Precise β branching-ratio measurement for the $0^+ \rightarrow 0^+$ superallowed decay of ³⁴Ar

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(Received 22 January 2020; accepted 18 March 2020; published 7 April 2020)

We have measured the branching ratio for the superallowed $0^+ \rightarrow 0^+ \beta$ transition from ³⁴Ar to be 0.9448(8), and determined its ft value to be 3058.1(28) s, a result with $\pm 0.09\%$ precision, which is a factor of 3 improvement over the previous result based on current world data. The ft-value ratio for the mirror pair of superallowed transitions ³⁴Ar \rightarrow ³⁴Cl and ³⁴Cl \rightarrow ³⁴S becomes the most precise yet measured and, in a sensitive test of the method used to calculate the isospin-symmetry-breaking correction, δ_C , it agrees well with the ratio as calculated with Woods-Saxon radial wave functions. This confirms the method used in the most recent survey of superallowed decays to extract V_{ud} , the up-down quark-mixing element of the Cabibbo-Kobayashi-Maskawa matrix. In addition, our branching-ratio results for the four observed Gamow-Teller branches to 1⁺ states in ³⁴Cl are shown to agree well with shell-model calculations based on the same effective interactions that were used in the determination of δ_C .

DOI: 10.1103/PhysRevC.101.045501

I. INTRODUCTION

Superallowed β decay between $J^{\pi} = 0^+$, T = 1 analog states has long been used to probe the universality of the weak interaction. Currently, an ensemble of 14 such transitions is the source of the most precise value for V_{ud} , the up-down quark-mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and a key ingredient in the most demanding test of that matrix's unitarity [1]. Uncertainties are all-important here since any deviation from unitarity would be a signal for new physics beyond the standard model. The *ft* values for these transitions have all been measured with high precision (± 0.01 –0.3%), their corrected $\mathcal{F}t$ values determined to be consistent with one another, and the implied value of V_{ud} established with $\pm 0.02\%$ precision.

As tiny as this uncertainty is, the experimental contribution to it is dwarfed by the contribution from theory. There are four small theoretical corrections—all of order 1%—that are required to convert ft into $\mathcal{F}t$, and then to extract V_{ud} from an average of all the $\mathcal{F}t$ values. Because these are by far the dominant contributors to the V_{ud} uncertainty, subsequent measurements of $0^+ \rightarrow 0^+$ decays can only contribute usefully to the test of CKM unitarity if they can improve the precision of the *theoretical* correction terms, for example by ruling out whole classes of models used to calculate the two sets of nuclear-structure-dependent correction terms (see Refs. [2,3]).

The superallowed $0^+ \rightarrow 0^+ \beta$ -decay branch from ³⁴Ar is one of the transitions already being used in the determination of $V_{\rm ud}$. However, its contribution has been minimal since its $\pm 0.3\%$ uncertainty marked it as being the least precise of all 14 contributing transitions [1]. This is particularly unfortunate, since the ³⁴Ar superallowed decay also can play a critical role in distinguishing among the various models that have been used to calculate the structure-dependent correction terms required to convert a measured *ft* value into a corrected $\mathcal{F}t$ value. This is because it is one of the relatively few measured superallowed transitions that have a mirror decay partner; in this case, the mirror partner to the ³⁴Ar \rightarrow ³⁴Cl decay is ³⁴Cl \rightarrow ³⁴S. It turns out that the ratio of *ft* values for such mirror pairs is a particularly sensitive test of the model used to calculate the structure-dependent corrections [2].

The *ft* value for the ${}^{34}\text{Cl} \rightarrow {}^{34}\text{S}$ superallowed transition is known to $\pm 0.03\%$ but, with the experimental uncertainty for ${}^{34}\text{Ar} \rightarrow {}^{34}\text{Cl}$ being ten times worse, the *ft* ratio for the *A* = 34 mirror pair was of no use in this test of the corrections. We report here a measurement of the superallowed branching ratio for ${}^{34}\text{Ar}$, which leads to a factor of 3 reduction in the corresponding *ft*-value uncertainty and produces an *ft*-value ratio that can now make a definitive choice among theoretical model options.

II. EXPERIMENT

We produced pure ³⁴Ar samples using a 30A-MeV ³⁵Cl beam from the K500 superconducting cyclotron at Texas A&M University to initiate the reaction $p(^{35}Cl, 2n)^{34}Ar$. The target was liquid-nitrogen-cooled hydrogen contained at 2.0-atm pressure in a thin-walled gas cell, which was located in the target chamber of the Momentum Achromat Recoil Spectrometer (MARS) [4]. The fully stripped reaction ejectiles were spatially separated in MARS by their charge-to-mass ratio, q/m, leaving a >99% pure ³⁴Ar beam to emerge from the focal-plane extraction slits. After exiting the vacuum

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FIG. 1. (a) The deposited energy vs position as obtained with the PSSD in the MARS focal plane. The spectrometer had already been optimized for ³⁴Ar production and the extraction slits (vertical dotted lines) had been set. All the impurity-isotope labels are placed where the peaks of the respective activities had been seen with the slits wide open. (b) The projection of total counts between the slits. From this and subsequent spectra recorded during our measurement, we established that the contribution of ³²Cl and ³¹S to the extracted ³⁴Ar beam was 0.10(5)% and 0.43(14)%, respectively.

system through a 51- μ m-thick Kapton window, this beam then passed through a 0.3-mm-thick BC-404 scintillator and a stack of aluminum degraders, before finally stopping in the 76- μ m-thick aluminized Mylar tape of a fast tape-transport system.

After the primary ³⁵Cl beam from the cyclotron had been fully tuned, we optimized the secondary ³⁴Ar beam purity through MARS before the measurement began. First, we inserted an attenuating grid into the cyclotron injection line to reduce the primary beam intensity, allowing us to place a 1mm-thick 16-strip position-sensitive silicon detector (PSSD) at the MARS focal plane. Then, with the low-current primary beam, we focused the ³⁴Ar beam, identified nearby reaction products and minimized those that could affect the purity of the beam. With optimization completed and the focal-plane acceptance slits set, we obtained the result shown in Fig. 1. As is clear from the figure, there are only two residual impurities of any significance, ³²Cl and ³¹S, both very weak: 0.10(5)% and 0.43(14)%, respectively, relative to ³⁴Ar. There are also some weak lighter-mass impurities cut off from the figure but they all have a much longer range than the ³⁴Ar, and pass entirely through the collection tape. None of these play any role in the measurement.

Following the optimization of MARS, the PSSD was removed from the beam path and the attenuating grid was withdrawn from the injection line, restoring the full primary beam intensity without any change in cyclotron or beam-line parameters. The measurement then began. We rechecked the composition of the beam daily during our measurement and again after it was completed by reinserting the attenuating grid and the PSSD, and recording the spectrum of deposited energy versus position each time. Small variations were observed and corrected from time to time; they were subsequently incorporated into our detailed off-line analysis. Data-taking was in repetitive cycles. First, ³⁴Ar was collected in the tape for 2.0 s, its rate of accumulation being measured by the BC-404 scintillator located at the exit of MARS. Then the beam was interrupted at the cyclotron, and the tape-transport system was activated to move the sample in 230 ms to a shielded counting location 90 cm away, where decay data were acquired for 1.93 s. With counting completed, the beam was restored and the cycle repeated. To obtain adequate statistics, this clock-controlled sequence of collect-move-count was repeated over 70 000 times, with the results collected into nearly 120 separate "runs," averaging ~600 cycles per run.

