

**$\Lambda$ -nucleon scattering in baryon chiral perturbation theory**X.-L. Ren<sup>1</sup>, E. Epelbaum<sup>1</sup> and J. Gegelia<sup>1,2</sup><sup>1</sup>*Ruhr University Bochum, Faculty of Physics and Astronomy, Institute for Theoretical Physics II, D-44870 Bochum, Germany*<sup>2</sup>*Tbilisi State University, 0186 Tbilisi, Georgia*

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We calculate the  $\Lambda$ -nucleon scattering phase shifts and mixing angles by applying time-ordered perturbation theory to the manifestly Lorentz-invariant formulation of SU(3) baryon chiral perturbation theory. Scattering amplitudes are obtained by solving the corresponding coupled-channel integral equations that have a milder ultraviolet behavior compared to their nonrelativistic analogs. This allows us to consider the removed cutoff limit in our leading-order calculations also in the  ${}^3P_0$  and  ${}^3P_1$  partial waves. We find that, in the framework we are using, at least some part of the higher-order contributions to the baryon-baryon potential in these channels needs to be treated nonperturbatively, and we demonstrate how this can be achieved in a way consistent with quantum field theoretical renormalization for the leading contact interactions. We compare our results with the ones of the nonrelativistic approach and lattice QCD phase shifts obtained for non-physical pion masses.

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Nuclear systems with nonvanishing strangeness play an important role in the study of nuclear, particle, and astrophysics. Hyperon-nucleon ( $YN$ ) interactions are crucial for understanding hypernuclear binding. Experiments investigating the  $YN$ , hyperon-hyperon ( $YY$ ), and cascade-nucleon ( $\Xi N$ ) interactions are carried out at various laboratories such as CERN, DAΦNE, GSI, JLab, J-PARC, KEK, MAMI, RHIC and will also be performed at the future FAIR facility. For reviews on the subject of hypernuclear physics see Refs. [1–4].

Lattice QCD is another valuable source of information on  $YN$  and  $YY$  interactions [5–17]. While some lattice simulations are approaching the physical values of the light quark masses, most of the available lattice QCD calculations still correspond to unphysically large values and, therefore, require extrapolations to their physical values.

Chiral effective field theory (ChEFT) is a natural framework for analyzing low-energy properties of (hyper)nuclei and performing chiral extrapolations. Chiral perturbation theory for systems involving two and more nucleons was initiated in Refs. [18,19]. In that formulation, power counting rules are applied to the effective potentials, and the scattering amplitude is then obtained by solving the Lippmann-Schwinger (LS) or Schrödinger equations. Reviews of ChEFT in the few-body sector can be found in Refs. [20–24].

Chiral EFT for baryon-baryon ( $BB$ ) interactions in the strange sector was formulated in Refs. [25–38] using the nonrelativistic framework. In these studies, the standard Weinberg power counting for nucleon-nucleon ( $NN$ ) interactions is extended to the nonzero strangeness sectors of the baryon-baryon interaction. Ultraviolet divergences of the LS equation have been taken care of by applying finite-cutoff regularization using exponential cutoffs in the range of 500–700 MeV.

For nucleon-nucleon scattering, a modified Weinberg approach with an improved ultraviolet behavior was proposed

in Ref. [39]. This novel framework uses time-ordered perturbation theory (TOPT) applied to the manifestly Lorentz-invariant effective Lagrangian and leads to the Kadyshevsky equation for the scattering amplitude [40]. It has been explored in the nonstrange sector [41–43] and also with a different treatment of Dirac spinors and using an alternative power counting [44]. First applications of a similar formalism to baryon-baryon systems with nonzero strangeness can be found in Refs. [45–49].<sup>1</sup>

Recently, we have worked out in details the diagrammatic rules of TOPT for particles with nonzero spin and for interactions involving time derivatives [51]. In this paper, we apply the resulting framework to the strangeness  $S = -1$  sector of baryon-baryon scattering and focus, in particular, on  $\Lambda$ -nucleon and  $\Sigma$ -nucleon scatterings.

Our paper is organized as follows: in Sec. II we specify the effective Lagrangian required for our calculations. In Sec. III, we consider the system of integral equations for baryon-baryon scattering and present the  $\Lambda$ -nucleon and  $\Sigma$ -nucleon scattering phase shifts. Next, in Sec. IV, we discuss the renormalization of the scattering amplitudes with the next-to-leading order (NLO) contact interaction potentials treated nonperturbatively. The results of our work are summarized in Sec. V.

**II. EFFECTIVE LAGRANGIAN**

The starting point of our analysis is the manifestly Lorentz-invariant effective Lagrangian of baryon chiral perturbation theory consisting of the purely mesonic, single-baryon, and two-baryon parts,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\phi + \mathcal{L}_{\phi B} + \mathcal{L}_{BB}. \quad (1)$$

<sup>1</sup>For an application of the modified Weinberg approach to hadronic molecules see Ref. [50].

