$^{2}\mathrm{H}(p,\gamma)^{3}\mathrm{He}$ cross section measurement using high-energy-density plasmas

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(Received 22 September 2019; revised manuscript received 7 February 2020; accepted 6 April 2020; published 27 April 2020)

An absolute cross section for the radiative capture reaction $^2\text{H}(p,\gamma)^3\text{He}$ has been measured at the OMEGA laser facility using inertially confined plasmas. These high-temperature plasmas are created by imploding a fuel-containing capsule using laser ablation, and are advantageous in that they better mimic astrophysical systems. We measure an S factor for this reaction of $0.429 \pm 0.026_{\text{stat}} \pm 0.072_{\text{sys}}$ eV b at $E_{c.m.} = 16.35 \pm 0.40$ keV, which is higher than the adopted evaluations. This reaction is important as a source of nuclear energy in protostars and brown dwarfs. It is also a critical reaction during big-bang nucleosynthesis, and an accurate cross section can be used as a constraint on cosmology.

DOI: 10.1103/PhysRevC.101.042802

Fusion of deuterons (²H, or D) and protons via the reaction

$$p + D \rightarrow {}^{3}\text{He} + \gamma \quad (Q = 5.5 \text{ MeV})$$
 (1)

is the primary reaction for generation of ³He in nucleosynthesis processes; this reaction is critically important for many astrophysical systems such as main-sequence stars, brown dwarfs, protostars, and the universe during the big bang.

Interstellar gas undergoing gravitational contraction can have a brief epoch of halted contraction due to burning of primordial material in the "protostar" phase [1]. Similarly, in "brown dwarf" stars, which are not massive enough to ignite hydrogen fusion ($\lesssim 0.07 M_{\odot}$) but larger than giant planets ($\gtrsim 0.01 M_{\odot}$), deuterium burning can generate substantial amounts of thermonuclear energy [2–4]. The internal dynamics of these objects depends sensitively on the pD fusion reactivity at extremely low temperatures.

This reaction is highly relevant for big-bang nucleosynthesis (BBN), in which the *p*D reaction consumes D and generates ³He needed for the reaction chain to proceed to heavier nuclei such as Li and Be [5,6]. The deuterium concentration in primordial material is well known. When combined with the relevant cross sections and a BBN model, strong constraints on, or consistency tests of, cosmological physics can be done [7–9]. An improved understanding of *p*D fusion for this purpose has been called for, e.g., by Marcucci *et al.* [8].

This reaction has been studied at low energy in several accelerator (beam-target) experiments [10–13] and in theoretical work [14–16]. Achieving low energies relevant for astrophys-

ical nucleosynthesis is a challenge for accelerator measurements, requiring long-duration runs in underground laboratories to reduce backgrounds. High-energy-density plasmas are an alternative technique, with several reaction cross sections recently reported for the first time [17,18], as well as studies of fundamental nuclear physics [19–25]. Cross section measurements in these plasmas require no corrections for beam stopping or electron screening, and thermal plasmas generate many reactions compared to a beam-target system.

In this paper we report the first measurements of the pD fusion cross section using inertially confined laboratory plasmas. Our measured S factor is higher than in reported evaluations [8,26,27]. This work represents the second use of high-energy-density laboratory plasmas to measure a cross section with direct relevance to current nucleosynthesis problems, a novel technique with significant potential applications to modern astrophysics. The data for this reaction could be extended to both higher and lower energies, relevant to BBN and protostars/brown dwarfs, respectively, and continue to improve BBN constraints on cosmology.

Spherical plastic (CH) shells filled with various fuel mixtures were imploded with the OMEGA laser [28]. The targets are shown in Fig. 1; the CH shells were nominally 15 μ m thick with an outer diameter of 860 μ m and a 0.1 μ m Al overcoating. The shell was driven by the laser irradiation at 3 ω (351 nm) using super-Gaussian (fourth-order) distributed phase plates [29], polarization rotation [30], and smoothing by spectral dispersion [31]. The pulse duration was 1 ns and the total energy delivered was approximately 25 kJ. Three different target fills were used, as shown in Fig. 1. The primary experiment, studying pD fusion, used an equimolar mix of deuterium (D₂) and hydrogen (H₂) fill gas with 5.33 atm of

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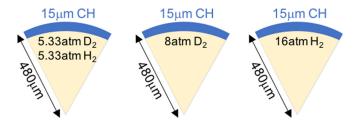


FIG. 1. Schematic of the capsules used in this work, which are 480 μ m radius spherical plastic (CH) shells, 15 μ m thick, filled with a variety of gaseous fuel mixtures of hydrogen (H₂) and deuterium (D₂).

each. Background characterization shots were conducted with either pure D_2 or pure H_2 fills, using a mass-equivalent fill of 8 atm of D_2 or 16 atm of H_2 gas. While pure "hydroequivalence" [32] is not possible between these gas mixes, keeping the mass density constant maintains similar hydrodynamics between the fills. Detailed setup parameters for each shot, including measured target information, are given in the Supplemental Material [33].