At the counting location were two detectors located on opposite sides of the tape. One, situated 3 mm away, was a 1-mm-thick BC-404 scintillator used to detect β^+ particles. Opposite it, at a nominal distance of 151 mm from the tape, was our high-purity germanium (HPGe) γ -ray detector, whose efficiency has been very precisely calibrated. The distance between the stopped tape and the HPGe detector was measured during the counting period of each cycle with a laser-ranging device [5] mounted next to the HPGe detector. The result, which was accurate to 30 μ m, was recorded with the data for that cycle. The measured distances were quite consistent from cycle to cycle, with the full width at half maximum (FWHM) of their distribution being 0.4 mm, with a centroid at 151.17 mm. Because our HPGe detector had been calibrated for efficiency at a source-detector distance of exactly 151.0 mm, we used the laser result to adjust the calibrated detector efficiency to correspond with the actual average source-detector distance. The difference being only 0.17 mm (0.1%), the adjustment was very small.

During the counting period of every cycle, our dataacquisition system generated a "master trigger" whenever a β particle and a γ ray were detected within $\sim 2 \mu s$ of one another. This signaled the occurrence of a β - γ coincident event and initiated acquisition. For each such event, we recorded the detected energy of both the β and γ rays, the precise time difference between their arrivals, and the time that the event itself occurred relative to the beginning of the counting period. For each cycle we also recorded the total number of β - and γ -ray singles, as well as the laser distance reading. The same discriminator signals used to scale the β singles were also used in creating the master triggers and establishing the occurrence of β - γ coincidences. Electronic dead times for the coincidence channel and the two singles channels were measured continuously throughout the measurement with pulser signals from a precise 1-MHz pulse generator being recorded in coincidence with the gating signals from each channel. Finally, we recorded with each cycle the rate of accumulation of ³⁴Ar ions in the tape as a function of time during the corresponding collection period.

We measured room background during the experiment to establish its contribution both to the β - γ coincidence spectrum and to the β -singles rate. To do this we used measurement cycles that were normal in every way except that the tape motion was disabled, so that the collected sample never reached the counting location. Under these conditions, essentially no β - γ coincidences were observed, and the β -singles rate dropped to 0.012% of the rate observed under



FIG. 2. Implantation profiles of 34 Ar (solid line) and the two contaminant activities 31 S and 32 Cl (dashed lines). The beam enters from the left. The shaded region shows the actual thickness of the Mylar collection tape. Our collected sample contains only those ions that are stopped in the tape.

normal conditions. Though very low, this room-background rate for β singles was incorporated into our analysis.

It is important to establish the role played by impurities in the collected samples. The two impurities identified in Fig. 1, ³²Cl and ³¹S, have similar ranges to ³⁴Ar and, although weak, could be of concern. As in our previous published measurements (see, for example [6]), we obtained implantation profiles based both on calculations with the SRIM code [7] and on the depth distribution we determined for ${}^{34}Ar$ by measuring its collected activity as a function of degrader thickness. The result is given in Fig. 2 where the relative intensities have been chosen to reflect the measured PSSD intensities. Evidently all the ³¹S ions stop in the tape and, with a half-life of 2.6 s, they must contribute to the β 's detected during the counting period. As for ³²Cl, only half of it stops in the tape but, more importantly, it has a half-life of 298 ms: Most of it will have decayed away long before the count time begins. Ultimately, the samples present in the tape were 99.6% pure ³⁴Ar, with ³¹S being the only perceptible impurity.

III. ANALYSIS

The β -decay scheme for ³⁴Ar is shown in Fig. 3. One branch is dominant: the strong superallowed transition that directly feeds the ground state of ³⁴Cl, which subsequently decays by another superallowed β transition to the ground state of ³⁴S. No γ rays whatsoever are emitted in this decay path. In contrast, all the allowed Gamow-Teller transitions from ³⁴Ar to excited 1⁺ states are followed by prompt γ rays, predominantly emitted in each case directly to the ground state of ³⁴Cl.

It is these latter transitions whose absolute intensity we can measure from the β -coincident γ -ray spectrum. The total of the Gamow-Teller branching ratios can then be subtracted from 100% to yield the superallowed branching ratio. While the relative weakness of these transitions—they total only 5.5%—has a negative impact on the counting statistics that can be obtained in a measurement, it actually works to our



FIG. 3. Beta-decay scheme of ³⁴Ar, showing the five observed β -delayed γ -ray transitions in ³⁴Cl. Each energy level is labeled with its (J^{π}, T) as well as its energy, expressed in keV, relative to the ³⁴Cl ground state. Basic content is from the most recent evaluation [8]; however, the branching percentages and the presence of a γ -ray transition between the 666- and 146-keV levels come from this measurement.

advantage in another way. The percentage uncertainty on the measured Gamow-Teller intensity is reduced by a multiplicative factor of 0.06 (=5.5/94.5) when it is applied to the superallowed branch. A measurement precision of $\pm 1.4\%$ for the Gamow-Teller branches leads to a $\pm 0.08\%$ result for the superallowed branch.

To determine precisely the branching ratio for the superallowed transition from ³⁴Ar, our first step was to establish the β -branching ratio to the 666-keV 1⁺ state in ³⁴Cl, the most intense branch observed. To do so, we determined the number of β -coincident 666-keV γ rays relative to the total number of positrons emitted from ³⁴Ar. Then, we used the relative intensities of all the other (weaker) observed γ -ray peaks to establish the total Gamow-Teller β branching to all 1⁺ states. Finally, the subtraction of this total from 100% resulted in the branching ratio for the superallowed transition.

To be more specific about our first step, we write the β^+ -branching ratio R_i for a pure β^+ transition populating the particular state *i*, which is deexcited by the emission of a single γ ray γ_i as follows:

$$R_i = \frac{N_{\beta\gamma_i}}{N_\beta \epsilon_{\gamma_i}} \frac{\epsilon_\beta}{\epsilon_{\beta_i}},\tag{1}$$

where $N_{\beta\gamma_i}$ is the total number of β - γ coincidences in the γ_i peak; N_{β} is the total number of beta singles corresponding to ³⁴Ar β decay; ϵ_{γ_i} is the efficiency of the HPGe detector for detecting γ_i rays; ϵ_{β_i} is the efficiency of the plastic scintillator for detecting the betas that populate state *i*; and ϵ_{β} is the average efficiency for detecting the betas from all ³⁴Ar transitions. Note that this equation only accounts for pure positron emission so, even though the contribution from electron capture for A = 34 is very small, it must be separately accounted for. Furthermore, another small correction must be applied to incorporate the effect of the weak γ transition

between the the 666- and 146-keV states in 34 Cl. Both these adjustments will be dealt with in Sec. IV.

In the immediately following sections, after describing some initial processing of the experimental data, we explain how all the factors on the right-hand side of Eq. (1) were obtained specifically for the β transition to the 666-keV state. Our approach is the same as the one we followed in previous measurements of the superallowed branching ratios from ²⁶Si [9] and ³⁸Ca [10] so we will be brief. The reader is referred to those earlier publications for more complete details.

A. Cycle selection

Before analysis began, we filtered our accumulated data by rejecting cycles that did not meet certain criteria. The first criterion we applied was the number of implanted ³⁴Ar ions detected by the BC-404 scintillator at the exit of MARS during each collection period. We rejected all cycles that had fewer than 1000 ions recorded, an indication of very little—or no—primary beam from the cyclotron. This removed $\sim 3\%$ of the total cycles.

