Shell-like quarteting in heavy nuclei: Algebraic approaches based on the pseudo- and proxy-SU(3) schemes

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Background: The semimicroscopic algebraic quartet model (SAQM) generates the excitation spectra of shell-like quartets, formed by two protons and two neutrons in a well-defined shell configuration. It is based on the SU(3) symmetry, which is valid only in light nuclei. Algebraic quartet models have been proposed for heavy nuclei, too, but their shell model background is not known.
Purpose: I wish to construct an algebraic approach for the description of shell-like quarteting in heavy nuclei.
Methods: The SAQM is extended based on the pseudo- and proxy-SU(3) symmetries.
Results: The procedure of the generalization is presented. The new models are applied for the description of the energy spectrum and electromagnetic transitions of the low-lying bands of ^{224}Th.
Conclusions: In the present case the performance of the two approximate symmetries is fairly similar due to the alike nature of the relevant shell orbits. The possibly similar or different results of the two symmetries are discussed.

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I. INTRODUCTION

The importance of quarteting in nuclei has long been known; its origin goes back to Wigner's supermultiplet theory [1]. Qualitatively it is easily understandable in light of the short-range and attractive nature of the nucleon-nucleon forces, and the fact that the exclusion principle allows two protons and two neutrons to occupy a single particle orbital.

Several quartet models have been formulated (see, e.g., [2]), and for a long time they were applied exclusively in the study of the binding energy of the ground state nuclei. The concept of quartet excitations was introduced in 1970 [3], and from then on one is interested in the quartet spectra, too. In [3] a quartet is defined as two protons and two neutrons on a single orbital (without specifying the coupling scheme), and the excitation occurs, via the jumping of the four nucleons to the next major shell. The corresponding energies were determined from mass relationships. Later this model was extended into several directions [4,5], including not only intershell, but also intrashell quartet excitations.

An important generalization was presented in [6] by incorporating any number of particle-hole excitations (in the language of the nucleon-shell model), contrary to the quartet-shell model of [3,5] which had only zero, four, eight,... excitation quanta (in terms of nucleon-shell model). This considerable extension of the quartet model space appeared due to the conceptual generalization of a quartet. Harvey defined it [6] as two protons and two neutrons having a quartet symmetry: permutational symmetry of {4}, and spin-isospin symmetry of {1,1,1,1}.

In these models a quartet had a well-defined shell configuration, but the models did not have detailed formalism to calculate the spectra (energies, transitions, etc).

Interacting boson type quartet models were also invented [7,8] for the description of quarteting in heavy nuclei. In [7] the basic building block quartets are treated as $l = 0$ (s) and $l = 2$ (d) bosons, and the model has a U(6) group structure, like the interacting boson model of the quadrupole collectivity [9]. This model describes a spectrum of positive parity states. In [8] the α-like correlation is treated in terms of bosons of nucleon pairs, but in addition to the s and d bosons another set of basic building blocks of $l = 0$ (s^*) boson and $l = 1$ (p) boson is included, therefore, negative parity states are also involved. These phenomenological models have the efficiency and elegance of the algebraic methods in generating the spectrum, e.g., they have dynamical symmetries as limiting cases, which provide us with exact solutions for the eigenvalue problem. On the other hand, the shell content of these boson models is not known exactly.

In [10] an algebraic approach was proposed, based on Elliott's SU(3) formalism [11], for the calculation of detailed spectra of the shell-like quartets defined in [3,6]. Actually two algebraic quartet models were introduced, a phenomenological (PAQM) one, which did not treat the nucleon degrees of freedom one-by-one, and a semimicroscopic (SAQM) one, in which the model space is fully microscopic, i.e., based on nucleon degrees of freedom.

Due to the definition of the quartet in terms of shell model symmetries of the nucleons, and the application of the SU(3) formalism, the semimicroscopic algebraic quartet model is in fact a symmetry-governed truncation of the (no-core) SU(3) shell model [12]. For an I quartet system ($A = 4I, N = Z = 2I$) the truncation means the $\{I, I, I, I\}$ Wigner-scalar sector, or the corresponding {4,4,...,4} permutational symmetry. This kind of shell model connection is not univocal; in other quartet models, based on other definitions of a quartet, the relation is different.

