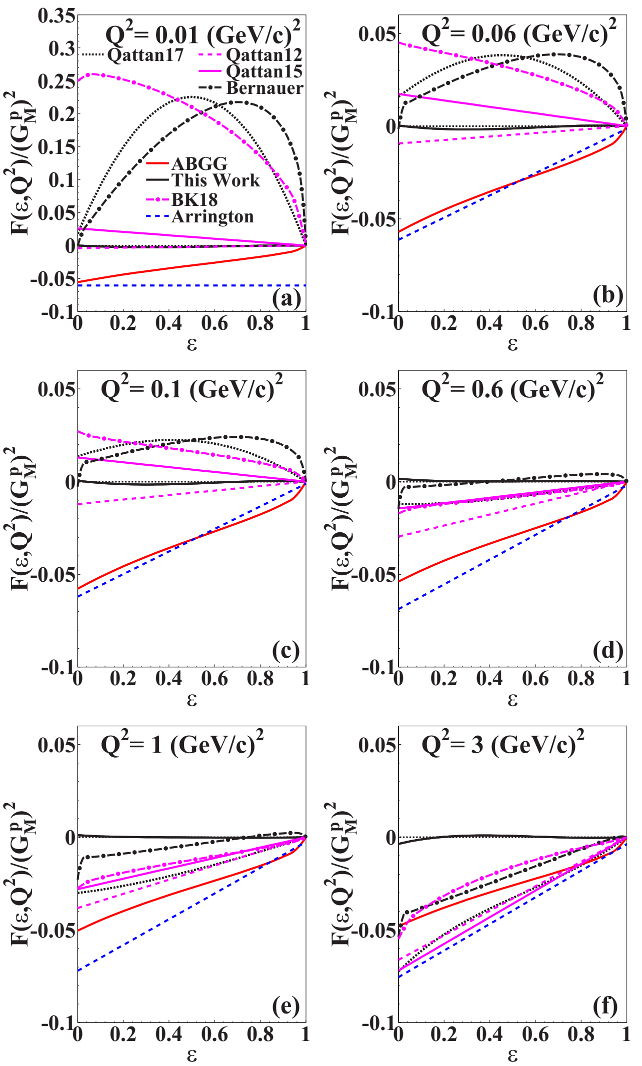

FIG. 1. The ratio $F(\epsilon, Q^2)/(G_M^p)^2$ as a function of ε as extracted from this work (solid black line) for a range of Q^2 values listed in the figure. Also shown are previous phenomenological extractions: Qattan17 [81] (dotted black line), Qattan12 [76] (dashed magenta line), Qattan15 [80] (solid magenta line), Bernauer [13] (dashed-dotted black line), ABGG [91] (solid red line), BK18 [87] (dashed-dotted magenta line), and Arrington [73] (dashed blue line).

