Issues with the search for critical point in QCD with relativistic heavy ion collisions

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(Received 18 December 2019; accepted 31 January 2020; published 18 March 2020)

A systematic search for a critical point in the phase diagram of QCD matter is under way at the Relativistic Heavy Ion Collider (RHIC) and is planned at several future facilities. Its existence, if confirmed, and its location will greatly enhance our understanding of QCD. In this article, we emphasize several important issues that are often not fully recognized in theoretical interpretations of experimental results relevant to the critical point search. We discuss ways in which our understanding on these issues can be improved.

DOI: 10.1103/PhysRevC.101.034913

Considerable experimental efforts have been made or are under way and several new facilities are being planned to search for a critical point in the QCD phase diagram with relativistic heavy ion collisions. If the existence and location of a critical point can be ascertained, it will become the second confirmed "point" in the QCD phase diagram that has a physical meaning, after the ground state of nuclear matter. It will act as a landmark in the QCD phase diagram, and our understanding of the phases of QCD will be much deepened. This is the reason why the critical point search is so important and attracts great scientific interest.

In order to understand the effects of the existence of a critical point on physical observables, it is necessary to understand the roles and features of each observable. In the search for critical point, fluctuations of conserved charges [1–4] are most frequently studied. However, as we will discuss in the following, the important feature of fluctuations of conserved charges, which motivated the original proposal, fluctuations of conserved charges as detectors of the change of the degrees of freedom [1,2], is often forgotten. As a result, conserved charge fluctuations, or more precisely their cumulants, are often not properly utilized in the current interpretation of experimental results. In this paper, we discuss how we should understand the experimentally observed conserved charge cumulants, focusing on three issues.

First, we argue at which freeze-out, chemical or kinetic, conserved charge cumulants decouple. Very often, conserved charge cumulants are assumed to decouple at the chemical freeze-out surface. However, this contradicts the notion of chemical freeze-out. Chemical freeze-out is where most inelastic reactions cease, i.e., where the changes of particle numbers and particle species stop. Examples of such reactions are $\pi\pi \leftrightarrow \pi\pi\pi\pi$ and reactions that change the number of strange and antistrange valence quarks, such as $\pi\pi \leftrightarrow K\bar{K}$ or $N\pi \leftrightarrow \Lambda K$. (Note that inelastic reactions which do not require much energy and have large cross sections, such as $n\pi^+ \leftrightarrow p\pi^0$ via the Δ^+ resonance and $p\bar{p} \leftrightarrow 3\pi$, are exceptions [5,6].) Observables involving conserved charges are "frozen" in this sense at all times as they cannot be changed by local processes. However, as emphasized in Refs. [1,2,7], fluctuations of conserved charges change through diffusion, which does not require inelastic scatterings and thus persists until kinetic freeze-out. (In Ref. [8] Albright *et al.* observed that the crossover equation of state can reproduce the data if the fluctuations are frozen at a temperature significantly lower than the average chemical freeze-out.) The change of fluctuations of conserved charges does not proceed by chemical reactions, because chemical reactions do not change the net charge. It is diffusion that changes the fluctuations of conserved charges. This was qualitatively discussed in Ref. [1] and more quantitatively shown in Ref. [7]. Thus, it is necessary to study the diffusion dynamics in the hadron phase between the neighborhood of the critical point and the kinetic freeze-out surface in order to explore the potential trace of the critical point in the observed conserved charge fluctuations.

Second, we discuss the relationship between the cumulants of conserved charges and the correlation length ξ of the order

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parameter field σ:

$$\langle\sigma(\mathbf{x})\sigma(\mathbf{y})\rangle - \langle\sigma\rangle^2 \propto e^{-|\mathbf{x}-\mathbf{y}|/\xi}. \quad (1)$$

Stephanov [9] showed that in equilibrium

$$K_n \propto \xi^{m_n}, \quad (2)$$

where K_n is the nth cumulant of a particle number to which the order parameter couples and $m_2 \simeq 2$, $m_3 \simeq 4.5$, $m_4 \simeq 7$, $m_5 \simeq 9.5$, $m_6 \simeq 12$. These relations, in particular, the increasingly large powers for the higher cumulants, caught the attention of experimentalists as they imply that higher cumulants are more sensitive to the critical point. Led by this expectation, many experimental efforts are being made to measure higher cumulants (fourth and sixth orders) of net protons as proxies of net baryon number cumulants.

Here we need to remember the following fact. In the case of conserved charge fluctuations, the left-hand side of Eq. (2) is a conserved quantity; it can change only through diffusion, i.e., by particle transport. Neither pair production nor pair annihilation changes its value. On the other hand, the right-hand side of Eq. (2) is not a conserved quantity; it can change due to propagation of information related to the order parameter. Thus, the left-hand side and right-hand side obey different equations of motion; if the conserved charges are associated with quasiparticles, the left-hand side is achieved only by particle transport, while the right-hand side can be changed by propagation of, e.g., the information of the amplitude of the order parameter field, which obeys a wave equation (see, e.g., Ref. [10]).