In the semimicroscopic algebraic quartet approach also the connection to clusterization is well-defined and transparent [13]. In [13,14], e.g., high-lying detailed cluster spectra of two different configurations could be predicted from the quartet model description of the low-lying bands. The predictions turned out to be in good agreement with the experimental observation.

Quartet condensates have also been considered [15–17], but I do not discuss them here, because for the present purpose the individual quartet picture is more relevant.

In this paper I introduce a new algebraic approach to quarteting in heavy nuclei. As for the definition of the quartet, the well-defined shell content of [3,6] is kept. In particular, two protons and two neutrons form a quartet, when they have permutational symmetry {4}. (In heavy nuclei it is not the spin-isospin scheme, which is appropriate, rather the proton-neutron one, that is why the permutational symmetry is applied here to define a quartet.)

The formalism of this new approach is that of SU(3) [11], i.e., I present here a generalization of the semimicroscopic algebraic quartet model (SAQM) of light nuclei [10]. The generalization is needed due to the well-known fact, that the original SU(3) symmetry is valid only in light nuclei, in heavy ones it is badly broken.

Though the real SU(3) symmetry is not good in heavy nuclei, nevertheless, other symmetries were found to be valid. I consider here the pseudo-SU(3) [18] and the proxy-SU(3) schemes [19]. Both of these symmetries are valid in subspaces of the full shell model space: some intruder orbits are omitted. The rearrangement of the shell scheme is, however, different in the two cases.

The harmonic oscillator shell structure is destroyed in heavy nuclei by the spin-orbit force. In particular, the $j = n + \frac{1}{2}$ orbitals of the nth oscillator shell are pushed down among the next lower shell. They have opposite parity, and are called unique parity states.

In the pseudo-SU(3) scheme all the unique parity orbitals are omitted, and the remaining orbitals can be rearranged into the pseudoshell of $(n-1)$ oscillator quanta.

In the proxy-SU(3) scheme only one of the unique parity orbitals is excluded: the highest lying state, e.g., $[505]_{11/2}$ in the 50–82 shell. (Here, the $[Nn_z\Lambda]_K$ quantum numbers of the asymptotic Nilsson state indicate the total number of oscillator quanta, the quanta along the symmetry axis, the projection of the orbital, and total angular momentum, respectively.) The rest of the intruder states are replaced by the highest j orbit from the next lower shell. The new set of orbits constitutes a full oscillator shell.

Here, I propose algebraic models for the shell-like quarteting in heavy nuclei, based on the pseudo- and proxy-SU(3) symmetries.

Since in heavy nuclei the proton and neutron valence shells are different, I use the permutational symmetry for the definition of the quartet, as mentioned above. When two protons and two neutrons sit in two different major shells with $\{2\}_p$ and $\{2\}_n$ symmetries, they may result in the quartet symmetry {4}. Similarly if eight protons and eight neutrons are there in two subsequent shells, then the $\{2,2,2,2\}_p$ and $\{2,2,2,2\}_n$ symmetries may combine to the {4,4,4,4} symmetry, corresponding to a quartet state. (This latter example is relevant for the ^{224}Th nucleus discussed below.)

In what follows first I recall some basic features of the pseudo-SU(3) and proxy-SU(3) schemes, and illustrate them by constructing the model space of the ^{224}Th nucleus. Then the spectrum and the $E2$ transition ratios are calculated, by applying the formalism of the semimicroscopic algebraic quartet model of Ref. [10].

The main purpose of the present paper is methodological. I wish to find how the algebraic description of the shell-like quarteting can be extended to heavy nuclei, based on the approximate symmetries of the pseudo- and proxy-SU(3) schemes. Furthermore, the two methods are compared, when they are applied for the treatment of a low-energy spectrum.

II. THE PSEUDO-SU(3) SCHEME

As is mentioned above, the subspace of the pseudo-SU(3) symmetry is obtained by excluding the intruder orbitals [20]. Under the action of the normal-to-pseudo (unitary) transformation the composition of the angular momentum changes from $j = l + s$ to $j = \tilde{l} + \tilde{s}$, where "tilde" ($\sim$) indicates pseudo [21,22].

