Conditions on the pion electroproduction and photoproduction form factors in the soft-pion limit

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Using the Bernard-Kaiser-Lee-Meissner (BKLM) parametrization of the hadronic vertex $\gamma^* N \rightarrow \pi N'$, with γ^* being a virtual photon, we analyze the restrictions imposed on the form factors (FF) in the limit at which the pion three-momentum and mass go to zero, which is known as the soft-pion limit. We obtain information about the normalization of the FF in this limit using current algebra methods and the hypothesis of partially conserved axial-vector current (PCAC) which is a way to implement chiral symmetry. We relate these FF with those of the $W^\pm N \rightarrow N'$ weak interaction. The present work provides a useful tool to decide whether a certain model is better than another, checking whether the corresponding FF satisfy the conditions imposed in the soft-pion limit, which are model independent.

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I. INTRODUCTION

The understanding of the baryon spectrum and the searching for the missing nucleon resonances and new exotic states are hot topics in hadronic physics. Several reactions on free nucleons involving leptonic and hadronic probes excite the Δ(1232)-MeV hadronic resonance as an intermediate state. This resonance, being the first excited on the nucleon (N), is the dominant degree of freedom in the pion (π)-production processes at invariant masses of πN pairs decaying with the Δ in the resonance region ($m_\Delta - \Gamma_\Delta$, $m_\Delta + \Gamma_\Delta$) until $\approx$1.4 GeV. A consistent model for including the Δ was successfully used for describing elastic ($\pi N \rightarrow \pi' N'$) and radiative ($\pi N \rightarrow \pi' N' \gamma$) πN scattering [1,2], π-photoproduction ($\gamma N \rightarrow N' \pi$) [3], and the single π-production in charged ($\nu N \rightarrow l N' \pi$) and neutral current ($\nu N \rightarrow \nu' N' \pi$) neutrino ($\nu$)-$N$ scattering reactions [4,5], in that region. Currently, the experiments reach energies of the order of 2 GeV, and for this reason other additional resonances belonging to the second resonance region need to be included in the analysis. The production of mesons in reactions induced by electromagnetic and weak probes has been extensively used in the study of the properties of nucleon resonances. These reactions are complicated since the initial and/or final πN states are built by the strong interaction. Particularly, we remark that all processes mentioned above have an hadronic vertex of the schematic form $JN \rightarrow \pi N'$, with J being the corresponding hadronic current. This vertex includes all the form factors (FF) allowed by the symmetries of the interaction to a determined order in the involved momenta. From here on, for simplicity and only to fix ideas, we will consider the case of pion electroproduction process $\gamma^* N \rightarrow \pi N'$ (with γ^* being the virtual photon coming from the electron), which reduces to the pion photoproduction $\gamma N \rightarrow \pi N'$ when the photon is real.

Several phenomenological models have been developed for studying the properties of nucleon resonances: partial wave analysis, isobar analysis, effective Lagrangian approach models, chiral effective Lagrangian models, etc. The theoretical models developed in the literature to describe the mentioned processes differ between themselves. The models for pion photo and electroproduction differ mainly in the treatment of the Δ resonance, particularly its propagator, the $\pi N \Delta$ vertex, the interference between the background and the resonant terms, etc. For example, some authors include only the on-shell part of the propagator and the vertex [6] and others consider the complete propagator, consistent with contact transformations [4,5]. On the other hand, some authors use the derivative coupling for the πNN vertex and others adopt only the pseudoscalar one. In this way, we can see that comparing different models means comparing the FF at a given finite pion momentum. Alternatively, if it would be possible to get information about the FF directly from the experiments, we could compare them with the theoretical predictions within each model (concerning the information about the FF from the experiments see, e.g., Refs. [7,8]).