From the purely mesonic sector, we only need the lowest-order Lagrangian [52]

$$\mathcal{L}_\phi^{(2)} = \frac{F_0^2}{4} \text{Tr}\{u_\mu u^\mu + \chi_\pm\}, \quad (2)$$

where

$$\begin{aligned} u_\mu &= iu^\dagger \partial_\mu U u^\dagger, \quad u^2 = U = \exp(\sqrt{2}i\phi/F_0), \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 s, \end{aligned} \quad (3)$$

and  $\phi$  is the irreducible octet representation of  $SU(3)_f$  for the Goldstone bosons,

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (4)$$

Here  $F_0$  stands for the meson decay constant in the chiral limit while  $s$  is the external scalar source that gives rise to the quark (and Goldstone-boson) masses. For the purposes of the current work we switch off all other external sources and consider the isospin-symmetric case of  $m_u = m_d \neq m_s$ . The constant  $B_0$  in Eq. (3) is related to the quark condensate.

The leading-order (LO) Lagrangian of the single-baryon sector is given by

$$\mathcal{L}_{\phi B}^{(1)} = \text{Tr}\{\bar{B}(i\gamma_\mu D^\mu - m)B\} + \frac{D/F}{2} \text{Tr}\{\bar{B}\gamma_\mu \gamma_5 [u^\mu, B]_\pm\}, \quad (5)$$

where  $B$  is the irreducible octet representation of  $SU(3)_f$  involving baryon fields,

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad (6)$$

$D$  and  $F$  are coupling constants corresponding to  $[\dots]_+$  and  $[\dots]_-$ , respectively, and  $D_\mu B = \partial_\mu B + [[u^\dagger, \partial_\mu u], B]$  is the covariant derivative.

The effective baryon-baryon Lagrangian contributing to the LO  $BB$  potential consists of the following terms [27]:

$$\begin{aligned} \mathcal{L}_{BB} &= C_i^1 \text{Tr}\{\bar{B}_\alpha \bar{B}_\beta (\Gamma_i B)_\beta (\Gamma_i B)_\alpha\} \\ &+ C_i^2 \text{Tr}\{\bar{B}_\alpha (\Gamma_i B)_\alpha \bar{B}_\beta (\Gamma_i B)_\beta\} \\ &+ C_i^3 \text{Tr}\{\bar{B}_\alpha (\Gamma_i B)_\alpha\} \text{Tr}\{\bar{B}_\beta (\Gamma_i B)_\beta\}, \end{aligned} \quad (7)$$

where  $C_i^1$ ,  $C_i^2$ , and  $C_i^3$  are the coupling constants,  $\alpha$  and  $\beta$  are the Dirac spinor indices, and

$$\Gamma_1 = 1, \quad \Gamma_2 = \gamma^\mu, \quad \Gamma_3 = \sigma^{\mu\nu}, \quad \Gamma_4 = \gamma^\mu \gamma_5. \quad (8)$$

Notice that the  $\Gamma_5 = \gamma_5$  term starts contributing at NLO.

### III. INTEGRAL EQUATIONS FOR BARYON-BARYON SCATTERING

The off-shell baryon-baryon scattering amplitude  $T$  satisfies the integral equation, which can symbolically be written as [51]

$$T = V + VGT, \quad (9)$$

where  $V$  is the effective potential and  $G$  is the two-baryon Green function. To obtain the scattering amplitudes of processes with strangeness  $S = -1$  in the isospin limit, Eq. (9) is understood as a  $2 \times 2$  matrix equation, where

$$\begin{aligned} T &= \begin{pmatrix} T_{\Lambda N, \Lambda N} & T_{\Lambda N, \Sigma N} \\ T_{\Sigma N, \Lambda N} & T_{\Sigma N, \Sigma N} \end{pmatrix}, \\ V &= \begin{pmatrix} V_{\Lambda N, \Lambda N} & V_{\Lambda N, \Sigma N} \\ V_{\Sigma N, \Lambda N} & V_{\Sigma N, \Sigma N} \end{pmatrix}, \\ G &= \begin{pmatrix} G^{\Lambda N} & 0 \\ 0 & G^{\Sigma N} \end{pmatrix}, \end{aligned} \quad (10)$$

and the two-body Green functions read

$$G^{IJ}(E) = \frac{1}{\omega(p_I, m_I)\omega(p_J, m_J)} \frac{m_I m_J}{E - \omega(p_I, m_I) - \omega(p_J, m_J) + i\epsilon}, \quad (11)$$

with  $m_I$  and  $\omega(p_I, m_I) = (\vec{p}_I^2 + m_I^2)^{1/2}$  being the mass and the energy of the  $I$ th baryon with four-momentum  $p_I$ . In the partial wave basis, Eq. (9) leads to the following coupled-channel equations with the partial wave projected potentials  $V_{l'l', s's', j}^{IJ, KL}(E; p', p)$ :

$$\begin{aligned} T_{l'l', s's', j}^{IJ, KL}(E; p', p) &= V_{l'l', s's', j}^{IJ, KL}(E; p', p) \\ &+ \sum_{l'', s'', P, Q} \int_0^\infty dk k^2 \frac{V_{l''l'', s''s'', j}^{IJ, PQ}(E; p', k)}{2\pi^2} \\ &\times G^{PQ}(E) T_{l''l'', s''s'', j}^{PQ, KL}(E; k, p), \end{aligned} \quad (12)$$

where  $IJ$ ,  $KL$ , and  $PQ$  label the initial, final and intermediate particles, respectively, and  $p \equiv |\vec{p}|$ ,  $p' \equiv |\vec{p}'|$ , and  $k \equiv |\vec{k}|$ . The indices  $l$ ,  $l''$ ,  $l'$  and  $s$ ,  $s''$ ,  $s'$  stand for their orbital angular momenta and spins, respectively, while  $j$  denotes the total angular momentum of the  $BB$  states. Compared to the corresponding Lippmann-Schwinger equation for the same potential, Eq. (12) with the Green functions of Eq. (11) has a milder UV behavior. Therefore, its solutions show less sensitivity to the variation of the cutoff parameter [51].