Our second criterion was based on the ratio of the number of β particles detected to the number of ³⁴Ar ions implanted for each cycle. We restricted this ratio for a given cycle to between 95% and 100% of the maximum value obtained for the whole run containing that cycle; this ensured that the tape-transport system had moved the ³⁴Ar sample into the designated counting position between the β detector and the HPGe detector. A serious deviation from this position could affect the effective efficiency of the HPGe detector for that cycle, so our selected limits were very conservative, and removed ~34% of the cycles.

Our third criterion was a limit set on the reading from our laser measurement of the distance between the tape and the HPGe detector. We accepted cycles only if the distance was within ± 0.34 mm of the central value. This removed only $\sim 3\%$ of the remaining cycles.

In the end, our selection criteria provided 41 466 good cycles, ~59% of the total cycles recorded. These good cycles contained approximately $1.4 \times 10^7 \beta - \gamma$ coincidences, corresponding to over $5.8 \times 10^8 \beta$ singles. All subsequent analysis incorporates only the data from these cycles.

B. Eliminating random coincidences

For each event, we recorded the time between the detection of a γ -ray and the subsequent arrival of an electronically delayed signal from the positron detector. The corresponding time spectrum for all identified events appears in the top panel of Fig. 4, in which the broad peak represents real (i.e., "prompt") coincident events, while the flat distributions to the left and right are from random coincidences. This time spectrum allowed us to produce a γ -ray spectrum free from random-coincidence events. We gated first on the part of the time spectrum that contains the prompt peak and then gated on the flat, random parts of the spectrum. The γ -ray spectrum obtained with the second gate, suitably normalized, was then subtracted from the spectrum obtained with the first gate. The result appears as Fig. 5. In addition to the annihilation



FIG. 4. (a) Spectrum of measured time differences between the arrival of a γ ray and that of a β particle for all identified coincidence events. Note that the β signal was electronically delayed so that the prompt-coincidence peak appears near the center of the spectrum. (b) Measured time-difference spectrum for events corresponding only to the 666-keV γ ray.

radiation, the spectrum exhibits four clear full-energy γ -ray peaks from the decay of ³⁴Ar, at 461, 666, 2580, and 3129 keV, as well as escape peaks and a weak peak from coincident summing of two annihilation photons, one backscattered. No other peaks are immediately visible.

It is evident in the top panel of Fig. 4 that the prompt peak there has a noticeable tail to the left. This is because it incorporates all coincident events, covering a wide range of γ -ray energies. Low-energy γ rays trigger the TDC later than higher energy ones, and thus lead to a tail towards shorter times. The bottom panel of Fig. 4 shows the time spectrum corresponding to the single γ -ray peak at 666 keV. The prompt peak in this case is much narrower (FWHM <10 ns) and has only a weak tail. It is spectra like this, each restricted to a narrow energy window around a single γ -ray peak, that we used in our final analyses for the contents of individual peaks.

C. Efficiency calibrations

From Eq. (1) it is evident that our determination of the superallowed branching ratio relies critically on a precise absolute efficiency for the γ -ray detector ϵ_{γ_i} and equally precise relative efficiencies for the β detector $\epsilon_{\beta}/\epsilon_{\beta_i}$.

Our HPGe detector has been meticulously calibrated at a source-to-detector distance of 151 mm. This was reported thoroughly more than a decade ago [11,12]. Initially [11], we



FIG. 5. Spectrum of γ rays observed in prompt coincidence with positrons from the decay of ³⁴Ar. The principal peaks are labeled with their energy in bold type and, where present, their corresponding single- and double-escape peaks are denoted by "SE" and "DE" subscripts, respectively. The small peak labeled "511 + 171" is due to the sum of two annihilation photons, one of which has backscattered into the detector.

used 10 different radionuclides with accurately known relative photon emission rates together with two ⁶⁰Co sources specially prepared by the Physikalisch-Technische Bundesanstalt [13], having activities certified to $\pm 0.06\%$. The ⁶⁰Co sources were used to anchor our absolute efficiency calibration, with cascaded γ -ray transitions from the other sources providing precise links covering an energy range from 22 to 1836 keV.

We also made a number of measurements designed to reveal the physical dimensions and location of the detector's Ge crystal in its housing. This information was then used as input to Monte Carlo calculations performed with the electron and photon transport code CYLTRAN [14]. With only the thicknesses of the detector's two dead layers as adjustable parameters we achieved excellent agreement ($\chi^2/N = 0.8$) between the Monte Carlo efficiency results and our 40 measured data points. A year later, with three additional sources we extended our region of calibration up to 3.5 MeV [12].

Ever since these calibrations were made, we have kept the detector continuously at liquid-nitrogen temperature to ensure that the internal dead layer did not expand, and we have also periodically remeasured one of the precisely calibrated ⁶⁰Co sources. No change in detector efficiency has been detected, so we continue to use CYLTRAN calculations to obtain our detector efficiency with $\pm 0.2\%$ uncertainty in the range 50–1400 keV, and with $\pm 0.4\%$ from 1400 to 3500 keV.

The third column of Table I gives the efficiencies for the four main γ rays of interest as well as for a fifth weak γ ray at 519 keV, which will be discussed later.

Our β detector is a 1-mm-thick Bicron BC-404 scintillator disk optically coupled to a cylindrical Lucite light guide, which is coupled in turn to a photomultiplier tube. Its response function has been extensively characterized as a function of β particle energy by a combination of GEANT4 [15] Monte Carlo simulations and measurements with ¹³³Ba, ¹³⁷Cs, and ²⁰⁷Bi sources, all three of which emit conversion electrons, and one, ¹³⁷Cs, emits β -decay electrons. The agreement between

TABLE I. Detector efficiencies are given for the γ rays γ_i that deexcite states E_{x_i} in ³⁴Cl and for β particles emitted in the decay branches β_i that populate states E_{x_i} . Calculated ratios of electron capture to positron emission (ec/ β^+) also appear for each decay branch. The values for ϵ_{γ_i} apply to our HPGe detector, and the ratios $\epsilon_{\beta}/\epsilon_{\beta_i}$ to our thin β scintillator.

$E_{x_i}^{a}$ in ³⁴ Cl (keV)	$E_{\gamma_i}^{a}$ for γ decay (keV)	$\epsilon_{\gamma_i} \ (\%)$	$E_{\beta \max}$ for β_i feeding (keV)	$\epsilon_{eta}/\epsilon_{eta_i}$	ec/β^+
0.0			5039.9	0.9996	0.0007
461.0	461.0	0.4165(8)	4578.9	0.9993	0.0009
665.6	519.2	0.3801(8)	4374.3	0.9989	0.0010
665.6	665.6	0.3200(6)	4374.3	0.9989	0.0010
2580.4	2580.4	0.1179(5)	2459.5	1.0153	0.0058
3129.1	3129.1	0.0989(4)	1910.8	1.0266	0.0124

^aValues taken from Ref. [8].

measurements and simulations was found to be excellent [16]. Since those studies were completed ten years ago, we have extended our source tests to ²²Na, with similar success; like ³⁴Ar, ²²Na is a positron emitter. We have also demonstrated that the EGSnrc Monte Carlo code [17] produces equally good agreement with measurements and runs faster than GEANT4, so we have used the former code in the present analysis.