The reason for the appearance of the pseudo-SU(3) symmetry is the following. The single-particle Hamiltonian

$$H = H_0 + Cls + Dl^2 + \cdots \quad (1)$$

transforms into the pseudo-counterpart

$$\tilde{H} = \tilde{H}_0 + \tilde{C}\tilde{l}\tilde{s} + \tilde{D}\tilde{l}^2 + \cdots \quad (2)$$

H and $\tilde{H}$ have the same excitation spectrum, when $\hbar\omega = \hbar\tilde{\omega}$, $\tilde{C} = (4D - C)$, $\tilde{D} = D$. It turns out that for the $A \geqslant 100$ nuclei $C \approx 4D$, so $\tilde{C} \approx 0$, i.e., the symmetry-breaking spin-orbit force is negligible [23]. Therefore, the pseudo-SU(3) symmetry is realized in heavy nuclei to a similar extent, or even better, than the real SU(3) in light nuclei.

The nucleons of the intruder orbital from the next higher oscillator shell are assumed to play only a passive role in the dynamics, because of their opposite parity. In particular, they are usually considered to form $J = 0$ coupled pairs contributing only to the binding energy. This arrangement is favored because the pairing gap is large as compared to the spacing of low-lying rotational levels, thus the assumption seems to be valid except for the backbending and related phenomena [23].

The pseudo-SU(3) scheme proved to be successful also in the microscopic description of the quadrupole collectivity, which is a multishell phenomenon [24]. In light nuclei the $E2$ electromagnetic transitions could be described correctly, without applying effective charge, by the symplectic generalization of the Elliott model [25,26]. It takes into account $2\hbar\omega$ major shell excitations. When it is extended to heavy nuclei, based on the pseudo-SU(3) scheme, one would expect different kinds of major shell excitations: those of normal parity, unique parity, and a mixing between the two. But it was found [23] that the latter two kinds are not necessary (except for backbending, etc). The reason is that the unique parity part of the wave function is dominated by pairing correlations, which differs from the quadrupole correlation, in that they do not require the involvement of higher shells.

The physical operators transform under the normal-to-pseudo mapping into their pseudo-counterpart plus small corrections:

$$O = \kappa \tilde{O} + \dots . \tag{3}$$

$\tilde{O}$ has the same tensorial character as O, while the other terms in the series have a different tensorial character and an expansion coefficient that is typically less than 10% of the leading term. These correction terms were found to yield only a minor (less than one percent) change in calculated results in eigenenergies and electromagnetic transition rates, therefore, they are usually ignored [23,24].

In constructing the quartet-model-space based on the pseudo-SU(3) scheme, one can assume a scenario similar to that of the pseudosymplectic model of the quadrupole collectivity: the dynamics is dominated by the normal-parity major shell excitations, while those of the unique parity orbitals play a passive role, and their effect can be taken into account by renormalization of model parameters [24]. Further model assumptions (again similarly to the pseudosymplectic model [27]) are as follows. i) The most important normal parity configurations are those with highest spatial symmetry, which implies that only pseudospin zero configurations are taken into account. ii) The leading pseudo-SU(3) irreducible representations (irreps) in the proton and neutron spaces dominate. Whether or not these assumptions are valid, can be checked by comparing the predictions of a model with the experimental data.

Now we summarize the scenario of the construction of the model space, which is similar in both the pseudo and the proxy cases.

(i) First one chooses the closed core of the shell model problem. Here, the core is the $^{208}_{82}\mathrm{Pb}_{126}$ nucleus.

(ii) The nucleons above the closed core are placed on the Nilsson orbitals of the corresponding deformation according to the energy minimum and the Pauli principle.

(iii) The pseudo- and proxy oscillator shells are obtained as a subset of the single particle orbitals above the closed shell, according to the truncation of the two schemes. In the pseudo formalism all the unique parity orbitals are excluded, while in the proxy one, only the highest-lying Nilsson orbital is omitted.

The following steps are made in each shell. For the $0\hbar\omega$ case it means a single proton and single neutron shell. For the major shell excitations it involves two or more shells (in one or both of the proton and neutron sectors).