We organize the  $BB$  potential by applying the standard Weinberg power counting to two-baryon irreducible TOPT diagrams. The LO potential consists of the short-range contact interaction part  $V_{LO, C}^{IJ, KL}$  and the one-meson exchange (OME) contribution [51]:

$$\begin{aligned} V_{LO, M_P}^{IJ, KL} &= -\frac{f_{IKP} f_{JLP} \bar{\mathcal{I}}_{IJ, KL}}{2\omega(q, M_P)} \left[ \frac{1}{\omega(q, M_P) + \omega(p_K, m_K) + \omega(p_J, m_J) - E - i\epsilon} \right. \\ &+ \left. \frac{1}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i\epsilon} \right] \frac{(m_I + m_K)(m_J + m_L)}{\sqrt{m_I m_J m_K m_L}} \\ &\times \frac{(m_K \vec{\sigma}_1 \cdot \vec{p}_I - m_I \vec{\sigma}_1 \cdot \vec{p}_K)(m_L \vec{\sigma}_2 \cdot \vec{p}_J - m_J \vec{\sigma}_2 \cdot \vec{p}_L)}{\sqrt{\omega(p_I, m_I) + m_I} \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}, \end{aligned} \quad (13)$$

TABLE I. The fitted LECs of the  $^1S_0$  and  $^3S_1$  partial waves in the  $\Lambda N$ - $\Sigma N$  scattering. The momentum cutoff is taken as  $\Lambda = 20$  GeV.

$C_{\Lambda N, \Lambda N}^{1S_0}$ ( $10^4$ GeV $^{-2}$ )	-0.0186	$C_{\Lambda N, \Lambda N}^{3S_1}$ ( $10^4$ GeV $^{-2}$ )	-0.0124
$C_{\Lambda N, \Sigma N}^{1S_0}$ ( $10^4$ GeV $^{-2}$ )	0.00361	$C_{\Lambda N, \Sigma N}^{3S_1}$ ( $10^4$ GeV $^{-2}$ )	-0.00499
$C_{\Sigma N, \Sigma N}^{1S_0}$ ( $10^4$ GeV $^{-2}$ )	-0.00849	$C_{\Sigma N, \Sigma N}^{3S_1}$ ( $10^4$ GeV $^{-2}$ )	-0.0129

where  $q = p_I - p_K = p_L - p_J$  is the four-momentum transfer. The isospin factors  $\mathcal{I}_{IJ, KL}$  and the values of  $f_{IKP}$  can be found in Refs. [26,29].

It is straightforward to obtain from the Lagrangian of Eq. (7) the expressions for the contact interactions, which we include at LO according to Ref. [51]. They are identical to those of the nonrelativistic approach and can be found in Refs. [25,27,44,45,48].

Due to the SU(3) flavor symmetry assumed in Ref. [25], there are two low-energy constants (LECs) in the contact terms of  $^1S_0$  and  $^3S_1$  partial waves for the  $\Lambda N$ - $\Sigma N$  coupled channels, respectively. However, as argued in Ref. [53], the SU(3) symmetry is always broken by the one-meson-exchange potential in calculations using the actual meson and baryon masses. Therefore, we have to introduce an extra contact term in each of the  $^1S_0$  and  $^3S_1$  partial waves in order to be able to carry out renormalization. Specifically, we consider the contact interactions of the following form:

$$V_{LO,C}^{1S_0} = \begin{pmatrix} C_{\Lambda N, \Lambda N}^{1S_0} & C_{\Lambda N, \Sigma N}^{1S_0} \\ C_{\Lambda N, \Sigma N}^{1S_0} & C_{\Sigma N, \Sigma N}^{1S_0} \end{pmatrix},$$

$$V_{LO,C}^{3S_1} = \begin{pmatrix} C_{\Lambda N, \Lambda N}^{3S_1} & C_{\Lambda N, \Sigma N}^{3S_1} \\ C_{\Lambda N, \Sigma N}^{3S_1} & C_{\Sigma N, \Sigma N}^{3S_1} \end{pmatrix}. \quad (14)$$

To determine these LECs we fit them to the low-energy phase shifts quoted in Ref. [53], where nonrelativistic chiral EFT potentials up to NLO were employed to describe the scattering observables. The values of LECs are listed in Table I.