As Eq. (1) makes clear, it is not the absolute efficiency of our β detector that is required, but rather how the efficiency changes as a function of the end-point energy, $E_{\beta max}$, which naturally is different for each β -decay branch feeding a state in ³⁴Cl. The energy dependence of our β -detection efficiency is caused principally by the fixed low-energy electronic threshold, which removes a slightly different fraction of the total β spectrum for different end-point energies. Since this affects the measured intensity of coincident γ rays following a β transition, it must be accurately accounted for even though the effect is very small.

Figure 6 presents the β -detector energy spectrum measured in coincidence with 666-keV γ rays. It singles out the β



FIG. 6. The measured energy deposition (dots with error bars) in the β detector for the decay of ³⁴Ar to the 666-keV state in ³⁴Cl is compared with the EGSnrc-simulated spectrum (solid line). The dashed vertical line at 50 keV indicates our electronic threshold. The difference between the simulated and measured spectra below 50 keV indicates the events lost because of the threshold.

transition from ³⁴Ar feeding the 666-keV state in ³⁴Cl and is compared in the figure with a Monte Carlo spectrum for the same transition, generated with EGSnrc. In the simulation we have included the transport tape, together with the rest of the nearby counting-location geometry. Clearly there is excellent agreement between the simulated and measured spectra above the electronic threshold, which gives us confidence that we can use the simulations to obtain reliable efficiency ratios, $\epsilon_{\beta}/\epsilon_{\beta_i}$, for the transitions of interest.

The fifth column of Table I lists our calculated results for the transitions of interest based on our electronic threshold being set at 50 keV. They appear without uncertainties since all are quite near unity and have uncertainties that are negligible in the present context. The calculated absolute β efficiency for the total of all decay branches from ³⁴Ar, ϵ_{β} , is approximately 45%. Its precise value is not required.

D. Beta singles

The N_{β} term in Eq. (1) refers to the total number of beta particles emitted in the decay of ³⁴Ar. The number we actually record from the β detector includes not only the β 's from ³⁴Ar but also those from its daughter ³⁴Cl. In addition there are very weak contributions from β particles emitted in the decay of the ³¹S impurity, and from γ rays in the ³⁴Ar decay chain registering in the β detector. We deal with these contributions individually.

1. Impurity

In Sec. II we established that the only impurity present in the tape during counting periods was ³¹S; and determined its average collection rate to be 0.43(14)% that of ³⁴Ar. Since ³¹S decays ~99% to the ground state of ³¹P, its contribution to the β -coincident γ -ray spectrum of Fig. 5 would be negligible. Not so for the β -singles count, N_{β} . Taking account of the half-lives of ³¹S, ³⁴Ar, and ³⁴Cl, we calculate that the eventual contribution of ³¹S to the total number of β particles recorded during the counting period must have been 0.18(6)%.

2. Gamma rays registering in the β detector

Gamma rays produced in the decay of ³⁴Ar have a very small probability of being counted in the β scintillator. For the strongest, 511-keV γ rays, this is irrelevant since annihilation radiation can be thought of as a surrogate for a β particle; in that case its presence would not alter the efficacy of Eq. (1). However it could be relevant for the transition γ rays in cases where they are detected but the corresponding β particle that feeds the transition is not. Using EGSnrc Monte Carlo simulations, we determined that 0.017(2)% of the total counts in the β detector are γ rays of this type. Although it has an essentially negligible effect, for completeness we reduced the recorded number of counts in the detector by this factor.

3. Parent β -decay fraction

Of much more significance is the contribution to the measured β singles from the decay of ³⁴Cl, the daughter of ³⁴Ar. This nuclide is not at all present in the implanted beam, but it naturally grows in the collected sample as ³⁴Ar decays. Fortunately, the half-lives of both parent and daughter are well



FIG. 7. Typical arrival-time spectrum of the collected ³⁴Ar ions measured over the course of one run. The initial drop in intensity is generated by the decrease in local density of the hydrogen in the target cell as the primary beam heats the gas around its path. A fan located inside the gas target mitigates the effect and ensures a rapid transition to stable conditions.

known, ³⁴Ar being 846.46(35) ms [18] and ³⁴Cl, 1526.55(44) ms [1]. Consequently, the ratio of their activities can easily be calculated if the ³⁴Ar implantation rate is known as a function of time during the collection period. To enable this calculation, we recorded the arrival-time spectrum of ions detected by the scintillator at the exit of MARS for each individual cycle. A typical result for a single run appears as Fig. 7.

For us to extract the activity ratio for the singles events that we actually recorded in our β detector, two other factors must be considered. The first is our relative detection efficiency. Although the $E_{\beta max}$ for ³⁴Cl decay is less than it is for the superallowed transition from ³⁴Ar, the other lower-energy branches from ³⁴Ar act to offset the efficiency difference that would otherwise be expected (see discussion in Sec. III C). As a result, the total efficiency for observing β 's from ³⁴Cl fortuitously turns out to be exactly the same as it is for observing them from ³⁴Ar.

The second factor to consider is the effect of the 519-keV γ -ray transition, which feeds the 32-min isomeric state at 146 keV in ³⁴Cl, and thus steals a tiny amount, 0.036(5)%, of the ³⁴Ar strength away from the ³⁴Cl ground-state decay. In effect, this reduces the efficiency for detecting ³⁴Cl β 's by the same amount.

We are now in a position to calculate what fraction of the true $A = 34 \beta$ -decay events recorded in our detector is due to ³⁴Ar decay. Using the measured arrival-time spectrum of the ³⁴Ar ions (see Fig. 7), together with the known half-lives of ³⁴Ar and its daughter ³⁴Cl, we calculated the total number of decays of each, integrated over the precisely delineated counting period. The fraction attributable to ³⁴Ar was thus found to be 0.4636(2). This includes the effect of the 519-keV transition.

4. Final result for N_{β}

The steps required to obtain N_{β} appear quantitatively in Table II. Starting from the total counts recorded in our β detector, we first remove room background, and then correct

TABLE II. Derivation of N_{β} from the total number of singles events recorded in the β detector.

Quantity	Value	Source
Total β -detector counts	$5.77359(24) \times 10^{8}$	
Background	$-6.803(26) \times 10^{4}$	Sec. II
β decay of impurities	-0.18(6)%	Sec. III D 1
Detected γ rays	-0.017(2)%	Sec. III D 3
³⁴ Ar fraction of β 's	$\times 0.4636(2)^{a}$	Sec. III D 3
$N_{\beta}(^{34}\mathrm{Ar})$	$2.6710(20) \times 10^{8}$	

^aCalculation takes account of the weak 519-keV γ -decay branch from the 666-keV state to the isomeric state at 146 keV, which does not feed the superallowed decay of the ³⁴Cl ground state.

for β -decay events from impurities and for γ rays counted in the β detector. Finally we apply the calculated ³⁴Ar β fraction. The final result for N_{β} appears on the bottom line of the table.