(iv) The permutational symmetry of the protons (or neutrons), obtained from step (iii) provides us with with the irreducible representation of the U(N) group, where N is the number of the single particle orbital in the shell. In particular, the representations of the permutational and U(N) groups are given by the same Young pattern.

(v) The allowed SU(3) irreps are obtained from solving the U(N) $\supset$ SU(3) representation problem [28].

TABLE I. The dominating $\widetilde{SU(3)}$ irreps of the ^{224}Th nucleus in the semimicroscopic algebraic quartet model based on the pseudo scheme. The superscript indicates multiplicity.

$\hbar\omega$	(λ, μ)
0	(38,6),(39,4),(40,2),(36,7),$(37,5)^2$, $(38, 3)^2$,...
1	(43,4),(44,2),(45,0),$(41,5)^2$, $(42, 3)^3$, $(43, 1)^2$,...
2	(48,2),$(46,3)^2$, $(44, 4)^3$, $(45, 2)^2$, $(46, 0)^1$, $(42, 5)^2$,...

(vi) The Pauli-allowed SU(3) representations are obtained as the outer product of those SU(3) representations of the different major shells, which belong to permutational symmetries such that their outer product gives the required quartet symmetry.

In this procedure each major shell is treated separately. The larger algebraic structure is $\mathrm{U}_s(2) \otimes \mathrm{U}(N)$ in each shell of protons or neutrons, where $\mathrm{U}_s(2)$ stands for the spin sector, while U(N) refers to the space part. The antisymmetrization requirement is taken into account in a two-step treatment: within each major shell the spin and space parts of the wave function carry symmetries which belong to conjugate Young patterns, and in building up the complete model space only those outer products are taken, which keep the total antisymmetrization for the many major shell problem.

Due to the nature of the different truncation schemes, the dimension of the pseudo-SU(3) representations are usually smaller than those of the proxy ones.

Now I illustrate the procedure by constructing the model space for the $^{224}_{90}\mathrm{Th}_{134}$ nucleus. It has eight protons and eight neutrons, which may form four quartets, outside the $^{208}_{82}\mathrm{Pb}_{126}$ core.

The eight protons (in the 82–126 shell) occupy the normal parity $\tilde{N} = 4$ pseudo-harmonic oscillator shell and the $1i_{13/2}$ intruder levels of opposite parity. The eight neutrons (in the 126–184 shell) are distributed on the normal parity $\tilde{N} = 5$ pseudoshell and, and the $1j_{15/2}$ unique parity orbitals.

The deformation parameter of the ground state is $\epsilon = 0.15$ [29], therefore, four protons sit in the $\tilde{N} = 4$ pseudoharmonic oscillator shell [27], having a leading SU(3) representation of (12,2), and four protons are in the $1i_{13/2}$ intruder level. The eight neutrons sit in $\tilde{N} = 5$ pseudoharmonic oscillator shell, having a leading representation of (26,4). (All the (λ, μ) irreps in this section refer to the pseudo-SU(3) scheme, thus, we can ignore, for the sake of simplicity, their "tilde", without causing any confusion.) The dominating SU(3) symmetries of the $0\hbar\omega$ subspace (in Table I) are obtained as a product: $(12, 2) \otimes (26, 4)$.

The leading $1\hbar\omega$ representations, when a proton or a neutron is excited to the next pseudomajor shell, are (15,1), and (31,2), respectively. Thus, the $\widetilde{SU(3)}$ irreps are obtained as $(15, 1) \otimes (26, 4)$ and $(12, 2) \otimes (31, 2)$. Please, note that the $1\hbar\omega$ irreps of Table I cannot be obtained from the $0\hbar\omega$ representations multiplied by the (1,0) center of mass motion, therefore, they are not contaminated by spurious excitations of the center of mass.