Our results for the baryon-baryon scattering phase shifts in strangeness  $S = -1$  sector with isospin 1/2 are plotted in Figs. 1 and 2.<sup>2</sup> The obtained phase shifts for partial waves without contact interactions at LO are similar to those of the nonrelativistic approach except for the  $^3P_0$  partial wave in the  $\Lambda N$  channel and the  $^3P_1$  partial wave in the  $\Lambda N$  and  $\Sigma N$  channels, which strongly deviate from the phase shifts of Ref. [53]. The phase shifts and mixing angles for the  $^3S_1$ - $^3D_1$  coupled channels are consistent with the NLO results of Ref. [53]. The observed large differences between our LO results and the phase shifts in the  $^1S_0$ ,  $^3P_0$ , and  $^3P_1$  channels as well as the large NLO corrections in these partial waves found in Ref. [53] suggest that certain contributions to the potential beyond LO in these channels need to be treated nonperturbatively. In the next section, we will show how

TABLE II. The scattering lengths and effective ranges in the  $\Lambda N$  scattering for the  $^1S_0$  and  $^3S_1$  partial waves. The nonrelativistic results from LO and NLO studies are also listed.

	Rel.-LO, $\Lambda = 20$ GeV	NR-LO [25], $\Lambda = 0.6$ GeV	NR-NLO [53], $\Lambda = 0.6$ GeV
$a_{1S_0}$ (fm)	-2.94	-1.91	-2.91
$r_{1S_0}$ (fm)	-1.44	1.40	2.78
$a_{3S_1}$ (fm)	-1.41	-1.23	-1.41
$r_{3S_1}$ (fm)	1.61	2.13	2.53

the contributions of the  $P$ -wave contact interactions can be resumed nonperturbatively in the way compatible with EFT. As for the  $^1S_0$  partial wave, a more involved treatment similar to that of Ref. [43] in the nonstrange sector is needed. This case will be treated in a separate publication.

We also calculated the scattering lengths and the effective range parameters of the  $\Lambda N$  scattering for the  $^1S_0$  and  $^3S_1$  partial waves. Our results, given in Table II, agree reasonably well with those of the nonrelativistic approach except for the effective range of the  $^1S_0$  partial wave, which has different sign, as is already visible from the corresponding phase shifts shown in Fig. 1.

Since we employ here an explicitly renormalizable formalism, there is no implicit quark-mass dependence of the renormalized LECs; see Ref. [41] for more details. This allows us to calculate the phase shifts of various processes for unphysical values of the quark masses and to confront them with the results of lattice QCD calculations. The  $^1S_0$  and  $^3S_1$  phase shifts of the  $\Lambda N$  scattering for various values of the pion mass are shown in Fig. 3 together with the results of the NPLQCD [13] and HAL QCD Collaborations [5]. Note that the values of masses of pseudoscalar mesons and octet baryons are taken from the LHPC [54] and PACS-CS [55] Collaborations, which are tabulated in Table III, because NPLQCD and HAL QCD employ the lattice configurations of LHPC and PACS-CS, respectively. For  $^1S_0$  partial wave, our results for large values of the pion mass are in good agreement with those found by the HAL QCD Collaboration,<sup>3</sup> and the ones for the  $^3S_1$  partial wave agree, within errors, with the prediction of the NPLQCD Collaboration.

#### IV. NONPERTURBATIVE INCLUSION OF HIGHER-ORDER CONTACT INTERACTION POTENTIALS IN $P$ WAVES

As already pointed out above, our results indicate that at least some parts of subleading contributions to the potential will have to be treated nonperturbatively. This then raises the question of whether the scattering amplitude can still be renormalized, and the infinite-cutoff limit can be taken. Fortunately, at least for the case of nonperturbatively treated short-range interactions, the renormalization program can still be carried

<sup>2</sup>In all figures in this paper,  $p_{lab}$  is defined as the momentum of the incoming  $\Lambda$  particle in the laboratory system with the nucleon at rest.

<sup>3</sup>Notice, however, that the applicability of chiral EFT for such large values of the pion mass is rather questionable.