Certainly, it would be useful to have a tool that allows us to decide if one model is better than another when trying to reproduce experimental $\gamma^* N \rightarrow \pi N'$ data and, in particular, if it is consistent with the information we have about the $W^\pm N \rightarrow N'$ axial-vector vertex. Several works were carried out some years ago in order to relate pion electroproduction with FF in weak interactions of nucleons, using methods of current algebra and chiral perturbation theory [9–23]. In these works, (i) the axial-vector constant is calculated within the soft-pion limit and beyond including the so-called hard pion corrections [23] and (ii) low-energy theorems relating the axial-vector coupling with the electromagnetic pion form factor are derived. However, (a) second class currents are neglected and (b) the case with photon low moments (called the "photon point" in Ref. [23]) is analyzed.

Based on previous motivations, we adopt the covariant Bernard-Kaiser-Lee-Meissner (BKLM) parametrization for the hadronic vertex in pion photo and electroproduction, which includes all the FF allowed by the symmetries of the interaction, and obtain information about the normalization of the FF when the pion mass and three-momentum go to zero (the so-called soft-pion limit) by adopting the partially conserved axial-vector current (PCAC) hypothesis that is a way of implementing chiral symmetry. We relate the FF in this limit with those of the $W^{\pm}N \to N'$ interaction. The restrictions obtained on the FF can be used to verify which of the models available in the literature are consistent with them.

The paper is organized as follows. In Sec. II, we present the BKLM parametrization of the vertex and we deduce the conditions obtained in the soft-pion limit. Concluding remarks are briefly drawn in Sec. III.

II. FORMALISM

A. Bernard-Kaiser-Lee-Meissner parametrization

The amplitude for the process $\gamma^*(q) + N(p) \to \pi(k) + N'(p')$ can be written as

$$\mathcal{M}(\gamma^* N \to \pi N') = E^{\mu}\langle N'(p')\pi(k)|V_{\mu}|N(p)\rangle \equiv E^{\mu}\bar{u}(p')\Gamma^{V}_{\mu}u(p), \tag{1}$$

with $u(p)$ [$u(p')$] being the Dirac spinor of the initial (final) nucleon and E_{μ} representing the photon polarization vector, ϵ_{μ}, for photoproduction, and the leptonic current, ℓ_{μ}/q^2, for electroproduction. Γ^{λ}_{V} is the hadronic vertex compatible with Lorentz covariance [note that only the vector (V) contribution participates, without any axial-vector part]. We remark here that we have four 4-momenta, p, q, k, and p', and one energy-momentum conservation condition: $p + q = p' + k$. Concerning the Lorentz scalars, we have at our disposal ten scalars: p^2, p'^2, k^2, q^2, $p\cdot p'$, $p'\cdot q$, $p'\cdot k$, $q\cdot k$, $p\cdot q$, and $p\cdot k$. Using the energy-momentum conservation, all of them can be written in terms of the nucleon and pion masses, $m_N^2 = m_{N'}{}^2$ and m_π^2, respectively, and the scalar products q^2, $p\cdot q$, and $p\cdot k$, as follows: $p^2 = p'^2 = m_N^2$, $k^2 = m_\pi^2$, $p\cdot p' = p\cdot q - p\cdot k + m_N^2$, $p'\cdot q = p\cdot k + \frac{q^2}{2} - \frac{m_\pi^2}{2}$, $p'\cdot k = p\cdot q + \frac{q^2}{2} - \frac{m_\pi^2}{2}$, and $q\cdot k = p\cdot q - p\cdot k + \frac{q^2}{2} + \frac{m_\pi^2}{2}$. Thus, the amplitude can be expressed in terms of three independent scalars, which we have selected as $p\cdot q$, $p\cdot k$, and q^2. In spite of being $q^2 = 0$ for pion photoproduction process, we will keep in mind the q^2 dependence because we are considering here the electroproduction process, which will allow us to analyze in Sec. II B the relation in the soft-pion limit with the process $W^{\pm}(q) + N(p) \to N'(p')$, in which case we will have $q^2 \neq 0$.