The leading $2\hbar\omega$ proton and neutron excitations are (18,0), and (36,0), respectively. When one proton and one neutron is excited: (15,1) and (31,2). Therefore, the $\widetilde{SU(3)}$ irreps are obtained as $(18, 0) \otimes (26, 4)$ for proton excitation, $(12, 2) \otimes (36, 0)$ for neutron excitation, and $(15, 1) \otimes (31, 2)$ for proton-neutron excitations. The multiplicity of some $2\hbar\omega$ irreps of Table I is decreased (in comparison with the result of the multiplications), due to $1\hbar\omega$ spurious excitations [e.g., the $1\hbar\omega$ excitation of the center of mass of the (44,2) states results in (45,2)]. The $0\hbar\omega$ and $2\hbar\omega$ states have positive, while the $1\hbar\omega$ excitations have negative parities.

III. THE PROXY-SU(3) SCHEME

The microscopic support for the proxy-SU(3) scheme is provided by the observation that the Nilsson diagrams for well-deformed nuclei obtained with the proxy-SU(3) symmetry are very similar to the traditional Nilsson diagrams [19]. In particular, the pairs of Nilsson orbits related by $[\Delta N \Delta n_z \Delta\Lambda]_{\Delta K} = [110]_0$ have very large overlap.

But the parity content of the shells is different. The replacement of the intruder orbits with those from the next lower shell changes the parity. When, however, the orbitals are double occupied it does not effect the parity of the nucleus.

Due to the approximation applied in the proxy-SU(3) scheme some selection rules (and avoided level crossing) are affected [19]. Furthermore, one has to be careful when applying this new symmetry to nuclei of odd mass number, due to the parity change between the real and proxy orbitals.

The proxy-SU(3) model space of the $^{224}_{90}Th_{134}$ nucleus is as follows.

The eight protons (in the 82–126 shell) occupy the $N = 5$ proxy harmonic oscillator shell. It is obtained by replacing the $i_{13/2}$ intruder orbitals, with $h_{11/2}$ ones, after excluding the $[606]_{13/2}$ highest-lying one, as described above. The excluded orbital is not occupied in the ground-sate region. The highest weight SU(3) symmetry of this configuration is (26,4).

The eight neutrons (in the 126–184 shell) are distributed on the $N = 6$ proxy shell. (The excluded $[707]_{15/2}$ orbit does not play an important role.) The highest weight symmetry of the neutron sector is (34,4).

Therefore, the dominating SU(3) symmetries of the $0\hbar\omega$ subspace (in Table 2) are obtained as a product: (26,4)⊗(34,4).

Please, note that the ambiguity between the highest weight representation and the leading one (corresponding to the largest eigenvalue of the second order Casimir), as discussed in detail in Ref. [30], is not disturbing here. The reason is that in the lower half of the major shells (which are relevant in the present case) the two representations coincide.

When major shell excitations take place, they can be different kinds. Nucleons may jump from normal parity to normal parity subshells, as well as between unique parity orbitals, and mixed ones. This situation is similar to that of the pseudo-SU(3) case. Here, again one can assume that the excitations between the normal parity states dominate the spectrum. The shell model dynamics seems to prefer these excitations [23], but in fact it is a model assumption.

The leading $1\hbar\omega$ representations, when a proton or a neutron is excited to the next major shell, are (31,2), and (39,2), respectively. Thus, the SU(3) irreps are obtained as $(31, 2) \otimes (34, 4)$ and $(26, 4) \otimes (39, 2)$. Here, again the $1\hbar\omega$ irreps cannot be obtained from the $0\hbar\omega$ representations multiplied by the (1,0) center of mass motion, therefore, they are not contaminated by spurious excitations.

The leading $2\hbar\omega$ proton and neutron excitations are (36,0) and (44,0), respectively. When one proton and one neutron is excited: (31,2) and (39,2). Therefore, the SU(3) irreps are obtained as $(36, 0) \otimes (34, 4)$, $(26, 4) \otimes (44, 0)$, and $(31, 2) \otimes (39, 2)$. The multiplicity of some $2\hbar\omega$ irreps of Table II is decreased (in comparison with the result of the multiplications), due to $1\hbar\omega$ spurious excitations.

The $0\hbar\omega$ and $2\hbar\omega$ states have positive, while the $1\hbar\omega$ excitations have negative parities (if each of them is a normal-to-normal rearrangement).

TABLE II. The dominating irreps of the proxy-SU(3) symmetry-based algebraic quartet model of the ^{224}Th nucleus. The superscript indicates multiplicity.