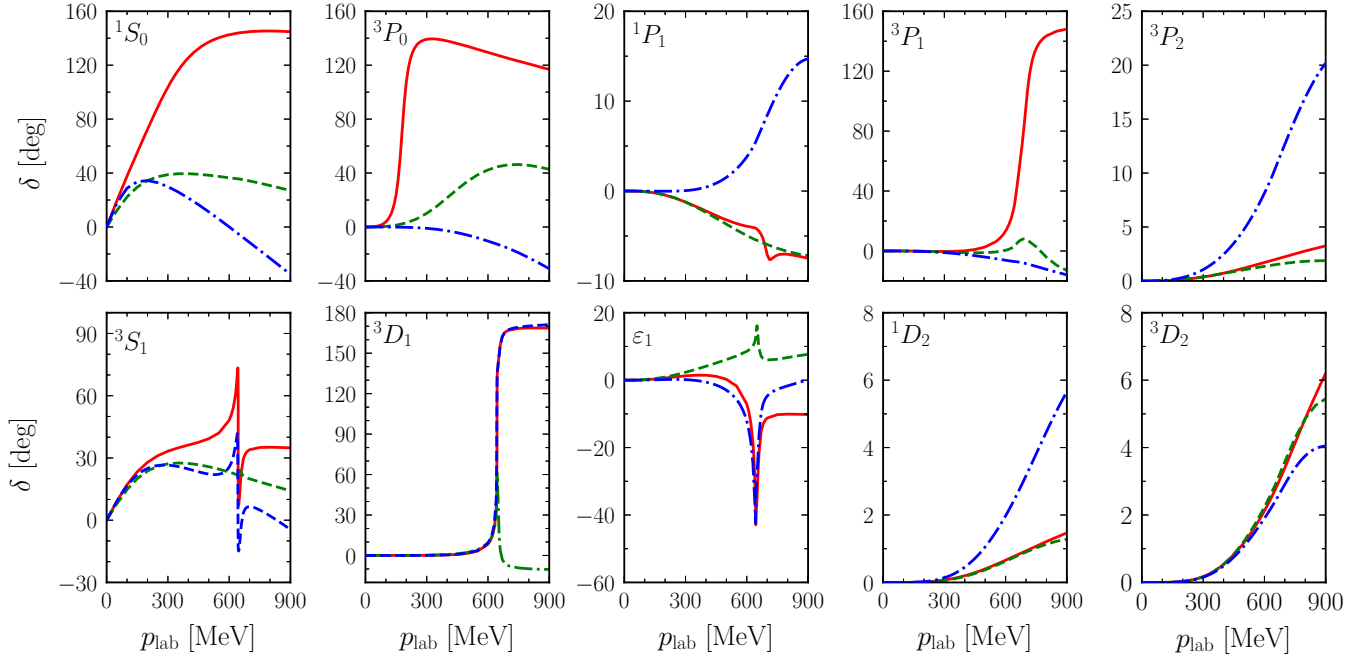


FIG. 1. Phase shifts of  $\Lambda N$  scattering. Red lines represent our LO results with the cutoff  $\Lambda = 20$  GeV, green dashed lines and blue dot-dashed lines denote the nonrelativistic results (obtained with  $\Lambda = 0.6$  GeV) at LO [25] and NLO [53], respectively.

out, as will be demonstrated below. We start by considering the case of the  ${}^3P_0$  partial waves and treat nonperturbatively the potential

$$V(p'_1, p'_2; p_1, p_2) = V_C(p'_1, p'_2; p_1, p_2) + V_{\text{LO}, M_P}^{JJ, KL}, \quad (15)$$

where  $V_{\text{LO}, M_P}^{JJ, KL}$  stands for the OME projected onto the  ${}^3P_0$  partial wave and the contact interaction potential is given

by

$$V_C = \xi(p'_1, p'_2) \mathcal{C} \xi(p_1, p_2),$$

$$\mathcal{C} = \begin{pmatrix} C_{\Lambda N, \Lambda N} & C_{\Lambda N, \Sigma N} \\ C_{\Lambda N, \Sigma N} & C_{\Sigma N, \Sigma N} \end{pmatrix}, \quad (16)$$