Each experiment is diagnosed with a suite of nuclear diagnostics. Neutrons are measured with the standard OMEGA time-of-flight suite [34] for spectra and yield plus a scintillator-based system to infer the peak nuclear burn time [35]. The γ rays produced by pD fusion are measured with gas Cherenkov detectors (GCDs) [36]. These detectors consisted of a converter foil where incident γ rays Compton scatter electrons into a gas cell, where Cherenkov light is produced if the Compton electron exceeds the local speed of light (c/n). The Cherenkov light is relayed to a photomultiplier tube (PMT) using Cassegrain optics, and the electrical signal is recorded on a high-bandwidth oscilloscope. The original GCD, as used in studies of tritium (T) plus 3 He fusion [17], was limited to 100 psi (absolute) of gas, corresponding to a threshold

 γ energy of 6.3 MeV when using CO₂. A recent upgraded system has a design that can operate at 400 psi (absolute) [37], reducing the threshold to 2.9 MeV and enabling these studies of pD fusion, which produces a γ ray at only 5.5 MeV. The response curves of these detectors are calculated with GEANT4 [38] and calibrated *in situ* [39].

The oscilloscope signal is corrected for electrical splits and attenuators used in the experiment to recover the signal level at the PMT output. The scope trace is aligned in time such that the peak Cherenkov signal, corresponding to peak nuclear production, is at t=0. The trace before and after the Cherenkov signal is used in a background subtraction. The Cherenkov signals for shots with D_2 , H_2 , and $D_2 + H_2$ are shown in Fig. 2 for each shot (numbered as 74xxx), normalized to the DD neutron yield on each shot. Each fuel type is shown by a corresponding color: HD in reds, D in blues, and H in greens.

The variance in signal level on the HD-filled shots is due to statistics in the detected number of γ 's: each shot has \approx 35 productive Compton electrons in the detector. The H₂-filled shots were designed as a background test for the Cherenkov detector, since no fusion reactions should occur in these implosions. As expected, all nuclear diagnostics showed no evidence of nuclear reactions in the H₂ implosions. Some background is observed on shots with only a D₂ fill. The H₂ null shot demonstrates that the signal observed is due to a nuclear process and not due to another process in the implosions (e.g., laser-plasma interactions generating hot electrons). Since the data are shown yield normalized, the DD curves represent the primary source of background in the pD measurement: due to the DD- γ (branching ratio $\approx 10^{-7}$), (n, γ) reactions from DD-n interacting with the surrounding material, or mixing of hydrogen from the shell into the fuel [40] generating pD reactions. The signal level is \approx 3–4 times too high to be explained by mixing of hydrogen from the CH shell into the fuel at a level consistent with Ref. [40], \approx 4%

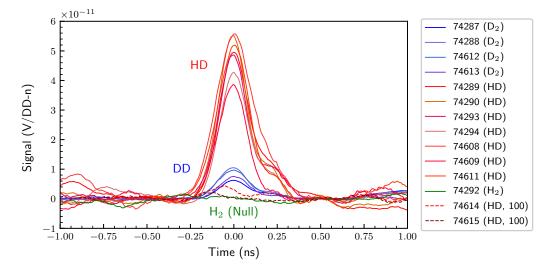


FIG. 2. Cherenkov detector data from each shot as PMT V per DD-n yield versus time, peak aligned at t=0. Different capsule gas fills [mixtures of deuterium (D) and hydrogen (H)] are distinguished by color: D_2 (blues), $H_2 + D_2$ (reds), and H_2 (green). Two shots with 100 psi (absolute) of CO_2 (74614 and 74615) instead of 400 psi (absolute) are shown by the dashed curves. The H_2 data is normalized to the average DD-n yield on the $H_2 + D_2$ experiments since that shot produces no nuclear yield.

atomic fraction of H from mixing. The estimated contribution from mixing and DD fusion nuclear reactions are treated separately when subtracting the background from the pD shots, including propagating uncertainty in the mix fraction and DD contribution. The remaining signal in the pD implosions is attributed to the $p + D \gamma$ ray [Eq. (1)] at 5.5 MeV.