E. β -coincident 666-keV γ rays

The starting point for us to obtain $N_{\beta\gamma_{666}}$, the number of β coincident 666-keV gamma rays, is the integrated area of the 666-keV γ -ray peak recorded in coincidence with the prompt peak in the γ - β time spectrum; see Fig. 4 and the description in Sec. III B. This peak area, like all others needed for this measurement, we determined using a modified version of GF3, the least-squares peak-fitting program in the RADWARE series [19]. A combined Gaussian and skewed Gaussian peak with a smoothed step function and a linear background in the peak region were sufficient to properly describe the detailed shape of all peaks of interest in the spectrum of Fig. 5. This was the same fitting procedure as was used in the original detector-efficiency calibration [11,12]. The number of counts we obtained in this way for the 666-keV peak appears in the top row of Table III.

Before this result can be used in Eq. (1), there are several small corrections that must be applied to account for coincidence summing, dead time and pileup. These corrections are outlined in the following sections.

1. Coincidence summing

The 666-keV state in ³⁴Cl is fed by a positron decay branch from ³⁴Ar. For the γ ray from its subsequent decay to be recorded in our measurement, the positron must have

TABLE III. Derivation of $N_{\beta\gamma_{666}}$ from the total number of events in the 666-keV peak in the β -coincident γ -ray spectrum.

Quantity	Value	Source
Area of 666-keV peak	20 224(171)	
511-keV summing	+581(145)	Sec. III E 1
Bremsstrahlung summing	+0.21(3)%	Sec. III E 1
Dead time/pileup	+1.43(10)%	Sec. III E 2
Random preemption	+0.35(2)%	Sec. III E 3
$N_{eta\gamma_{666}}$	21 220(235)	

appeared in our β detector in coincidence with the γ ray in our HPGe detector. Because the positron generally annihilates in or near the β detector, there is a non-negligible probability that one of the resulting 511-keV photons will also be recorded in the HPGe detector, and will sum with the 666-keV γ ray, thus removing some of the latter events from the full-energy peak. The resultant sum peak at 1177 keV (666 + 511) is too small to be visible in the β -coincident γ -ray spectrum shown in Fig. 5, but it is identifiable as a peak above background and we could determine its area, albeit with a rather large uncertainty.

The sum peak accounts only for a portion of the total events lost from the peak at 666 keV. To determine the total losses, we must incorporate the complete 511-keV response function, since losses from the 666-keV peak also result from summing with signals from 511-keV photons that Compton scatter in the HPGe crystal and deposit less than their full energy. This requires the ratio of the total efficiency of our detector to its full-energy-peak efficiency—the total-to-peak ratio—for 511-keV photons, a ratio we have already determined for our experimental conditions using a ⁶⁸Ge source [10]. After a small adjustment to incorporate the effects of annihilation in flight that apply specifically to ³⁴Ar decay, we determine the ratio to be 3.63(3).

This total-to-peak ratio multiplied by the area of the 1177-keV sum peak establishes the total losses from the 666-keV peak due to summing with annihilation radiation to be 581(145) counts. This result appears as a correction to the 666-keV peak area in Table III.

External bremsstrahlung, emitted when positrons from ³⁴Ar slow down in the β detector or its surroundings, is another source of coincidence summing, but one that does not leave a characteristic sum peak. It results instead in a continuous energy spectrum indistinguishable from the summed Compton distributions resulting from detected γ rays. To arrive at the total contribution from bremsstrahlung in the spectrum of Fig. 5, we first took the areas of all γ -ray peaks, including the 511-keV peak, multiplied each by its corresponding totalto-peak ratio, summed the results, and subtracted the sum from the total number of counts in the spectrum. We took this difference to be the contribution from bremsstrahlung. Combining this number with the full-energy-peak efficiency of our detector for 666-keV γ rays, we could calculate the probability for coincidence summing between those γ rays and the bremsstrahlung. We determined the resultant loss from the 666-keV peak to be 42(6) counts or 0.21(3)% of the total. This amount appears as a small applied correction in Table III.

An alternative approach is to determine the total coincidence summing—annihilation plus bremsstrahlung—in a single step. In this approach, only the true γ -ray peaks (i.e., excluding the annihilation radiation), multiplied by their respective total-to-peak ratios, are subtracted from the total number of counts in the coincident γ -ray spectrum. The difference, when divided by the area of the 511-keV peak, yields a redefined "total-to-peak" ratio for that peak, which now includes in the "total" the effects from annihilation in flight and bremsstrahlung, in addition to Compton scattering. Not surprisingly, this leads to a correction that is statistically consistent with the sum of the two correction terms—511-keV summing, and bremsstrahlung summing—that are given individually in Table III.

2. Dead time and pileup

Dead time in the β -detection system is only 450 ns per event, and since it affects both the numerator and denominator in Eq. (1) equally, there is no need to consider it any further. However, dead time and pileup do affect the much slower signals from the HPGe detector, and their impact depends not only on the rate of coincident γ rays, which averaged 190 counts/s, but also on the singles γ rate, which averaged 685 counts/s. Furthermore, the rate during each cycle naturally decreased with time.

The dead time per event for encoded coincident γ rays was measured online to be 25.6 μ s, a value that encompasses the pileup time. For singles γ rays, which were not encoded, pileup is the dominant effect, with its time determined from the signal pulse shape to be 17 μ s per event. Since both dead time and pileup remove legitimate signals, we treat them together. We calculated the total losses from both sources by integrating over the whole counting period, incorporating the decrease in rate caused by the decay of ³⁴Ar, and the growth and decay of ³⁴Cl. Our result, appearing in Table III, is that losses due to the combination of dead time and pileup amount to 1.43(10)%.

3. Random preemption of real coincidences

There is a small probability that coincidences get lost as a result of a random coincidence preempting a real one. This can occur if a master trigger is generated by a real β - γ coincidence, which starts our timing clock (the TDC), but a random β event stops the clock before the true coincident β does. This effect can easily be calculated from the known rate of β signals and the time between the clock start and the appearance of the prompt peak; see Fig. 4(b). We calculated the loss from this effect to be 0.35(2)%, the amount shown in Table III.

4. Final result for $N_{\beta\gamma_{666}}$

All the corrections to the measured area of the 666-keV peak listed in Table III are additive, since they account for various identified losses. We first add back the counts lost to coincidence summing, then correct for losses due to dead time, pileup, and random preemption of true coincidence events. The resultant value for $N_{\beta\gamma_{666}}$ appears in the last row of the table. It is the last piece of input data required to complete the right-hand side of Eq. (1).

It has already been noted, though, that Eq. (1) only yields the β branching ratio under very restrictive conditions. In fact, in our case it yields the β -branching ratio for production of 666-keV γ rays. This is only equal to the β -branching ratio to the 666-keV level if that state is solely populated by β decay and is depopulated by a single γ transition. This is almost, but not entirely, true. As described in the next section, there are other weak γ transitions that must be considered and, if necessary, accounted for before the true β -branching ratio to this state, or any of the other 1⁺ states, can be established.



FIG. 8. Partial level scheme of ³⁴Cl, showing the excited states populated by the β decay of ³⁴Ar and the γ transitions that occur or may occur following the β decay. The four transitions shown with solid lines are the strongest ones; they correspond to the γ -ray peaks identified in Fig. 5. The weak 519-keV transition, shown dashed, is masked by the tail of the 511-keV peak in the spectrum in Fig. 5 but its strength has been determined. All five of these transitions appear in the decay scheme in Fig. 3. The dotted lines indicate possible weaker transitions, for which we set only upper limits.