$$\xi(p_1, p_2) = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}.$$

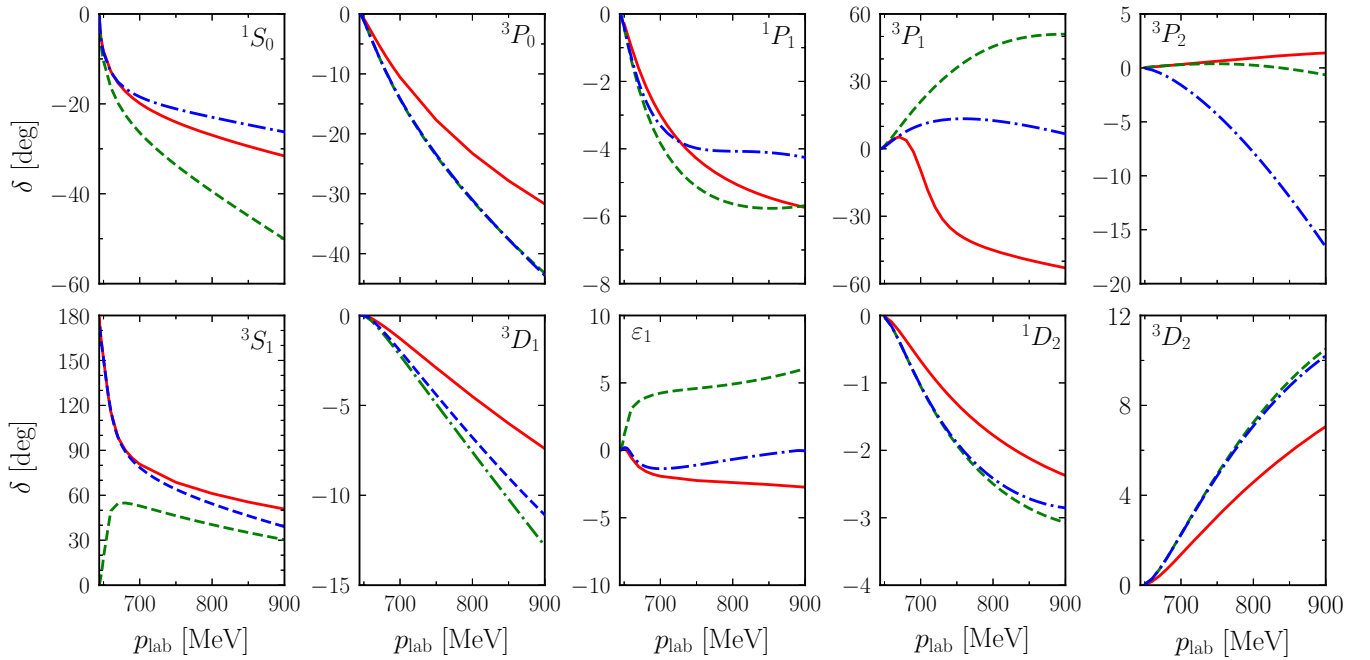


FIG. 2. Phase shifts of  $\Sigma N$  scattering; same description of curves as in Fig. 1.

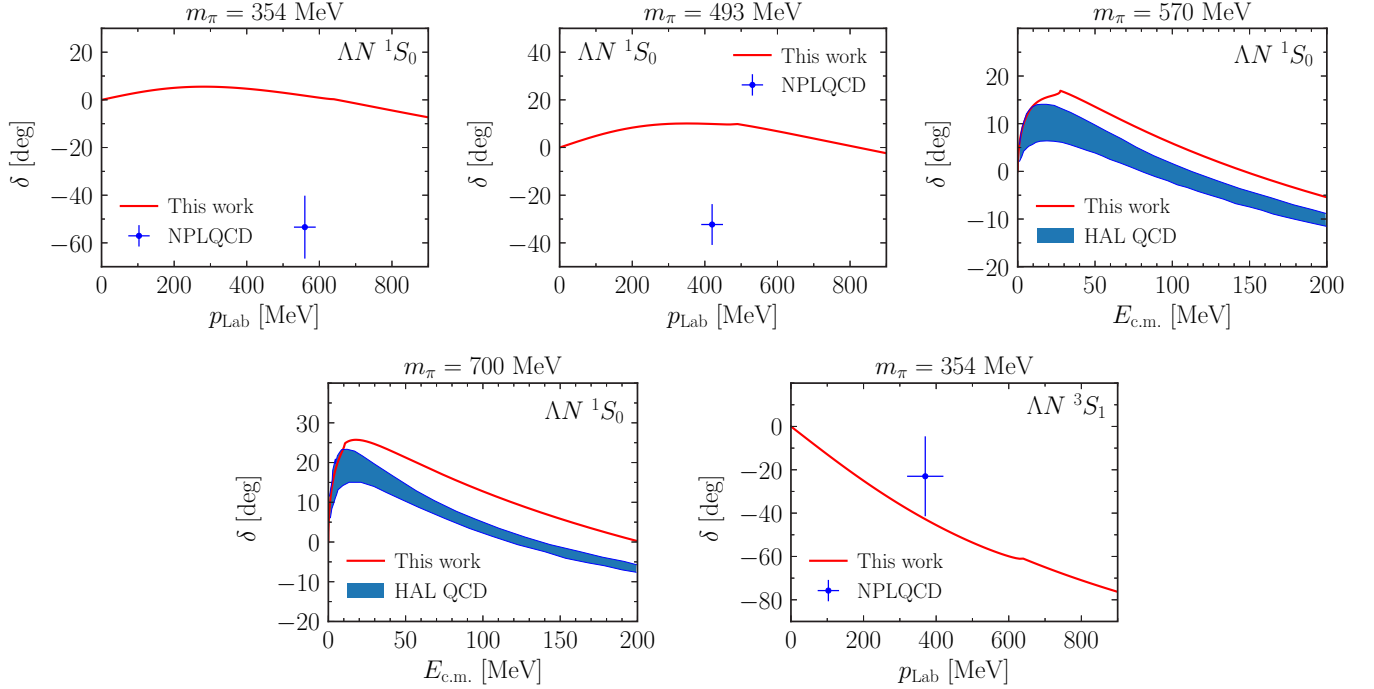


FIG. 3.  $^1S_0$  and  $^3S_1$  phase shifts of the  $\Lambda N$  scattering for various values of the pion mass.

Using the results of Ref. [56], we write the solution to the system of integral equations for the potential of Eq. (15) in a form which allows one to carry out the subtractive renormalization. We start by writing the integral equations (12) symbolically as

$$T = V + VGT, \quad (17)$$

and rewrite it in the form [43]

$$T = T_M + (1 + T_M G)T_C(1 + GT_M), \quad (18)$$

where the amplitude  $T_M$  satisfies the equation

$$T_M = V_{LO,Mp} + V_{LO,Mp} GT_M. \quad (19)$$

For the contact interaction potential of Eq. (16) the amplitude  $T_C$  is given as

$$T_C(p'_1, p'_2; p_1, p_2) = \xi(p'_1, p'_2)\mathcal{X}\xi(p_1, p_2), \quad (20)$$

where

$$\mathcal{X} = [C^{-1} - \xi G \xi - \xi GT_M G \xi]^{-1}. \quad (21)$$

Thus, the final expression for the amplitude  $T$  has the form

$$T = T_M + (1 + T_M G)\xi \mathcal{X} \xi(1 + GT_M). \quad (22)$$

Analogously to Refs. [43,51] we apply the subtractive [Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) type] renormalization; i.e., we subtract *all* divergences in *all* loop diagrams and regard the coupling constants as renormalized (i.e., cutoff-independent but renormalization scheme dependent) finite quantities; see, e.g., Ref. [57] for details of the BPHZ renormalization. Subtractive renormalization of the amplitude in Eq. (22) corresponds to the inclusion of contributions of an infinite number of counter terms generated by an infinite number of bare parameters of the effective Lagrangian [58].