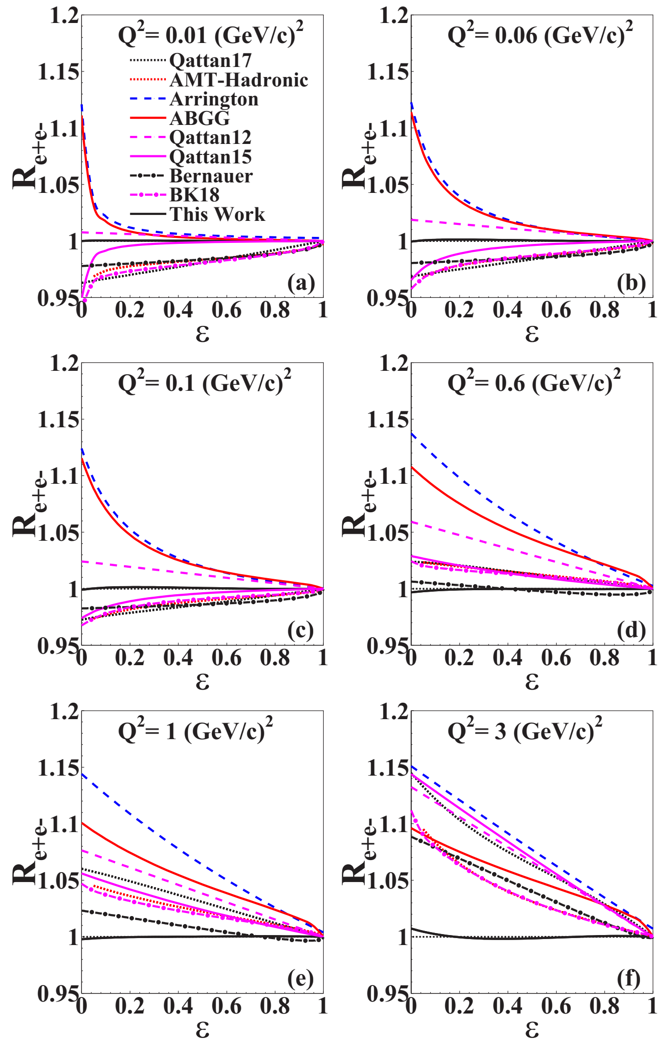

FIG. 2. The ratio $R_{e^+e^-}$ as a function of ε as extracted from this work (solid black line) for a range of Q^2 values listed in the figure. Also shown are TPE hadronic calculations AMT [98] (dashed red line) and previous phenomenological extractions [curves as in Fig. 1]: Qattan17 [81] (dotted black line), Qattan12 [76] (dashed magenta line), Qattan15 [80] (solid magenta line), Bernauer [13] (dashed-dotted black line), ABGG [91] (solid red line), BK18 [87] (dashed-dotted magenta line), and Arrington [73] (dashed blue line).

TPE corrections. On the other hand, the ABGG parametrization is the only extraction that shows no Q^2 dependence.

The ratio $R_{e^+e^-}$ as a function of ε extracted from this work is shown in Fig. 2 for a range of Q^2 values. We also compare our results to all phenomenological extractions discussed before and to TPE hadronic calculations from Ref. [98] "AMT" (short-dashed red line). Note that the curves shown are the same as in Fig. 1. The ratio $R_{e^+e^-}$ as extracted from this work is consistent with unity showing little sensitivity to ε and essentially no Q^2 dependence suggesting a small contribution from hard-TPE effect to $R_{e^+e^-}$. Contrary to our results, all other phenomenological extractions and TPE hadronic calculations predict a sizable hard-TPE effect as the ratio clearly deviates from one. At low Q^2, and while the Qattan12, Arrington, and ABGG extractions predict a ratio above unity, with the Arrington and ABGG extractions showing strong nonlinearity at low Q^2 and low ε, the remaining extractions predict a ratio below unity, and roughly linear in ε, except for the Qattan15 and BK18 extractions which show strong nonlinearity as well at low ε. Note, however, that extractions which assume linear or roughly linear TPE correction in ε times $(G_M^p)^2$ will yield strong linearities in the ratio $R_{e^+e^-}$ at low Q^2. As Q^2 increases, the ratio $R_{e^+e^-}$ starts to increase slowly, change sign (above unity), and become linear in ε as predicted by all phenomenological extractions and TPE hadronic calculations, except for the ABGG extraction which shows no sensitivity to Q^2.