This can be verified with a Cherenkov threshold test. By reducing the GCD gas fill pressure from 400 to 100 psi, the γ -ray energy threshold increases to 6.3 MeV. This threshold scan is shown in Fig. 2 by shots 74614 and 74615 (dotted lines), which were HD-filled implosions conducted with the increased threshold, showing that the Cherenkov signal disappears in the HD implosions as the threshold is increased. This demonstrates that the signal is due to a low-energy γ ray, i.e., the p+D fusion γ .

The γ -ray yield can be calculated from the integrated signal (Vs) using the detector equation

$$Y = \frac{Vs}{\Omega Re \, OE \, G} \frac{\chi}{C},\tag{2}$$

where Ω is the detector solid angle, R is the scope input impedance (50 Ω), e is the fundamental charge, QE is the PMT quantum efficiency, and G is the PMT gain. C is the effective GCD response for the γ energy measured in photons collected per incident γ , and χ is the overall detector efficiency calibration factor [39].

On each HD implosion the γ yield is calculated using Eq. (2), the 5.5 MeV γ -ray response, and the calibration factor. The statistical uncertainty in γ yield results from uncertainty in the integrated signal, DD background subtraction, statistical uncertainty in the calibration, and a treatment of the Cherenkov detection statistics (see Supplemental Material [33]). The systematic uncertainty results from the uncertainty in the DD background subtraction and from the calibration.

The S factor for the pD reaction can be calculated from the pD- γ yield using the DD reaction as a reference. The yields for the two reactions are

$$Y_{pD} = \int d\vec{r} \, dt \, n_H n_D \langle \sigma v \rangle_{pD}, \tag{3}$$

$$Y_{DD} = \int d\vec{r} \, dt \, \frac{n_D^2}{2} \langle \sigma v \rangle_{DD}, \tag{4}$$

where n_H and n_D are the proton and deuteron number densities, respectively, and $\langle \sigma v \rangle$ is the fusion reactivity as a function of temperature; n and T_i are typically unknown functions of space and time. The ratio of the two yields is

$$\frac{Y_{pD}}{Y_{DD}} = 2 \int d\vec{r} \, dt \, \frac{n_H}{n_D} \frac{\langle \sigma v \rangle_{pD}}{\langle \sigma v \rangle_{DD}}.$$
 (5)

If there are no "profile" or species-separation effects occurring in these implosions, as demonstrated in the Supplemental Material [33], then the yield ratio is given by

$$\frac{Y_{pD}}{Y_{DD}} = 2 \frac{n_H}{n_D} \frac{\langle \sigma v \rangle_{pD}}{\langle \sigma v \rangle_{DD}}.$$
 (6)

where the reactivities are burn averaged. This assumes that the density and temperature profiles for each reaction are the same, thus the burn duration and volume are also identical. Since these are both nonresonant reactions in this energy range, the reactivity can be written

$$\langle \sigma v \rangle = \frac{8}{\pi \sqrt{3}} \frac{\hbar}{m_r Z_1 Z_2 e^2} S \xi^2 e^{-3\xi},\tag{7}$$

which is Eq. 1.56 in Atzeni [41], where Z_1 and Z_2 are the reactant charges, m_r is their reduced mass, S is the S factor, and ξ is related to the Gamow energy (ϵ_G):

$$\xi = \left(\frac{\epsilon_G}{4k_BT}\right)^{1/3} \tag{8}$$

$$= \left[\frac{(\pi \alpha Z_1 Z_2)^2 2 m_r c^2}{4 k_B T} \right]^{1/3} \tag{9}$$

$$= 6.2696(Z_1Z_2)^{2/3}A_r^{1/3}T^{-1/3}, (10)$$

where A_r is the reduced mass in atomic units and T is the temperature in keV. Using this formalism, the yield ratio can then be written as

$$\frac{Y_{pD}}{Y_{DD}} = 2 \frac{n_H}{n_D} \frac{A_{pD}^{-1} S_{pD} \xi_{pD}^2 e^{-3\xi_{pD}}}{A_{DD}^{-1} S_{DD} \xi_{DD}^2 e^{-3\xi_{DD}}},\tag{11}$$

which is inverted to give an expression for the pD S factor in terms of the DD S factor:

$$S_{pD} = S_{DD} \frac{Y_{pD}}{Y_{DD}} \left[\frac{n_D}{2n_H} \frac{A_{pD}}{A_{DD}} \frac{\xi_{DD}^2 e^{-3\xi_{DD}}}{\xi_{pD}^2 e^{-3\xi_{pD}}} \right]. \tag{12}$$

This expression includes a correction for the fact that the DD and pD reactions are expected to have different Gamow peak energies in a thermal plasma, as shown by the dependence on ξ_{pD} and ξ_{DD} in Eq. 12. Plasma screening at these conditions is totally negligible ($\leq 0.02-0.03\%$) [27,42].