$\hbar\omega$	(λ, μ)
0	$(60,8),(61,6),(62,4),(63,2),(64,0),(58,9),(59,7)^2,...$
1	$(65,6)^2, (66,4)^2, (67,2)^2, (63,7)^2, (64,5)^4, (65,3)^4,...$
2	$(70,4)^3, (68,5)^3, (69,3)^3, (66,6)^2, (67,4)^2, (68,2)^2,...$

IV. CALCULATION OF THE SPECTRUM

I apply a simple Hamiltonian, expressed in terms of the invariant operators of the U(3) ⊃ SU(3) ⊃ SO(3) algebra chain:

$$\hat{H} = (\hbar\omega)\hat{n} + a\big(\hat{C}^{(2)}_{SU3} + -0.004\hat{C}^{(3)}_{SU3}\big) + d\hat{L}^2. \quad (4)$$

The first term is the harmonic oscillator Hamiltonian [linear invariant of the U(3)] with a strength obtained from the systematics [31] $\hbar\omega = 6.73$ MeV. The last one is the rotational term with a parameter to fit. The remaining parts were written in terms of the second ($\hat{C}^{(2)}_{SU3}$) and third order ($\hat{C}^{(3)}_{SU3}$) invariant of the SU(3). The former one accounts for the quadrupole-quadrupole interaction, and the latter one distinguishes between the prolate and oblate shapes. This Hamiltonian turned out to be successful in describing the low-energy quartet spectra of light nuclei [10,13], and predicting from them the high-lying cluster spectra [13,14].

In the ^{224}Th nucleus two bands are known experimentally, as shown in the central part of Fig. 1 [32]. Their description in the pseudo-SU(3) and proxy-SU(3) schemes turns out to be fairly similar, with the sets of parameter $a = -0.0500$ MeV, $d = 0.00872$ MeV and $a = -0.148$ MeV, $d = 0.00872$ MeV, respectively.

The intraband $B(E2)$ value is given by [10,33]

$$B(E2, I_i \to I_f) = \frac{2I_f + 1}{2I_i + 1}\alpha^2 |\langle(\lambda, \mu)KI_i, (11)2||(\lambda, \mu)KI_f\rangle|^2 C^{(2)}(\lambda, \mu), \quad (5)$$

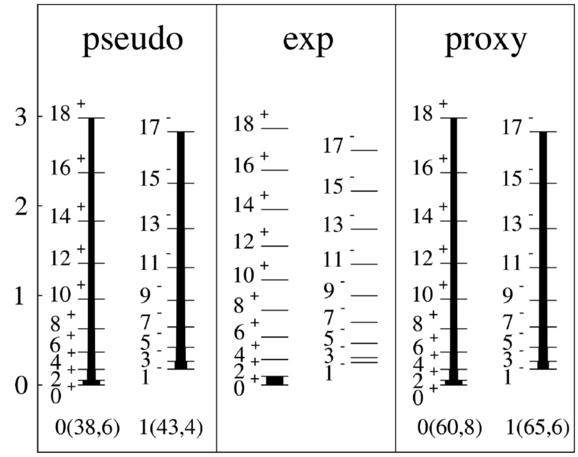

FIG. 1. Experimental bands in ^{224}Th and their description in the pseudo and proxy-SU(3) quartet models. The energy is measured in MeV, and the thickness of the arrows are proportional to the $E2$ transition rates.

where $\langle(\lambda,\mu)KI_i,(11)2||(\lambda,\mu)KI_f\rangle$ is the SU(3) $\supset$ SO(3) Wigner coefficient [28], and α is a parameter fitted to the the experimental value of the $2_1^+ \rightarrow 0_1^+$ transition of 96 W.u. [34]: α^2(pseudo)= 0.0536 W.u., α^2(proxy)= 0.0226 W.u. The interband transition rate is zero.

In order to see, how the other parts of the low-lying model spectra compare to each other the energies of the low-lying band heads were calculated. They are shown in Fig. 2. In particular, the pseudo-SU(3) spectrum was obtained with the parameters above, and the parameters $a = -0.05441$ MeV, $d = 0.00872$ MeV of the proxy-SU(3) Hamiltonian were fitted to the energies of the pseudo spectrum (in order to check how similar the two spectra can be).