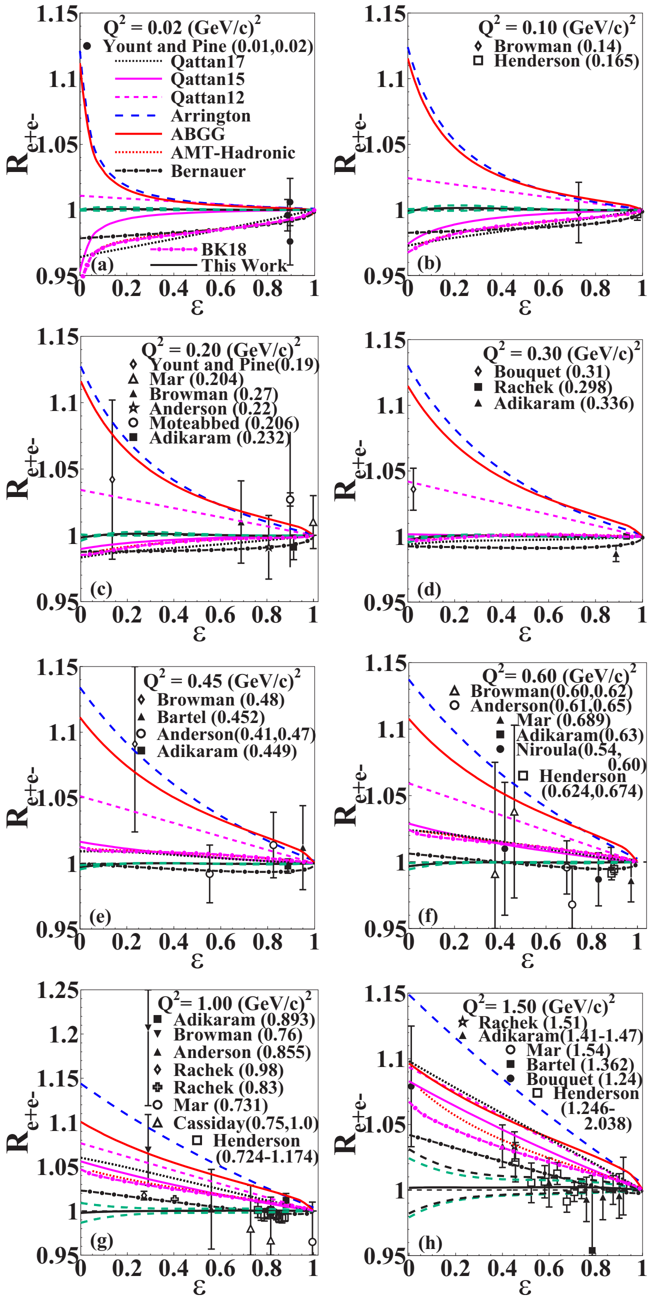

FIG. 3. The ratio $R_{e^+e^-}$ as a function of ε as extracted from this work (solid black line) at the Q^2 values listed in the figure. The error bands on $R_{e^+e^-}$ are shown as very-long-dashed dark-green lines. In addition, the error bands calculated for $Q^2 = 2.0\ (\mathrm{GeV}/c)^2$ (very-long-dashed black lines) are also shown in panel (h). Also shown are TPE hadronic calculations AMT [98] (dashed red line), and previous phenomenological extractions [curves as in Figs. 1 and 2]: Qattan17 [81] (dotted black line), Qattan12 [76] (dashed magenta line), Qattan15 [80] (solid magenta line), Bernauer [13] (dashed-dotted black line), ABGG [91] (solid red line), BK18 [87] (dashed-dotted magenta line), and Arrington [73] (dashed blue line). The data points are direct measurements of $R_{e^+e^-}$ from Refs. [99–111]. For the world data, the measurement and the Q^2 value(s) are given in $(\mathrm{GeV}/c)^2$.

In Fig. 3 we compare our extractions of the ratio $R_{e^+e^-}$ along with their associated error bands, shown as very-long-dashed dark-green lines, as computed by propagating the errors on $\delta_{2\gamma}$ in Eq. (4) by using the covariance matrix of the fits listed in Table I, and the uncertainty on the recoil polarization ratio δ_{R_p} (very negligible), to the world data from Refs. [99–111] including the very recent direct measurements from the CLAS collaboration [109], VEPP-3 collaboration [110], and OLYMPUS collaboration [111] at the Q^2 value listed in the figure. Note that for the point $Q^2 = 1.50\ (\mathrm{GeV}/c)^2$, Fig. 3(h), we also show the error bands calculated at $Q^2 = 2.0\ (\mathrm{GeV}/c)^2$ (very-long-dashed black lines) as the OLYMPUS measurements in the ε range of 0.456–0.581 were taken in the Q^2 range of 1.718–2.038 $(\mathrm{GeV}/c)^2$. We also show all previous phenomenological extractions discussed above from Refs. [13,73,76,80,87,91], as well as TPE hadronic calculations from Ref. [98] for comparison with data.

All "old" previous world data on the ratio $R_{e^+e^-}$ were limited to either low Q^2 or large ε where TPE contribution is very small. The previous data also suffered from the following: (1) it was not clear whether the charge corrections were applied to all measurements, (2) the details of the applied radiative corrections were also not available, and (3) the large uncertainties associated with the data. Therefore, these measurements were not able to provide a clear evidence of nonzero TPE contribution. Recently, three new precise measurements by the CLAS collaboration [109], VEPP-3 collaboration [110], and OLYMPUS collaboration [111] measured the ratio $R_{e^+e^-}$ for $Q^2 < 2.1\ (\mathrm{GeV}/c)^2$ which is still below where the discrepancy on the ratio $\mu_p R_p$ is significant. The CLAS and VEPP-3 data have provided precise measurements at $Q^2 \approx 1.0$ and $1.5\ (\mathrm{GeV}/c)^2$. The reported ratio $R_{e^+e^-}$ is larger than unity and shows clear ε dependence consistent with the form factor discrepancy at Q^2 values of 1.0–1.6 $(\mathrm{GeV}/c)^2$. The two collaborations concluded that their data provided evidence for a sizable TPE contribution at larger-Q^2 values, with clear deviation and change of sign from the exact calculations, high-proton-mass limit, at $Q^2 = 0$ [112], and finite-Q^2 calculations for a point-proton [21]. However, the $R_{e^+e^-}$ ratio as measured by the VEPP-3 collaboration shows a sharper Q^2 dependence which tends to disappear when the results are compared with calculations that increase with Q^2 value. The OLYMPUS collaboration measured the ratio at Q^2 values of 0.165–2.038 $(\mathrm{GeV}/c)^2$ and the reported ratio is below unity at high ε. The ratio then changed sign and started to increase gradually to about 2% at $\varepsilon = 0.46$. The results of the three recent measurements are very much in good agreement with each other within statistical and systematic uncertainties.