To apply subtractive renormalization to Eq. (22) we notice that the amplitude  $T_M$  and expressions  $\bar{\Xi}(p'_1, p'_2) = (1 + T_M G)\xi$  and  $\Xi(p_1, p_2) = \xi(1 + GT_M)$  are finite. Therefore we need to apply subtractions only to the quantity  $\mathcal{X}$ . While the  $\xi GT_M G \xi$  term in Eq. (21) contains only the overall logarithmic divergences, the term  $\xi G \xi$  is quadratically

TABLE III. Masses of the pseudoscalar mesons and octet baryons obtained by the LHPC [54] and PACS-CS [55] Collaborations. The first error is the statistical uncertainty and the second is determined by the lattice spacing.

$m_\pi$ (MeV)	$M_K$ (MeV)	$m_N$ (MeV)	$m_\Lambda$ (MeV)	$m_\Sigma$ (MeV)
LHPC				
355.9	602.9	1157.8(6.4)(23.1)	1280.2(4.8)(25.6)	1350.2(4.8)(27.0)
495.1	645.2	1288.2(6.4)(25.8)	1369.3(4.8)(27.4)	1409.1(6.4)(28.2)
596.7	685.6	1394.8(6.4)(27.9)	1440.9(8.0)(28.8)	1463.1(9.5)(29.2)
PACS-CS				
569.7	713.2	1411.1(12.2)(21.8)	1503.8(9.8)(23.2)	1531.2(11.1)(23.6)
701.4	789.0	1583.0(4.8)(24.4)	1643.9(5.0)(25.4)	1654.5(4.4)(25.5)

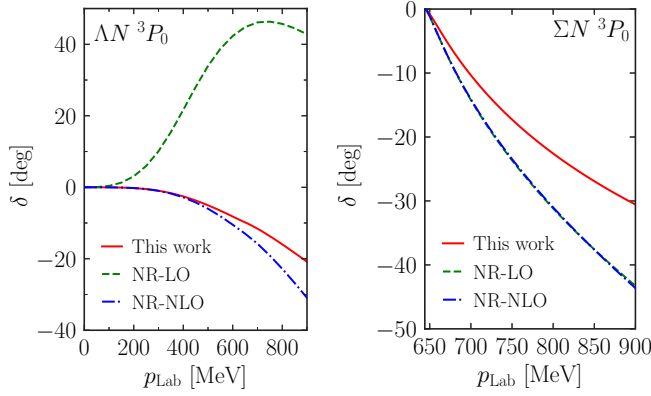


FIG. 4.  ${}^3P_0$  phase shifts of  $\Lambda N$  and  $\Sigma N$  scatterings with the subtractive renormalization.

divergent and, therefore, requires two additional BPHZ subtractions.

After carrying out the subtractions, the final expression of the amplitude reads

$$\begin{aligned} T(p'_1, p'_2; p_1, p_2) &= T_M(p'_1, p'_2; p_1, p_2) + \bar{\Xi}(p'_1, p'_2) \\ &\times \frac{1}{\mathcal{C}_R^{-1} - (\xi G \xi)^R - (\xi G T_M G \xi - \alpha)} \Xi(p_1, p_2), \end{aligned} \quad (23)$$

where the counter term matrix  $\alpha$  subtracts the overall divergences of  $\xi G T_M G \xi$ . The subtracted expression of  $(\xi G \xi)^R$  is given by

$$(\xi G \xi)^R = \begin{pmatrix} I_{\Lambda N} & 0 \\ 0 & I_{\Sigma N} \end{pmatrix}, \quad (24)$$

where the integrals  $I_{IJ}$ , subtracted at  $E = E_0 < m_I + m_J$ , are given by

$$\begin{aligned} I_{IJ} &= \int \frac{dk k^2}{2\pi^2} \\ &\times \frac{k^2(E_0 - E)^3 m_I m_J}{\sqrt{m_i^2 + k^2} \sqrt{m_j^2 + k^2} (E_0 - \sqrt{m_i^2 + k^2} - \sqrt{m_j^2 + k^2})^3} \\ &\times \frac{1}{(E - \sqrt{m_i^2 + k^2} - \sqrt{m_j^2 + k^2} - i\epsilon)}. \end{aligned} \quad (25)$$