For the hadronic vertex, we will adopt the covariant BKLM parametrization from Ref. [22]. From Eqs. (2.4) and (2.5) in that reference, we can express the transition current matrix element in terms of six independent form factors, conventionally denoted by A_i, $(i = 1, \ldots, 6)$, as follows:

$$\mathcal{M}^{\rm BKLM} = iE_{\mu}\bar{u}(p')\gamma_5\left(\sum_{i=1}^{6}A_i\mathcal{M}_i^{\mu}\right)u(p), \tag{2}$$

with

$$\begin{aligned}
\mathcal{M}_1^{\mu} &= \tfrac{1}{2}(\gamma^{\mu}\not{q} - \not{q}\gamma^{\mu}),\\
\mathcal{M}_2^{\mu} &= \tfrac{1}{2}(p+p')^{\mu}(2q\cdot k - q^2) - \tfrac{1}{2}(p+p')\cdot q(2k-q)^{\mu},\\
\mathcal{M}_3^{\mu} &= \gamma^{\mu}q\cdot k - \not{q}k^{\mu},\\
\mathcal{M}_4^{\mu} &= \gamma^{\mu}(p+p')\cdot q - \not{q}(p+p')^{\mu} - m_N\gamma^{\mu}\not{q} + m_N\not{q}\gamma^{\mu},\\
\mathcal{M}_5^{\mu} &= q^{\mu}q\cdot k - k^{\mu}q^2,\\
\mathcal{M}_6^{\mu} &= q^{\mu}\not{q} - \gamma^{\mu}q^2.
\end{aligned} \tag{3}$$

We remark that (i) all these structures $\mathcal{M}_i^{\mu}$'s are individually gauge invariant (it is very simple to check that $q_{\mu}\mathcal{M}_i^{\mu} = 0$) irrespective of whether $q^2 = 0$ or $q^2 \neq 0$ and (ii) the BKLM form factors, A_i, could be evaluated within a given model.

B. Restrictions on form factors in the soft-pion limit

We will analyze here the behavior of the amplitude (2) in the limit where the pion momentum and mass go to zero ($k \to 0$, $m_\pi \to 0$) and we will refer to this limit as the soft-pion limit (spl). This spl implies $p' - p \to q$, $E\cdot(p'-p) \to E\cdot q$, $q\cdot(p'-p) \to q^2$, $p\cdot k \to 0$, $p'\cdot k \to 0$, $q\cdot k \to 0$, $k\cdot E \to 0$, $p\cdot p' \to m_N^2 - \frac{q^2}{2}$, $p\cdot q \to -\frac{q^2}{2}$, and $p'\cdot q \to \frac{q^2}{2}$. Thus, the amplitudes can be expressed as a function of q^2 in this limit. Using the definition $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$, the Dirac matrix properties, the Dirac equations $\not{p}u(p) = m_N u(p)$ and $\bar{u}(p')\not{p}' = m_N\bar{u}(p')$ to write $\bar{u}(p')(\not{p}' - \not{p})\gamma_5 u(p) = 2m_N\bar{u}(p')\gamma_5 u(p)$, and the Gordon identity $\bar{u}(p')(p+p')^{\mu}\gamma_5 u(p) = -i\bar{u}(p')\sigma^{\mu\nu}(p'-p)_{\nu}\gamma_5 u(p)$, from (2) we obtain

$$\mathcal{M}^{\rm BKLM} \overset{spl}{\to} iE_{\mu}\bar{u}(p')\gamma_5\left(\sum_{i=1}^{6}A_i^{(\rm spl)}(m_{t_N}, m_{t_{N'}})(\mathcal{M}_i^{\mu})^{(\rm spl)}\right) \times u(p), \tag{4}$$