It is seen, that though the two spectra are not identical, the difference is not very big. In particular, the spin-parity content of the bands are the same for the low-lying states, but their energy-distribution is somewhat different. This can be understood as a consequence of the similarity of the two symmetries. In both cases the spectrum is dominated by a few low-lying single-particle orbitals of two subsequent major shells for the protons and neutrons. In the ground state each orbitals is double occupied. In particular the ground-state shell model configuration is $(\tilde{4})_p^4\ (\tilde{5})_n^8$ in the pseudo-SU(3) model, and $(5)_p^8\ (6)_n^8$ in the proxy-SU(3) model. In spite of the considerable difference in the (λ and μ) quantum numbers of the pseudo and proxy schemes, the low-lying spectra are fairly similar.

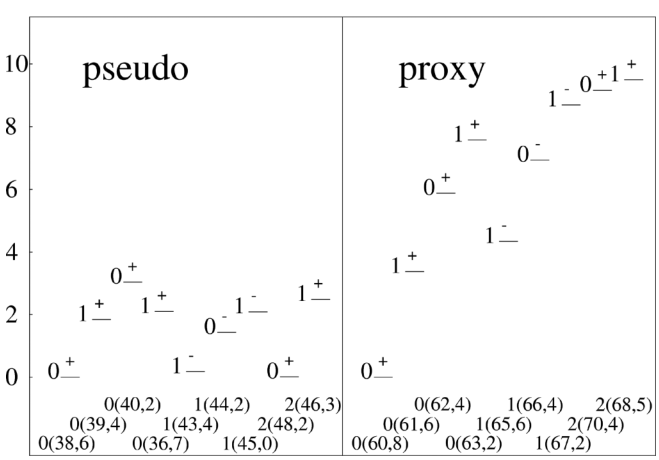

FIG. 2. Low-lying band heads of the ^{224}Th from the pseudo- and proxy-SU(3) quartet models.

The quadrupole deformation is more sensitive to the actual value of the symmetry quantum numbers. The pseudo-SU(3) symmetry of (38,6) corresponds to quadrupole deformation parameters [35] of $\beta = 0.11$, $\gamma = 7.2$ (deg), while the proxy-SU(3) symmetry (60,8) gives $\beta = 0.17$, $\gamma = 6.2$ (deg).

V. SUMMARY AND CONCLUSIONS

In this paper I have proposed two methods for the extension of the semimicroscopic algebraic quartet model [10] to heavy nuclei. This model describes the shell-like quarteting (of two protons and two neutrons in a well-defined shell configuration). In its original form it is based on the SU(3) symmetry [11], therefore, it is applicable to light nuclei. The generalization is carried out by applying the pseudo-SU(3) [18] and proxy-SU(3) [19] schemes.

Both of these symmetries are approximate ones, which are valid in a subspace of the full shell model space. The truncations they use are different, and so are the limitations of their applicability. The proxy scheme keeps more single particle orbitals to contribute to the SU(3) symmetry than the pseudo scheme, thus it results in larger dimension representations.

In the pseudo-SU(3) quartet model I applied similar model assumptions, like those of the pseudo symplectic model of the quadrupole collectivity [24]. The reason is that both models deal with multishell problem.

It seems that both the pseudo and the proxy-SU(3) symmetry is applicable for the description of shell-like quarteting of heavy nuclei. Their performance for the low-lying spectra of the ^{224}Th nucleus shows a considerable similarity. Such a similarity is expected to happen in many other cases, too. It is, however, obviously not valid in any mass region. To what extent the results of the two schemes are similar or different depends on the details of the shell model configurations, which are relevant for the low-lying spectrum.

Formulating the same statement from a different angle we can say that the low-lying energy spectrum is not necessarily sensitive to the difference between the two approximate SU(3) symmetry schemes. The quadrupole deformation parameters show bigger differences. And the most sensitive physical quantities may very well be those ones, which are related to the SU(3) selection rules, e.g., allowed and forbidden clusterizations, or in general: cluster spectroscopic factors.

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