The DD S factor is well known from accelerator experiments; for our normalization we use the evaluation of Bosch and Hale [43] with a 5% uncertainty in the DD S factor as given in that work. The two yields are measured experimentally, and the factor in square brackets is calculated using the DD-n Doppler ion temperature, which is \approx 0.1 for $T_i \approx 5$ keV. The average center-of-mass energy ($E_{c.m.}$) for the reactions, the Gamow peak energy, is $\xi k_B T$ and thus is calculated for the DD and pD reactions separately, though using the same thermal temperature. An S factor is calculated for each of the seven HD data shots, which are then used to calculate a weighted mean value:

$$S_{pD} = 0.429 \pm 0.026_{\text{stat}} \pm 0.072_{\text{sys}} \text{ ev b},$$
 (13)

at $E_{c.m.} = 16.35 \pm 0.4$ keV. This new result is shown as the red point in Fig. 3, compared to previous data from accelerator

¹The evaluation in Ref. [43] is mostly constrained for the D(D,n) ³He reaction by the data by Brown and Jarmie [44] which is a direct measurement. At lower energies than those considered in this work, reference data for the DD reaction are often taken using the Trojan horse method, which are relative measurements and require absolute direct measurements for normalization. See the Supplemental Material [33] as well as Refs. [45–63] for more details on the reference data.

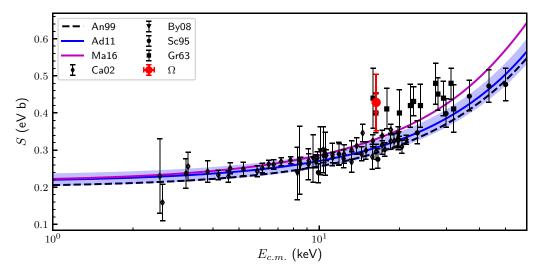


FIG. 3. Our measurement (red point), compared to previous data from accelerator experiments [10–13] (black symbols), as well as the evaluated *S* factors reported [8,26,27] (black dashed, blue, and purple curves respectively). The smaller error bar on our measurement is statistical only, while the larger is the combined statistical and systematic.

experiments [10–13], as well as the evaluated S factors reported [8,26,27]. Our value is higher than previously reported data at this $E_{c.m.}$, with the exception of Ref. [10],² and higher than the adopted evaluations.

In conclusion, in this Rapid Communication we report the first measurement of the $D(p,\gamma)^3$ He S factor measured from inertially confined high-energy-density plasmas, which is only the second cross section measurement directly relevant to nucleosynthesis made with this technique. These plasmas are generated by using intense laser irradiation to implode a fuel-containing capsule, reaching temperatures where thermonuclear reactions occur. Reactions occurring in these plasmas are a more direct surrogate for astrophysical systems, including the universe during the epoch of big-bang nucleosynthesis. We find a cross section higher than most data reported by accelerator measurements in this range, and higher than recent evaluations including the solar fusion II compilation

[27] and a recent paper by Marcucci *et al.* [8]. In addition to relevance for brown dwarfs and protostars, an accurate cross section for *p*D fusion from fundamental laboratory measurements is critically important as it enables precision checks of the consistency between big-bang nucleosynthesis models and cosmology [8]. Our measurements suggest that additional work on this topic is required, since Ref. [8] still observes a systematic difference between cosmology and big-bang calculations that could be attributed to uncertainties in these underlying cross sections. This technique can produce additional data at both higher and lower center-of-mass energies with further experiments, to continue to constrain astrophysical nucleosynthesis models and cosmological physics.

We thank the operations crews and engineering staff at OMEGA for supporting these experiments. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory in part under Contract No. DE-AC52-07NA27344 and supported by the U.S. DOE Early Career Research Program (Fusion Energy Sciences) under FWP SCW1658.

²We note that the data from Ref. [10] have been called into question, for example in Ref. [27], due to concerns over use of an inaccurate stopping power.

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