with

$$\begin{aligned}
&\left(\mathcal{M}_1^{\mu}\right)^{(\rm spl)} = -i\sigma^{\mu\nu}q_{\nu}, \quad \left(\mathcal{M}_2^{\mu}\right)^{(\rm spl)} = \frac{q^2}{2}i\sigma^{\mu\nu}q_{\nu},\\
&\left(\mathcal{M}_3^{\mu}\right)^{(\rm spl)} = 0, \quad \left(\mathcal{M}_4^{\mu}\right)^{(\rm spl)} = 0,\\
&\left(\mathcal{M}_5^{\mu}\right)^{(\rm spl)} = 0, \quad \left(\mathcal{M}_6^{\mu}\right)^{(\rm spl)} = -2m_N q^{\mu} - \gamma^{\mu}q^2.
\end{aligned} \tag{5}$$

Here (spl) refers to the soft-pion limit [$A_i^{(\rm spl)}(m_{t_N}, m_{t_{N'}}) \equiv \lim_{k\to 0} A_i(k, m_{t_N}, m_{t_{N'}})$]. Additionally, we have indicated the isospin dependence of the FF by adding explicitly the isospin projection of the initial and final nucleons, m_{t_N} and $m_{t_{N'}}$, respectively. Then Eq. (4) can be more conveniently written as

$$\begin{aligned}
\mathcal{M}^{\rm BKLM} \overset{spl}{\to}\ & iE_{\mu}\bar{u}(p')\gamma_5\\
&\times\left\{\left[\frac{q^2}{2}A_2^{(\rm spl)}(m_{t_N}, m_{t_{N'}}) - A_1^{(\rm spl)}(m_{t_N}, m_{t_{N'}})\right]i\sigma^{\mu\nu}q_{\nu}\right.\\
&\left. - A_6^{(\rm spl)}(m_{t_N}, m_{t_{N'}})(2m_N q^{\mu} + \gamma^{\mu}q^2)\right\}u(p).
\end{aligned} \tag{6}$$

This result indicates that the limit $k \to 0$, $m_\pi \to 0$ in the process $\gamma^*(q) + N(p) \to \pi(k) + N'(p')$ does not lead to the process $\gamma^*(q)+N(p)\to N(p')$, because this limit does not mean to "annihilate" the pion. In fact, after taking this limit, we have still information in Eq. (4) on the pion: It is in the γ_5 matrix which remind us that the interaction is "vector like," unlike the "axial-vector" $W^{\pm}(q)+N(p)\to N(p')$ vertex. Therefore, we can say that

$$\mathcal{M}(\gamma^*N\to\pi N')\overset{spl}{\to}E^{\mu}\mathcal{M}_{\mu}(W^{\pm}N\to N'), \tag{7}$$

where

$$\begin{aligned}\mathcal{M}^{\mu}&(W^{\pm}N\to N')\\ &\equiv i\bigg\{\bigg[\frac{q^2}{2}A_2^{(\mathrm{spl})}(m_{t_N},m_{t_{N'}})-A_1^{(\mathrm{spl})}(m_{t_N},m_{t_{N'}})\bigg]i\sigma^{\mu\nu}q_{\nu}\\ &\quad-A_6^{(\mathrm{spl})}(m_{t_N},m_{t_{N'}})(2m_Nq^{\mu}-\gamma^{\mu}q^2)\bigg\}\gamma_5u(p).\end{aligned} \tag{8}$$

Note that in the spl, the electroproduction amplitude is related to the weak vertex, in which case is also $q^2\neq 0$. Therefore, the amplitude and the FF will depend on q^2. In order to carefully analyze the soft-pion limit, we will compare here the limit $k\to 0$, $m_\pi\to 0$ of the vector vertex given in Eq. (2) for pion electroproduction process, $\gamma^*(q)+N(p)\to\pi(k)+N'(p')$ [which, in spite of being a vector vertex, behaves as an axial-vector one because of the "memory" about of the pion which contributes with the γ_5 in Eq. (4), as mentioned previously] with the usual parametrization of the axial-vector vertex for the nucleon decay process, $W^{\pm}(q)+N(p)\to N'(p')$. The matrix element for the axial-vector current A_μ is