F. Relative γ -ray intensities

In Fig. 5 we have identified four prominent γ -ray peaks from the decay of ³⁴Ar. The levels in the daughter, ³⁴Cl, are well known and, as illustrated in Fig. 3, the observed γ rays deexcite four 1⁺ states directly to the ground state. These being the only 1⁺ states known in this energy region [8], we would not anticipate the appearance of any other allowed Gamow-Teller β transitions, and forbidden transitions would be too weak to be relevant. However, the four states that are populated by allowed β decay do have other γ -decay options that are energetically available to them. These are explicitly identified in Fig. 8.

The energy response of our HPGe detector was determined before our experiment began with a 152 Eu calibration source, and reinforced by the well known energies of the four principal γ rays observed during the experiment. With this

TABLE IV. Relative intensities of β -delayed γ rays from the β^+ decay of ³⁴Ar.

			I_{γ}	
E_{γ} (keV)	Ref. [20]	Ref. [21]	This work	Adopted
205		< 0.010	< 0.013	< 0.010
315		< 0.0018	< 0.0086	< 0.0018
461	0.365(36)		0.353(9)	0.354(9)
519		< 0.010	0.029(10)	0.0146(19) ^a
549			< 0.0045	< 0.0045
666	1	1	1	1
693		< 0.010	< 0.0026	< 0.0026
948		< 0.015	< 0.0033	< 0.0033
971		< 0.015	< 0.0034	< 0.0034
1242		< 0.010	< 0.0056	< 0.0056
1350		< 0.0034	< 0.0041	< 0.0034
1899		< 0.010	< 0.0055	< 0.0055
1915		< 0.0068	< 0.0032	< 0.0032
2119		< 0.0068	< 0.0035	< 0.0035
2434		< 0.0068	< 0.0026	< 0.0026
2464		< 0.015	< 0.0027	< 0.0027
2580	0.345(10)		0.338(8)	0.341(6)
2668		< 0.010	< 0.0029	< 0.0029
2983		< 0.010	< 0.0022	< 0.0022
3129	0.521(12)		0.511(10)	0.515(8)

^aResult taken from Ref. [23].

calibration, we searched carefully in our data for γ -ray peaks corresponding to any of the potential transitions presented in Fig. 8, but only one could be identified: the 519-keV peak corresponding to the transition between the 666- and 146-keV levels. For all other potential transitions, we have set upper limits on their intensity.

The relative intensities and limits we obtained for all the β -delayed γ rays observed in the decay of ³⁴Ar are listed in the fourth column of Table IV. In determining each γ -ray intensity in the table, we have incorporated the β -detector efficiency given in column 5 of Table I for the β transition that populates the state from which each originates. Furthermore, since we recorded only β -coincident γ rays, we have also corrected the relative intensities to include the small calculated contributions from electron-capture decay (see column 6 of Table I).

The 519-keV transition plays a critical role in our determination of the β -decay branching to the 666-keV level since it participates directly in the deexcitation of that level. Unfortunately, the area of the 519-keV γ -ray peak is difficult to quantify precisely in our data since it lies in the tail of the strongly dominant 511-keV annihilation peak, as can be seen in Fig. 9. This severely limits the precision of the relative intensity value we quote for the 519-keV γ ray under "this work" in Table IV.

In addition to our results, Table IV includes results from the only previous ³⁴Ar β -decay measurement with useful precision [20], together with limits derived principally from (p, γ) results as summarized by Endt in Table 34.17 of Ref. [21]. Our results are more precise than, but agree well with, the previous measurement of the principal γ rays, while our limits are gen-



FIG. 9. Portion of the summed β -coincident γ -ray spectrum of Fig. 5, which focuses on the 511-keV annihilation peak and the weak 519-keV peak from the transition between the 666- and 146-keV states in ³⁴Cl. The inset focuses further onto the 519-keV peak. The histogram represents the data, while the smooth curve is the function we used to fit the 511-keV peak shape.

erally tighter than the previous ones for all unobserved transitions. The "adopted" values in the last column of Table IV are either an average of measured quantities, or the lower of the two limits where only limits have been determined. Note that there is one slight disagreement with a previous result: the intensity we measured for the all-important 519keV peak is well above the upper limit previously quoted in Ref. [21] though, as it happens, it is somewhat below an earlier measurement [22] apparently rejected by Endt [21].

This discrepancy and the importance of the 519-keV peak intensity to our analysis led us to measure, in a different collaboration, the decay of the 666-keV state in ³⁴Cl, which was produced in the ³³S(p, γ)³⁴Cl reaction. The intensity ratio of the 519- and 666-keV peaks obtained in this measurement [23] is more than an order-of-magnitude more precise than any previous result so we adopt it unmodified in the fifth column of Table IV.

IV. RESULTS

A. Gamow-Teller branching ratios

In Secs. III C, III D, and III E we have obtained values for all the quantities on the right-hand side of Eq. (1). Using these results we determine that

$$R_{666}' = 0.024\,80(28),\tag{2}$$

where we adopt the prime on *R* to signify that R'_{666} refers only to the probability for producing a 666-keV γ ray from the β decay of ³⁴Ar. As illustrated in Fig. 8, the 666-keV state has two measured γ -ray branches, at 519 and 666 keV, and has a third 205-keV option, on the intensity of which we have set an upper limit. The state also can be fed, in principle, by γ -ray transitions of 1915 and 2464 keV; for those too we have upper limits on their intensities. All relevant quantities appear in Table IV. Taking account of all these values and limits, and continuing to use the same normalization as that used in the table, we determine the total relative β^+ feeding to the 666-keV level to be $1.015(^{+10}_{-5})$.

$E_{\rm r}$	Egmax	$(\beta^+ + ec)$		
(keV)	(keV)	Relative	Absolute	$\log ft$
0	5039.9		0.9448(8)	3.4855(4)
461.0	4578.9	$0.354(^{+9}_{-14})$	$0.0088(^{+2}_{-4})$	$5.324(^{+20}_{-10})$
665.6	4374.3	$1.015(^{+10}_{-5})$	$0.0252(^{+4}_{-3})$	$4.776(^{+5}_{-7})$
2580.4	2459.5	$0.341(^{+9}_{-8})$	$0.0085(^{+3}_{-2})$	$4.122(^{+11}_{-16})$
3129.1	1910.8	$0.515(^{+14}_{-8})$	$0.0128(^{+4}_{-2})$	$3.466(^{+7}_{-14})$

TABLE V. Measured β -branching ratios to all the states in ³⁴Cl populated by the β decay of ³⁴Ar.

One last requirement is to take account of electron-capture feeding of the 666-keV state. Recognizing that both the numerator and denominator in Eq. (1) need to be corrected for missing electron-capture decays, we multiply the result for R'_{666} by $(1 + \xi_{666})/(1 + \xi)$, where ξ_{666} is the electron-capture-to-positron ratio (ec/β^+) for the β transition populating the 666-keV state and ξ is that ratio for the total decay of ³⁴Ar. The ec/β^+ ratios for all the transitions of interest appear in column 6 of Table I and yield the value $(1 + \xi_{666})/(1 + \xi) = 1.001 \ 04/1.00089) = 1.000 \ 15$. This correction is negligible in the present context but is incorporated for completeness. The total relative $(\beta^+ + ec)$ feeding to the 666-keV level is $1.015(^{+10}_{-15})$. This result appears in the third row, third column of Table V.

The final branching ratio for the $(\beta^+ + ec)$ transition to the 666-keV state is the product of R'_{666} , from Eq. (2), with the relative $(\beta^+ + ec)$ feeding value just derived. The result is

$$R_{666} = 0.025 \, 17 \binom{+38}{-30},\tag{3}$$

which also appears in the third row, fourth column of Table V.