In nonequilibrium or in dynamical situations as in relativistic heavy ion collisions, there is no guarantee that the proportionality relation (2) holds (see, e.g., Refs. [11,12]). The left-hand side changes by diffusion and evolves only slowly. This observation motivated the proposal that fluctuations may be probes of quark deconfinement [1,2]. The change of the right-hand side is not constrained by particle diffusion but governed by the dynamics of the critical mode.

As the evolution of a conserved quantity is much slower than that of the order parameter field σ, on longer timescales the order parameter field follows the evolution of the conserved quantity and plays no independent role in the dynamics [13,14]. Assuming this separation of the timescales, the evolution of the conserved quantity near the critical point has been studied recently [12,15].

Because of the finiteness of the reaction time and critical slowing down, however, the growth of the correlation length in relativistic heavy ion collisions is limited even if the system passes right through the critical point [14,16–18]. In other words, even the fast mode, the order parameter field σ, does not reach equilibrium; neither do quantities related to the slow mode, the cumulants of the conserved quantity. In order to confirm to what extent the relation (2) holds or is violated in relativistic heavy ion collisions, dynamical calculations that treat the fast order parameter field and the slow hydrodynamical fields properly, as described in Refs. [14,17] for the static case, will be needed.

To summarize, Eq. (2) does not hold in general in relativistic heavy ion collisions. Combining this observation and the argument in the previous paragraph, we conclude that one needs to follow the evolution of cumulants from the quark phase down to kinetic freeze-out by properly taking account of the diffusive property of conserved charge fluctuations without resort to relation (2) in order to compare theoretical expectations and experimental results.

Finally, we need to recognize that experimentally observed cumulants are measured in momentum space with an acceptance, e.g., in the case of STAR $|\eta| \leqslant 1.6$, while theoretical cumulants are usually calculated in coordinate space. In order to compare experimental results and theoretical calculations, one needs a map from coordinate space to momentum space. Only in limited cases, such as for a one-dimensional boost-invariant (Bjorken) expansion, the result in coordinate space is identical to the one in momentum space. Note that even in this case, the identity holds only up to thermal smearing [19]. For this purpose, i.e., to have the map, construction of a dynamical model of the reaction is inevitable. In the energy range of Beam Energy Scan (II), obviously the Bjorken picture is not appropriate. It may be worthwhile considering the case of the Landau-Fermi picture as an opposite extreme to the Bjorken scenario.

To sum up, the collision geometry and its time evolution are necessary to understand the final-state fluctuations, which can be compared with experimental results. Note that it is not sufficient to know only the chemical and kinetic freeze-out surfaces, since the evolution of conserved charge fluctuations should be traced all the way from the initial state to kinetic freeze-out as we discussed above. These three issues are often not adequately taken into account in the interpretation of the experimental results for conserved charge cumulants.

At the end of this article, we point out two related issues. One concerns the net proton number cumulants. These are often considered as proxies for the net baryon number cumulants, which are conserved quantities. As we discussed above and as Refs. [5,6] pointed out, net proton number cumulants are not conserved locally in the hadron phase because they can change through reactions such as $p\pi^- \to n\pi^0$. These reactions do not require much energy and continue to occur after what is commonly called chemical freeze-out. The same argument should also be applied when one regards the net kaon number cumulants as proxies of the cumulants of net strangeness.

The other is the initial condition in the Landau-Fermi picture. At the collision energies that produce near-critical point QCD matter, the transverse correlation length of nuclear energy density is probably best estimated using the wounded nucleon model, implying a correlation length comparable to the nucleon radius, approximately 1 fm. This correlation evolves while the two nuclei collide, overlap, and eventually stop. When the two nuclei have stopped, the typical correlation length in the transverse direction is estimated to be of order $2R/c_s\gamma$. Here R is the radius of the colliding nuclei and $c_s \approx 0.4$ is the speed of sound in baryon-rich QCD matter. This correlation could be realized in simulations as correlated domains whose transverse size is of the order of $2R/c_s\gamma$ with a finite transverse flow gradient in the hydrodynamic initial conditions. This would give an improved initial condition for the Landau-Fermi picture implementing baryon stopping.

We thank Swagato Mukherjee (Brookhaven National Laboratory, BNL) for discussion of the topics discussed here. M.A. thanks BNL for support during his visit. M.A. and M.K. acknowledge support from JSPS KAKENHI Grants No. JP18K03646, No. JP17K05442 and No. JP19H05598, respectively. B.M. acknowledges support from US Department of Energy Grant No. DE-FG02-05ER41367.

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