$$\langle N'(p')|A_{\mu}|N(p)\rangle=\bar{u}(p')\Gamma_{\mu}^{A}u(p), \tag{9}$$

where

$$\begin{aligned}\Gamma_{\mu}^{A}=&\bigg[g_1(m_{t_N},m_{t_{N'}})\gamma_{\mu}+\frac{g_2(m_{t_N},m_{t_{N'}})}{2m_N}i\sigma_{\mu\nu}q^{\nu}\\ &+\frac{g_3(m_{t_N},m_{t_{N'}})}{2m_N}q_{\mu}\bigg]\gamma_5,\end{aligned} \tag{10}$$

with $q=p'-p$. Note that this "axial-vector" vertex does not satisfy the electromagnetic gauge invariance condition $q^{\mu}A_{\mu}=0$, as is the case with Eq. (2). It is important to mention that we do not neglect here the second-class currents, as done in Eq. (12) from Ref. [11], where the authors take $g_2=0$. We remark here that this form factor is only required to vanish by G parity (see, for example, Eq. (6.115) from Ref. [24]).

In the following, we intend to find the relation of the FF given in Eq. (2) in the soft-pion limit with those given in Eq. (10), by looking at the quark content of the currents. We have the following four pion electroproduction channels: $\gamma^*p\to n\pi^+$, $\gamma^*p\to p\pi^0$, $\gamma^*n\to p\pi^-$, and $\gamma^*n\to n\pi^0$, where the quark content of the involved baryons and mesons is

$$\begin{aligned}&|p\rangle=|uud\rangle,\quad |n\rangle=|udd\rangle,\quad |\pi^+\rangle=|u\bar{d}\rangle,\\ &|\pi^-\rangle=-|d\bar{u}\rangle,\quad |\pi^0\rangle=\frac{1}{\sqrt{2}}(|u\bar{u}\rangle-|d\bar{d}\rangle).\end{aligned} \tag{11}$$

The current responsible for those processes is the electromagnetic one (see Eq. (3.1) from Ref. [25]),

$$V_{\mu}^{e.m.}=V_{\mu}^{3}+\frac{1}{\sqrt{3}}V_{\mu}^{8}, \tag{12}$$

where we have introduced the vector (V) and axial-vector (A) currents:

$$V_{\mu}^{k}=\tfrac{1}{2}\bar{q}\gamma_{\mu}\lambda^{k}q,\quad A_{\mu}^{k}=\tfrac{1}{2}\bar{q}\gamma_{\mu}\gamma_5\lambda^{k}q. \tag{13}$$

Here $q=\{\begin{smallmatrix}u\\ d\\ s\end{smallmatrix}$ is the quark spinor and λ^k are the usual Gell-Mann matrices (satisfying the conmutation relation $[\lambda_i,\lambda_j]=2if_{ijk}\lambda_k$, with f^{ijk} being the totally antisymmetric SU(3) structure constants).

Now, we follow the procedure of Sec. V from Ref. [25] for the evaluation of matrix elements of processes involving hadrons A and B: $A\to B\pi^a(k)$, $a=0,\ \pm$, using current algebra methods. From Eq. (5.4) in that reference, after taking the soft-pion limit for the pion electroproduction process, we have

$$\begin{aligned}\mathcal{M}_{\mu}(A\to B\pi^a)\overset{spl}{\to}&-\frac{i}{f_\pi}\int d^3x\delta(x_0)\\ &\times\langle B|\big[A_0^a(x),V_{\mu}^{e.m.}(0)\big]|A\rangle,\end{aligned} \tag{14}$$

with A_0^a given in Eq. (13). In the present calculation, we assume the PCAC hypothesis, which relates the divergency of the axial-vector current with the pion fields (see Eq. (4.4) from Ref. [25]). Additionally, we use the current algebra (commutation relations between charges and between charges and currents; see, for example, Sec. III on pp. 18–23 in Ref. [25]). The axial A_0^a current in (14) comes from the PCAC hypothesis.