The branching ratios for the Gamow-Teller transitions to other levels in ³⁴Ar can be derived in the same way from the intensities of the γ rays that populate and depopulate those levels. The results appear in column 3 of Table V, where we have maintained the same normalization to the intensity of the 666-keV γ ray as in Table IV, and made use of Fig. 8, which shows the placement in the decay scheme of all the observed γ rays. After multiplying by the value for R'_{666} in Eq. (2), we arrive at the final branching ratios for the three remaining Gamow-Teller transitions from ³⁴Ar, which are listed in the fourth column of the table.

The derived log ft values for these transitions are given in column five of the table. To obtain them we used the energies $E_{\beta \text{max}}$ from the second column of the table, combined with the ³⁴Ar half-life of 846.46(35) ms [18]. These data were used as input to the log ft calculator available at the National Nuclear Data Center (NNDC) [24] web site. The results obtained appear in the fifth column of Table V. They range from 3.5 to 5.3, which is well within the range that characterizes allowed $0^+ \rightarrow 1^+$ transitions [25].

B. Branching ratio for the superallowed transition

Summing the relative branching ratios in column 3 of Table V for the four Gamow-Teller transitions, we obtain $2.225(^{+21}_{-19})$. This result multiplied by R'_{666} from Eq. (2) yields

TABLE VI. Uncertainty budget for ³⁴Ar branching ratios.

	Uncertainty (%)				
Source	\sum GT branches	$0^+ \rightarrow 0^+$ branch			
Counting statistics					
γ_{666} and β singles	1.10	0.065			
$\sum \gamma / \gamma_{666}$	0.76	0.044			
HPGe detector efficiency	0.20	0.011			
Dead time	0.09	0.005			
Contaminant contribution to β singles	0.05	0.003			
³⁴ Ar component of β singles	0.04	0.002			
Bremsstrahlung coincidence summing	0.04	0.002			
Peak-to-total ratio for 511-keV γ 's	0.02	0.001			
Random preemption of real coincidences	0.02	0.001			
Total uncertainty	1.36	0.080			

0.0552(8), the total absolute (β^+ + ec) branching ratio to the 1⁺ states in ³⁴Cl.

Before proceeding, it is important to be sure that there is not a large number of unobserved weak transitions to higher excited states that could sum to appreciable missed strength: the pandemonium effect [26,27]. Shell-model calculations to be described in Sec. V B, which show excellent agreement with the observed transition intensities, limit possible unobserved intensity feeding higher excited states in ³⁴Cl to being less than 50 parts per million, more than an order-of-magnitude less than our quoted uncertainty. We can safely conclude that the Gamow-Teller sum we have obtained accounts for all the nonsuperallowed strength in the decay of ³⁴Ar.

The branching ratio for the superallowed $0^+ \rightarrow 0^+$ transition to the ground state is thus 0.9448(8), a result we obtain simply by subtracting the total Gamow-Teller (β^+ + ec) branching ratio, quoted above, from unity. This value, which has a precision of $\pm 0.08\%$ appears in the top line of the fourth column of Table V. The log *ft* value for the transition appears next to it in column 5. In this case, where the greatest precision is required, we used the full calculation for the statistical rate function *f*, as given in Ref. [1].

C. Uncertainty budget

A detailed uncertainty budget for our ³⁴Ar branching-ratio measurement appears in Table VI, where we present two relative uncertainties (in percent) for each contribution. The first is expressed relative to the total intensity of all Gamow-Teller branches; this is the uncertainty associated with the measurement itself. The second is expressed relative to the superallowed branching ratio, which is the derived quantity of principal interest.

Evidently, counting statistics are by far the largest contributors to the total uncertainty. The 666-keV γ ray, the strongest one we observe, follows a 2.5% Gamow-Teller β decay branch from ³⁴Ar. Its intensity is thus 160 times less than that of the 511-keV positron-annihilation photons, which arise from all decay branches of both ³⁴Ar and its daughter

TABLE VII. Comparison of our measured (β^+ + ec) branching ratios from ³⁴Ar with the only previous measurement.

$E_{\rm r}$	$(\beta^+ + ec)$ branching					
(keV)	Ref. [20]	This work				
0	$0.9444(^{+23}_{-26})$	0.9448(8)				
461.0	0.0091(10)	$0.0088(^{+2}_{-4})$				
665.6	$0.0249(^{+13}_{-10})$	$0.0252(^{+4}_{-3})$				
2580.4	$0.0086(^{+10}_{-4})$	$0.0085(^{+3}_{-2})$				
3129.1	$0.0130(^{+12}_{-6})$	$0.0128(\overset{+4}{-2})$				

³⁴Cl. We have to limit our counting rate so as to keep dead time and other corrections to a manageable size, but it is the annihilation rate that determines the limit. The relatively large counting-statistical uncertainties reflect the relatively few 666-keV γ -ray events that could be accumulated in a week-long measurement.

All the remaining contributions can be classified as systematic uncertainties. Altogether their contribution to the total is barely perceptible.

V. DISCUSSION

A. Superallowed decay branch

The branching ratios for the $(\beta^+ + ec)$ transitions from ³⁴Ar have been measured only once before, 45 years ago [20]. Remarkably, those earlier results agree completely with our new, more precise ones, as can be seen from the detailed comparison in Table VII. Since a weighted average of the two measurements of the superallowed transition is essentially the same as our new result, in the following development we will simply use the latter.

There have been no new measurements of the $Q_{\rm EC}$ value for the superallowed β branch from ³⁴Ar, so we adopt the *f* value, 3410.97(61), given for it in the most recent survey of superallowed $0^+ \rightarrow 0^+$ nuclear β decays [1]. However, there has been a new half-life measurement [18], which replaces the previously dominant one. We use the new value, 846.46(35) ms. In combination with our new branching-ratio measurement, these results yield

$$ft = 3058.1(28) \text{ s.} \tag{4}$$

The relationship between an ft value and the $\mathcal{F}t$ value used to extract V_{ud} is given by

$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{\rm NS}-\delta_C),\tag{5}$$

where δ_C is the isospin-symmetry-breaking correction and the terms δ'_R and δ_{NS} comprise the transition-dependent part of the radiative correction, the former being a function only of the decay energy and the atomic number *Z* of the daughter nucleus, while the latter, like δ_C , depends in its evaluation on the details of nuclear structure. Taking the values for these three small correction terms from Table IX in Ref. [1], we obtain the result,

$$\mathcal{F}t = 3074.0(34) \,\mathrm{s.}$$
 (6)

With 0.11% precision, this result for ³⁴Ar is competitive with $\mathcal{F}t$ values for the previously best-known $T_Z = 0$ superallowed emitters, all of which have branching ratios that are greater than 99.97% and thus did not require such a challenging measurement. Our new $\mathcal{F}t$ value for ³⁴Ar decay is entirely consistent with 3072.27(62) s, the average $\mathcal{F}t$ value quoted in Ref. [1] for all 14 superallowed emitters well known at the time.