While all of the three recent direct measurements have accounted for δ_{soft} contributions in the radiative correction procedure applied, the splitting of δ_{odd} into $\delta_{2\gamma}$ and δ_{soft} contributions was not done uniformly and can differ from one calculation to another [113–116]. Therefore, a difference on the order of 1% to 2% between the different measurements can be attributed to the different procedures applied to correct $R^{\mathrm{meas}}_{e^+e^-}$ for δ_{odd}, knowing that the different procedures applied differ only in some finite expression which depends on kinematical invariants.

To address this issue, Bytev and Tomasi-Gustafsson [85] presented an updated analysis of the ε and Q^2 dependence of the ratio $R_{e^+e^-}$ as measured by these new direct measurements. They removed the δ_{odd} correction from the calculations applied to the data and proceeded from $R^{\text{meas}}_{e^+e^-}$ to $R_{2\gamma}$ as

$$R_{2\gamma} = \frac{1 - A_{\text{odd}}(1 + \delta_{\text{even}}) + \delta_{\text{soft}}}{1 + A_{\text{odd}}(1 + \delta_{\text{even}}) - \delta_{\text{soft}}} = \frac{1 - \delta_{2\gamma}}{1 + \delta_{2\gamma}}, \quad (20)$$

where $A_{\text{odd}} = \delta_{\text{odd}}/(1 + \delta_{\text{even}}) = (R_{2\gamma} - 1)/(R_{2\gamma} + 1)$ is the charge-asymmetry which includes both soft- and hard-TPE contributions to first order in α taken from Ref. [117]. For δ_{soft}, the calculations by Mo and Tsai [114], and Maximon and Tjon [113] were used separately for comparison. For each of these measurements and following the procedure above, the measured (uncorrected) $R^{\text{meas}}_{e^+e^-}$ ratio, the ratio before applying radiative corrections, which deviates from unity only due to odd terms (soft-TPE and hard-TPE corrections) was in good agreement with the theoretical prediction for $R^{\text{meas}}_{e^+e^-}$ based on the calculation of A_{odd} from Ref. [117]. The radiatively corrected ratio $R_{2\gamma}$, which deviates from unity only in the presence of hard-TPE contributions which were not accounted for in the radiative corrections applied to data, was also determined after subtracting the soft correction from Mo and Tsai [114] and Maximon and Tjon [113] separately for comparison. The calculated ratio $R_{2\gamma}$ fell within the errors of most data points for all measurements. For each measurement, a point-by-point comparison between experimental data points and calculations was also done. The ε and Q^2 dependence of $R_{2\gamma}$ was performed where the ε (Q^2) dependence of the ratio was fit to a linear function of the form $c_0 + c_1\varepsilon$ ($d_0 + d_1Q^2$). The average ratio was found to be consistent with unity within errors, except for the VEPP data which showed a χ^2 value different from one. Overall, the point-by-point difference between the data and calculations was at the per-thousand level for most cases, and the data within their precision limit do not show evidence of TPE contribution beyond the expected percent level. Therefore, experimental evidence for a large hard-TPE contribution is not found.