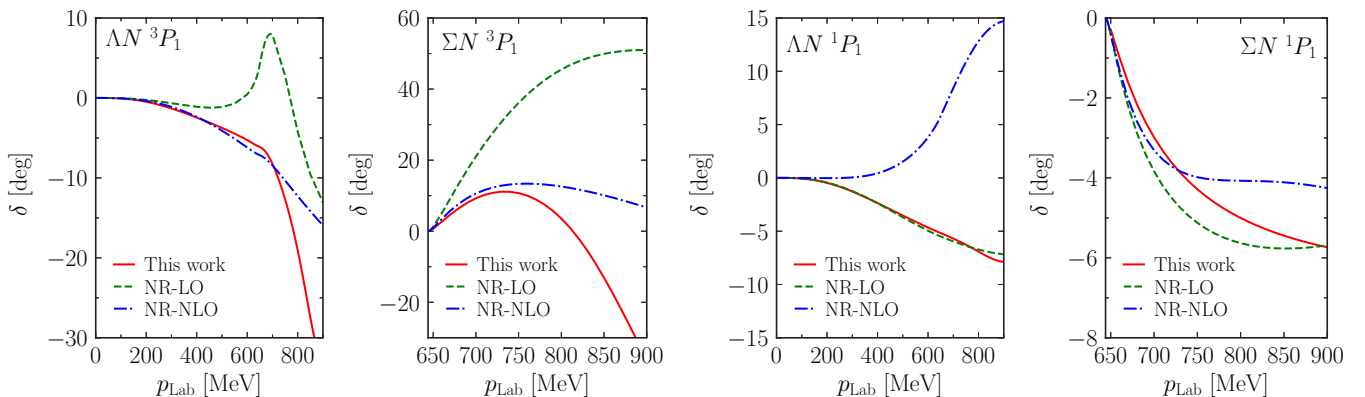


FIG. 5.  ${}^3P_1$  phase shifts of  $\Lambda N$  and  $\Sigma N$  scatterings with the subtractive renormalization.

The imaginary part of this integral has the form

$$\text{Im}(I_{IJ}) = -\frac{m_I m_J [m_I^4 - 2m_I^2(E^2 + m_J^2) + (E^2 - m_J^2)^2]^{3/2}}{16\pi E^4}. \quad (26)$$

In practice, we fix the bare constant matrix  $1/C = 1/C_R + \alpha$  as a function of the cutoff numerically in such a way that it cancels the divergent part of  $\xi G T_M G \xi$ , and the resulting cutoff-independent coupled-channel scattering amplitudes describe the phase shifts for a fixed value of the energy. The three renormalized LECs are fixed by the low-energy  ${}^3P_0$  phase shifts of the  $\Lambda N$  and  $\Sigma N$  scatterings and the inelasticity parameters of Ref. [53]. The results are shown in Fig. 4. The description of the  $\Lambda N$   ${}^3P_0$  phase shifts is satisfactory while the results of  $\Sigma N$   ${}^3P_0$  phase shifts are similar to the ones of the OME potential.

We also extended the renormalization procedure to the  ${}^1P_1$  -  ${}^3P_1$  coupled channels. We treat nonperturbatively three contact interactions for the coupled particle channels in the  ${}^3P_1$  partial wave and fix the corresponding renormalized LECs via the description of the low-energy  ${}^3P_1$  phase shifts of Ref. [53]. As seen from the results in Fig. 5, a rather good description of  ${}^3P_1$  partial wave phase shifts is achieved. One should also notice that there is small effect on the  ${}^1P_1$  partial wave phase shifts.

## V. SUMMARY

In this paper we calculated the  $\Lambda$ -nucleon scattering amplitude of the strangeness  $S = -1$  sector in the framework of manifestly Lorentz-invariant formulation of SU(3) baryon chiral perturbation theory by applying time-ordered perturbation theory [51].

For the case of baryon-baryon scattering, the relative importance of time-ordered diagrams can be determined using the Weinberg's power counting rules [18,19]. To sum up the relevant contributions it is convenient to define the effective potential as a sum of all two-baryon irreducible contributions to the scattering amplitude within TOPT. The scattering amplitudes are obtained as solutions of a system of the coupled-channel integral equations with the potentials at the corresponding order. These equations represent a

coupled-channel generalization of the Kadyshevsky equation [40] and feature a milder ultraviolet behavior as compared to their nonrelativistic analogs. By solving the integral equations for the LO amplitudes and including corrections perturbatively one can remove cutoff dependence from the physical amplitudes. Also for higher-order contact interactions included nonperturbatively one can remove all divergences by performing BPHZ type subtractions. This corresponds to taking into account an infinite number of counterterms of higher orders.

The large discrepancy between the results of our LO calculations and the phase shifts from Ref. [53] suggest that certain contributions to the  $BB$  potential beyond LO must be treated nonperturbatively in the  ${}^3P_0$  and the  ${}^3P_1$  partial waves. Thus, we have extended our calculations to include the NLO short-range interactions in these partial waves and carried out subtractive renormalization in a way consistent with EFT. The resulting phase shifts are found to be in a good agreement with the corresponding ones from the nonrelativistic approach

of Ref. [53]. We also studied the quark mass dependence of the  ${}^1S_0$  and  ${}^3S_1$  phase shifts and compared the resulting LO predictions with the available results from the lattice QCD simulations.

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