Next, we analyze each one of the pion electroproduction processes separately.

1. $\gamma^*p\to n\pi^+$ process

In particular, for the process $\gamma^*p\to n\pi^+$ we can use Eq. (14) and write [being $\pi^+=|u\bar{d}\rangle=\frac{1}{2}\bar{q}(\lambda^1-i\lambda^2)q$, we made $A_0^a\to\frac{1}{2}(A_0^1-iA_0^2)$]

$$\begin{aligned}\mathcal{M}_{\mu}&(p\to n\pi^+)\overset{spl}{\to}-\frac{i}{2f_\pi}\int d^3x\delta(x_0)\\ &\times\langle n|\bigg[(A_0^1(x)-iA_0^2(x)),V_{\mu}^3(0)+\frac{1}{\sqrt{3}}V_{\mu}^8(0)\bigg]|p\rangle.\end{aligned} \tag{15}$$

Using the current algebra conmutators (see Eq. (3.14) from Ref. [25]),

$$\big[A_0^a(x),V_{\mu}^k(y)\big]=if^{akd}A_{\mu}^d(x)\delta(x-y), \tag{16}$$

we get

$$\begin{aligned}\mathcal{M}_{\mu}&(p\to n\pi^+)\overset{spl}{\to}\mathcal{M}_{\mu}(W^-p\to n)\\ &\equiv-\frac{i}{2f_\pi}\delta(x_0)\langle n|A_{\mu}^1(0)-iA_{\mu}^2(0)|p\rangle.\end{aligned} \tag{17}$$

On the right hand side, we have the axial-vector matrix element for the $W^-p\to n$ process given in (9), for which we assume the form given in (10). Thus, comparing the results shown in Eqs. (8) and (17) [via Eq. (10)], both valid in the soft-pion limit, we have that

$$i\left\{\left[\frac{q^2}{2}A_2^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right)-A_1^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right)\right]i\sigma^{\mu\nu}q_\nu - A_6^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right)(2m_Nq^\mu-\gamma^\mu q^2)\right\}\gamma_5u(p)$$
$$= -\frac{i}{2f_\pi}\left[g_1\left(\frac{1}{2},-\frac{1}{2}\right)\gamma_\mu+\frac{g_2\left(\frac{1}{2},-\frac{1}{2}\right)}{2m_N}i\sigma_{\mu\nu}q^\nu + \frac{g_3\left(\frac{1}{2},-\frac{1}{2}\right)}{2m_N}q_\mu\right]\gamma_5, \tag{18}$$

which leads to the relations

$$q^2A_6^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right) = -\frac{1}{2f_\pi}g_1\left(\frac{1}{2},-\frac{1}{2}\right),$$
$$\frac{q^2}{2}A_2^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right)-A_1^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right) = -\frac{1}{2f_\pi}\frac{g_2\left(\frac{1}{2},-\frac{1}{2}\right)}{2m_N},$$
$$-2m_NA_6^{(\mathrm{spl})}\left(\frac{1}{2},-\frac{1}{2}\right) = -\frac{1}{2f_\pi}\frac{g_3\left(\frac{1}{2},-\frac{1}{2}\right)}{2m_N}. \tag{19}$$

2. $\gamma^*n \to p\pi^-$ process

Following the same procedure, in the soft-pion limit we have [being $\pi^- = -|u\bar{d}\rangle = -\frac{1}{2}\bar{q}(\lambda^1+i\lambda^2)q$, we made $A_0^a \to -\frac{1}{2}(A_0^1+iA_0^2)$]

$$\mathcal{M}_\mu(n\to p\pi^-) \overset{spl}{\to} \frac{i}{2f_\pi}\int d^3x\delta(x_0) \times\langle p|\left[A_0^1(x)+iA_0^2(x),V_\mu^3(0)+\frac{1}{\sqrt{3}}V_\mu^8(0)\right]|n\rangle. \tag{20}$$