The ratio $R_{e^+e^-} = R_{2\gamma}$ as extracted from our work is consistent with unity and supports a small hard-TPE contribution to $R_{e^+e^-}$ in agreement with the findings and conclusion of Ref. [85]. Our extractions are in generally good agreement with existing world data, including the recent direct measurements [109–111] at large ε and for all the Q^2 points shown within the statistical and systematic uncertainties of these measurements. On the other hand, and at $Q^2 = 1.0$ GeV$/c)^2$ as shown in Fig. 3(g), the low ε measurements of $R_{e^+e^-}$ taken at $\varepsilon = 0.272$ and 0.404 by the VEPP-3 collaboration [110] are within 3.0σ from our error band. For Q^2 points in the range of 1.5–2.0 (GeV$/c)^2$, Fig. 3(h), and while the OLYMPUS measurements [111] taken in the ε range of 0.456–0.581 are clearly within our error band, the CLAS measurement [109] at $\varepsilon = 0.40$, and the VEPP-3 measurement [110] at $\varepsilon = 0.452$ are within 1.5σ and 2.0σ from our error band, respectively.

Finally, our results within their error bands, and in particular at high-Q^2 values, do not rule out any hard-TPE contribution to $R_{e^+e^-}$ but only suggest that the size of the hard-TPE correction is relatively smaller than that predicted by several previous phenomenological extractions and TPE hadronic calculations. Therefore, the suppression of hard-TPE corrections to $R_{e^+e^-}$ as suggested by our work is driven mainly by the well-justified constraints imposed on the TPE amplitudes Y_E and Y_3, magnetic FF squared $(G_M^p)^2$, and the ratio R_p. The smaller hard-TPE contribution our extractions suggest compared with those of previous phenomenological extractions is largely driven by the different assumptions used and constraints imposed in the analysis. Therefore, calculating the size of hard-TPE correction in a model-independent way is very difficult. Consequently, the assumption that hard-TPE corrections could account for the discrepancy on the ratio $\mu_p R_p$ is still an open question.

IV. CONCLUSIONS

In conclusion, we presented a new extraction of the positron-proton and electron-proton elastic-scattering cross section ratio $R_{e^+e^-} = R_{2\gamma}$ based on a new parametrization of the TPE corrections to ep elastic-scattering cross section. We also compared our results to several previous phenomenological extractions from Refs. [13,73,76,80,81,87,91], TPE hadronic calculations from Ref. [98], and world data on $R_{e^+e^-}$ including the recent direct measurements from Refs. [99–111]. As the ratio $R_{2\gamma}$ deviates from unity only in the presence of hard-TPE contributions, the ratio $R_{2\gamma}$ as extracted from this work is consistent with unity, showing little sensitivity to ε and essentially no Q^2 dependence, suggesting a small contribution from hard-TPE effect in agreement with the findings and conclusion of Ref. [85].

In general, our results and within their error bands do not rule out any hard-TPE contribution to $R_{e^+e^-}$, but they only suggest that the size of hard-TPE correction, as can be seen clearly in Fig. 3, is relatively smaller than that previously predicted by several phenomenological extractions and TPE hadronic calculations. Our results are in generally good agreement with existing world data, including the recent direct measurements [109–111] at large ε and for all the Q^2 points shown within the statistical and systematic uncertainties of these measurements. At large Q^2 and low ε points, our results are also in agreement with the OLYMPUS measurements [111] within the error band of our extractions. On the other hand, the VEPP-3 measurements [110], and the CLAS measurements [109] are within 1.5σ, and 2.0σ-3.0σ from our error band, respectively.

It should be emphasized here that it is difficult to calculate the size of hard-TPE corrections in a model-independent way. Therefore, the sizable hard-TPE contribution predicted by all phenomenological extractions shown is driven mainly by the assumptions used and constraints imposed in their analysis. In our case, and knowing that the two TPE amplitudes Y_E and Y_3 were reported to have opposite sign to each other, and with the tendency to partially cancel each other [74,81], we show that it is possible to suppress $F(\varepsilon, Q^2)$, the TPE correction to σ_R, by constraining Y_E and Y_3 in Eq. (13) to $Y_3(\varepsilon, Q^2) = -\gamma(Q^2)Y_E(\varepsilon, Q^2)$, or effectively $\beta_k(Q^2) = -\gamma(Q^2)\alpha_k(Q^2)$ ($k = 1, 2$), provided that both $(G_M^p)^2$, and the ratio R_p are also constrained to their values as given by Eqs. (9) and (14), respectively. Based on our extractions and the analysis presented in Ref. [85], we believe that the assumption that hard-TPE corrections could account for the discrepancy on the ratio $\mu_p R_p$ is still an open question.

ACKNOWLEDGMENT

The authors acknowledge the support provided by Khalifa University of Science and Technology.

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