What makes the new ft value for the ³⁴Ar superallowed decay particularly interesting is the fact that it becomes onehalf of what is now the most precisely known mirror pair of superallowed transitions, viz. ³⁴Ar \rightarrow ³⁴Cl and ³⁴Cl \rightarrow ³⁴S. If the constancy of $\mathcal{F}t$ is taken as a premise, then the ratio of ftvalues from a mirror pair relates directly to the calculated correction terms δ'_R , δ_{NS} , and δ_C through the following equation:

$$\frac{ft^a}{ft^b} = 1 + \left(\delta_R^{\prime b} - \delta_R^{\prime a}\right) + \left(\delta_{\rm NS}^b - \delta_{\rm NS}^a\right) - \left(\delta_C^b - \delta_C^a\right), \quad (7)$$

where, in the present case, the superscript "*a*" denotes the decay ³⁴Ar \rightarrow ³⁴Cl and "*b*" denotes ³⁴Cl \rightarrow ³⁴S. As explained in Ref. [3], the crucial advantage offered by Eq. (7) is that the theoretical uncertainty on a difference term such as $(\delta_C^b - \delta_C^a)$ is significantly less than the uncertainties on δ_C^b and δ_C^a individually. This means that the experimental *ft*-value ratio can provide a sensitive and independent test of the veracity of the correction terms, particularly δ_C .

Taking ft^a from Eq. (4), and ft^b from the most recent review of superallowed $0^+ \rightarrow 0^+$ decays [1], we determine the ratio for the A = 34 pair to be $ft^a/ft^b = 1.0028(10)$. This value agrees with 1.0017(3), the ratio obtained if Woods-Saxon radial wave functions are used to calculate the δ_C values, and disagrees significantly with the ratio 0.9997(4) calculated if Hartree-Fock radial wave functions [3] are used.

The measured ratios for all three known mirror pairs—the present A = 34 result, together with the previously published results for A = 26 [9] and A = 42 [10]—are compared with calculated values [3] in Fig. 10. Taken collectively, the data are seen to strongly favor the Woods-Saxon-based calculations; quantitatively, the normalized χ^2 for the Woods-Saxon comparison is 0.84 (confidence level, CL = 43%) and that for the Hartree-Fock is 6.22 (CL = 0.20%). This is a definitive selection between the two.

B. Gamow-Teller branches

From the nuclear-model perspective, both ³⁴Ar and ³⁴Cl are well described by *sd* shell orbitals. In Table VIII we show the results of *sd* shell-model calculations for 1⁺ states in ³⁴Cl, involving the full *sd* shell with USD effective interaction of Wildenthal [28] and two more recent updates, USD-A and USD-B, of Brown and Richter [29]. In all cases we use a quenched value for the axial-vector coupling constant, $g_{A,eff} = 1.0$, which Brown and Wildenthal [30] demonstrated to be appropriate for use in calculations truncated to just *sd*-shell configurations.

These calculations identify eight 1⁺, T = 0 states in ³⁴Cl below the 6062-keV Q_{EC} value for ³⁴Ar decay [1]. The calculated energies and β -decay branching ratios are compared with experiment in Table VIII, from which it is evident that



FIG. 10. Mirror-pair ft^a/ft^b values for A = 26, 34, 38, and 42, where the "a" and "b" superscripts denote decays of the $T_Z = -1$ and $T_Z = 0$ parents, respectively. The black and grey bands connect calculated results that utilize Woods-Saxon (WS) and Hartree-Fock (HF) radial wave functions, respectively. The measured results for A = 26, 34, and 38 appear as open circles with error bars.

there is very good correspondence between experiment and theory for the four lowest-energy 1^+ states, whose branching ratios have been measured. Overall, the USD effective interaction gives the closest match to the experimental results, but the other two interactions show acceptable agreement.

For precise β -decay studies such as the one reported here, it is essential to ensure that no decay strength remains unaccounted for. In particular, one must rule out—or correct for low-energy β transitions to highly excited states, transitions that could be too weak to be observed individually but are numerous enough that their total intensity is of significance [26,27]. As seen in Table VIII, our calculations predict four more 1⁺ states at excitation energies above the four we have observed. The total predicted feeding of these states differs slightly from calculation to calculation, but is never higher than 65 ppm. Even this is an overestimate. Not all that strength would have been missed in our β -decay measurement, since some of the deexcitation γ -ray intensity feeds one of the four lower-lying 1^+ states. By also calculating the γ -ray deexcitation of these states, we determined that missed strength would actually be less than 50 ppm. This has negligible impact on our measurement.

VI. CONCLUSIONS

We report a precise measurement of the branching ratio for the superallowed $0^+ \rightarrow 0^+ \beta$ decay of ³⁴Ar. It is not the first measurement of this quantity but its precision is a significant improvement over the only previous measurement. As a result, the corresponding improvements in the *ft* and *Ft* values for this transition promote it to prominence among the most precisely known transitions of its type. The most powerful outcome is that the A = 34 mirror pair of $0^+ \rightarrow 0^+$ superallowed transitions, ³⁴Ar \rightarrow ³⁴Cl and ³⁴Cl \rightarrow ³⁴S, becomes the most precisely characterized mirror pair, with its ratio of *ft* values providing key confirmation for the use of Woods-Saxon wave functions in calculations of the isospinsymmetry-breaking correction δ_C .

Not only is the superallowed branching-ratio measurement valuable, but also there is good agreement between our results and shell-model calculations for the Gamow-Teller decay branches to 1⁺ states in ³⁴Cl. This is an important result since the same effective interactions were used in the shell-model contributions to the calculation of both δ_C and δ_{NS} . Our results demonstrate their efficacy for these nuclei.

The currently accepted value of V_{ud} , the up-down quarkmixing element of the CKM matrix, is determined from the average $\mathcal{F}t$ value for 14 0⁺ \rightarrow 0⁺ superallowed transitions [1], and each individual $\mathcal{F}t$ value that contributes to the average depends critically on its calculated δ_C and δ_{NS} correction terms. The correction terms used in the most recent survey [1] used the shell model to determine configuration mixing and Woods-Saxon radial wave functions to evaluate the radial mismatch between parent and daughter states. Our results for ³⁴Ar decay convincingly support these methods for calculating the structure-dependent correction terms.

ACKNOWLEDGMENTS

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award No. DE-FG03-93ER40773, and by the Welch Foundation under Grant No. A-1397.

TABLE	VIII.	Experimental	and	theoretical	excitation	energies	and	β -decay	branching	ratios	R to	the	daughter	1^{+} s	states	in ³⁴	Cl. '	Гhe
theoretical	values	were obtained	l from	n an <i>sd</i> shel	l-model ca	lculation	with	effective	interaction	ns USD	, US	D-A	, and USD	-B.				

	Expt.		US	D	USE	D-A	USD-B		
State	$\overline{E_x}$ (keV)	<i>R</i> (%)							
$1_1^+, T = 0$	461	0.88	320	0.28	550	0.47	330	0.26	
$1_2^+, T = 0$	666	2.52	660	2.24	270	2.09	520	6.42	
$1^+_3, T = 0$	2580	0.85	2520	0.69	2260	0.92	2370	0.28	
$1_4^+, T = 0$	3129	1.28	3250	0.85	3150	0.93	3050	0.90	
$1_{5}^{+}, T = 0$			3880	0.0003	3690	0.0028	3730	0.0010	
$1_{6}^{+}, T = 0$			3950	0.0017	4060	0.0022	3800	0.0015	
$1_7^+, T = 0$			4980	0.0008	4830	0.0015	4880	0.0008	
$1_8^+, T = 0$			5110	0.0005	5170	0.0000	5020	0.0006	

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