The current algebra commutators lead to

$$\mathcal{M}_\mu(n\to p\pi^-) \overset{spl}{\to} \mathcal{M}_\mu(W^+n\to p) \equiv -\frac{i}{2f_\pi}\delta(x_0)\langle p|A_\mu^1(0)+iA_\mu^2(0)|n\rangle. \tag{21}$$

The axial-vector matrix element on the right-hand side, $\langle p|A_\mu^1(0)+iA_\mu^2(0)|n\rangle$, is given in Eq. (9). Comparing (8) with (21) in the soft-pion limit will lead to the same relations shown in Eq. (19), but with different isospin projections in the FF:

$$q^2A_6^{(\mathrm{spl})}\left(-\frac{1}{2},\frac{1}{2}\right) = -\frac{1}{2f_\pi}g_1\left(-\frac{1}{2},\frac{1}{2}\right),$$
$$\frac{q^2}{2}A_2^{(\mathrm{spl})}\left(-\frac{1}{2},\frac{1}{2}\right)-A_1^{(\mathrm{spl})}\left(-\frac{1}{2},\frac{1}{2}\right) = -\frac{1}{2f_\pi}\frac{g_2\left(-\frac{1}{2},\frac{1}{2}\right)}{2m_N},$$
$$-2m_NA_6^{(\mathrm{spl})}\left(-\frac{1}{2},\frac{1}{2}\right) = -\frac{1}{2f_\pi}\frac{g_3\left(-\frac{1}{2},\frac{1}{2}\right)}{2m_N}. \tag{22}$$

3. $\gamma^*N \to N\pi^0$ process

Following the same procedure, in the soft-pion limit we have [being $\pi^0 = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) = \frac{1}{\sqrt{2}}\bar{q}\lambda^3q$, we made $A_0^a \to \frac{1}{\sqrt{2}}A_0^3$]

$$\mathcal{M}_\mu(N\to N\pi^0) \overset{spl}{\to} -\frac{i}{\sqrt{2}f_\pi}\int d^3x\delta(x_0) \times\langle N|\left[A_0^3(x),V_\mu^3(0)+\frac{1}{\sqrt{3}}V_\mu^8(0)\right]|N\rangle. \tag{23}$$

The current algebra commutators lead to

$$\left[A_0^3(x),V_\mu^3(0)+\frac{1}{\sqrt{3}}V_\mu^8(0)\right]=0, \tag{24}$$

which gives

$$\mathcal{M}_\mu(N\to N\pi^0) \overset{spl}{\to} 0, \tag{25}$$

leading to the relations (for $m_{t_N} = \pm\frac{1}{2}$)

$$\frac{q^2}{2}A_2^{(\mathrm{spl})}(m_{t_N},m_{t_N})-A_1^{(\mathrm{spl})}(m_{t_N},m_{t_N})=0,$$
$$A_6^{(\mathrm{spl})}(m_{t_N},m_{t_N})=0. \tag{26}$$

These relations, together with those given in Eqs. (19) and (22) that can be written as

$$q^2A_6^{(\mathrm{spl})}(m_{t_N},-m_{t_N}) = -\frac{g_1(m_{t_N},-m_{t_N})}{2f_\pi},\quad A_1^{(\mathrm{spl})}(m_{t_N},-m_{t_N})-\frac{q^2}{2}A_2^{(\mathrm{spl})}(m_{t_N},-m_{t_N}) = \frac{g_2(m_{t_N},-m_{t_N})}{4f_\pi m_N}, -2m_NA_6^{(\mathrm{spl})}(m_{t_N},-m_{t_N}) = -\frac{g_3(m_{t_N},-m_{t_N})}{4f_\pi m_N}, \tag{27}$$

establish all the conditions that the matrix elements should satisfy in the unphysical point $k^2 = m_\pi^2 = 0$ obtained after taking the soft-pion limit. The physical point with $k^2 = m_\pi^2 \neq 0$ will be determined within a particular model (effective Lagrangian approach, chiral perturbation theory, etc.) and the relations (26) and (27) give a condition that the FF should satisfy. In fact, the FF that describe the $\gamma^*N \to \pi N'$ process depend on several variables at finite values of the pion momentum. However, our Eqs. (26) and (27) show that in the soft-pion limit the FF are related to the weak vertex of the nucleon (including second class currents). For completeness, we mention here that one can find experimental information on g_1 and g_3 in the literature (see, for example, Ref. [24] and references therein). Otherwise, g_2 is suppressed by isospin symmetry and also because the difference between momentum is not so big. However, there exist theoretical information on g_2 (see also Ref. [24]). In other words, the pion electroproduction FF must be normalized in such a way to be consistent with the relations (26) and (27) in the soft-pion limit.

III. CONCLUDING REMARKS

We have analyzed the behavior of the hadronic vertex $\gamma^* N \to \pi N'$. Adopting the Bernard-Kaiser-Lee-Meissner parametrization of the hadronic vertex, we have considered the soft-pion limit $k \to 0$, $m_\pi \to 0$ in the process $\gamma^*(q) + N(p) \to \pi(k) + N'(p')$ and we have learned that it does not lead to the process $\gamma^*(q) + N(p) \to N'(p')$ because the vertex is vectorial in the first case and axial-vector in the second one, due to the "memory" about the pion which contributes with the γ_5 in Eq. (4). For this reason, we have analyzed carefully the soft-pion limit using current algebra methods. We obtained information about the normalization of the FF when the pion mass and three-momentum go to zero by adopting the PCAC hypothesis that is a way of implementing chiral symmetry. The FF in this limit were related with those of the $W^{\pm}N \to N'$ interaction, including second-class currents, through Eqs. (26) and (27). These relations should be understood as conditions "independent of the model" that the form factors A_1, A_2, and A_6 should satisfy in the soft-pion limit. They will allow us to decide if a given model is better than another to reproduce the experimental data. We remark here that soft-pion limit does not impose any condition on the form factors A_3, A_4, and A_5.

In conclusion, if it were possible to determine the experimental value of $A_i(\sqrt{s}, \cos\theta)$ [which we will refer to as $A_i^{\rm exp}(\sqrt{s}, \cos\theta)$] as a function of $\sqrt{s}$ and the c.m. angle θ between π and N', we could proceed as usual, analyzing which model including the dominant resonances participating up to a certain scale of energy is able to better reproduce $A_i^{\rm exp}(\sqrt{s}, \cos\theta)$ but looking at the same time if the relations (26) and (27) are satisfied in the soft-pion limit. On the other hand, even if we do not have experimental values $A_i^{\rm exp}(\sqrt{s}, \cos\theta)$ at our disposal, we can use the obtained relations to verify if different analytical calculations of the amplitudes A_1, A_2, and A_6, obtained within different theoretical models, are consistent with the known values of the g_i form factors when we analyze those amplitudes in the soft-pion limit. In this sense, if the soft-pion limit of those amplitudes obtained within different theoretical models (for example, with a different treatment of resonance Δ) is different, the relations obtained in (26) and (27) will be useful for distinguishing which of the models is better, simply observing which of the theoretical limits best reproduces the experimental data of the axial form factors g_i.

In summary, we have discussed how we can use the conditions (26) and (27), valid in the soft-pion limit, to guarantee that a given model can reproduce the experimental data when they are available and/or to compare different alternative theoretical models with the purpose of discarding those that are not consistent with those conditions. Thus, our relations (26) and (27) provide a tool to compare alternative models for pion photo and electroproduction.

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