Rapid Communications

^{9}Li(d,p) reaction as a specific probe of ^{10}Li, the paradigm of parity-inverted nuclei around the $N = 6$ closed shell

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We show, within the framework of renormalized nuclear field theory and of the induced reaction surrogate formalism, that the highly debated ^{10}Li structure, studied in a recent high statistics ^{9}Li(d, p) ^{10}Li one-neutron transfer experiment, is consistent with, or better, requires, the presence of a virtual $1/2^+$ state of similar single-particle strength than that of the $1/2^-$ resonance at 0.45 ± 0.03 MeV. Based on continuum spectroscopy self-energy techniques, we find that the physical mechanism responsible for parity inversion in $^{10}_{3}$Li is the same as that at the basis of the similar phenomenon observed in $^{11}_{4}$Be and as that needed in ^{11}Li to have an important s-wave ground-state component. In particular the strong dynamical coupling between the $s_{1/2}$ and the $d_{5/2}$ states, mediated by the quadrupole vibration of the core ^{9}Li. A phenomenon which also affects the strength distribution of the $d_{5/2}$ state, in particular, in the energy range of 3–4.5 MeV. Furthermore, this mechanism is also consistent with the (normal) sequence of the $1p_{1/2}$ and $2s_{1/2}$ levels in the $N = 7$ isotones $^{12}_{5}$B and $^{13}_{6}$C. The main aim of the present Rapid Communication is that of treating structure and reactions on equal footing and in a common language. In other words, the calculation of the ^{9}Li(d, p) ^{10}Li reaction as a single conceptual step from individual single-particle motion and collective vibrations to absolute double differential cross sections of renormalized virtual and resonant final states, which can be directly compared with experiment.

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Introduction. Seven decades have elapsed since the seminal papers in which Mayer [1,2] and Mayer and Teller [3] (see also, Ref. [4]) and Jensen and coworkers [5] introduced the shell model of the atomic nucleus. Much work on the subject of the magic numbers has been dedicated ever since [6–11]. Despite this, the quest for these pillars of nuclear structure is far from completed, being very much an open question reserving surprises and challenges [12–16].

Novel magic numbers: parity inversion. The first two Meyer-Jensen magic numbers are 2 and 8 for both protons and neutrons. Increasing the number of neutrons of a normal nucleus, Pauli principle forces them into states of higher momentum. When the core becomes neutron saturated, the nucleus expels most of the wave function of the last neutrons outside to form a halo which, because of its large size, can have a lower momentum, that is less kinetic energy of confinement. The system $^{11}_{4}$Be$_7$ [$(N - Z)/A \approx 0.36$] constitutes a much studied example of a one-neutron halo nucleus ([17–19] and references therein). In principle, one could have expected that because the $1s_{1/2}$ and $1p_{3/2}$ levels are filled, the last neutron occupies a substate of the $1p_{1/2}$ orbital. However, the ground state (gs) of ^{11}Be has spin and parity $1/2^+$, implying inversion in the sequence of the $1p_{1/2}$ and $2s_{1/2}$ orbitals. Because the $1/2^+$ (-0.50 MeV) and $1/2^-$ (-0.180 MeV) levels are very close to each other and separated from the $3/2^-$ level by about 3 MeV, the $N = 8$ role of magic number has been taken over by $N = 6$. In other words, ^{11}Be can be viewed as a one-neutron system outside closed shell, the reaction ^{10}Be (d, p) ^{11}Be being, thus, the specific probe of such a system. It is, furthermore, of note that closely associated with the parity inversion phenomenon, the dipole transition between the $1/2^+$ and the $1/2^-$ states carries about one Weisskopf unit, being the strongest $E1$ transition between bound states of the whole mass table [20]. A piece of information which can be used at profit in connection with the position of the $1/2^+$ and $1/2^-$ states in ^{11}Li.

A substantial set of experimental data [21–42] and theoretical insight [43–60] exists on the unbound isotone of $^{11}_{4}$Be, namely, $^{10}_{3}$Li which indicates parity inversion also in this case. This scenario is, furthermore, consistent with—required by—the bound two-neutron halo system $^{11}_{3}$Li$_8$ (Ref. [61] and references therein). The presence of a low-lying dipole resonance with $\approx$6–8% of the dipole energy weighted sum rule, $\approx$ 0.5-MeV width, and centroid energy $\lesssim$1 MeV (Ref. [39] and references therein) and, thus, carrying about one Weisskopf unit, implies the presence of a particle-hole dipole excitation with energy not much larger than 0.3–0.5 MeV. Furthermore, the value of the absolute differential two-neutron pickup cross section associated with the reaction ^{1}H[^{11}Li, ^{9}Li(gs)] ^{3}H [42] implies that the $|s^2_{1/2}(0)\rangle$ and $|p^2_{1/2}(0)\rangle$ configurations enter the ^{11}Li ground state with about the same amplitude $(0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle$ [55], the $1p_{3/2}(\pi)$ odd proton being considered as a frozen spectator, is not explicitly written). The above requirement implies that the energies of the two configurations are not too different, likely within 0.5–0.6 MeV.

The generally accepted picture was recently set in doubt as a result of a one-nucleon transfer experiment [62] which led to the conclusion that "…the level sequence in the ^{10}Li system may not show the shell inversion features observed in other $N = 7$ isotones, such as ^{11}Be." The specificity of the ^{9}Li(d, p) reaction to provide insight into the level sequence of a nuclear system combined with the high statistics of the experiment, implied that the above conclusions constituted a serious question mark on the validity of the entire picture of shell evolution leading to the $N = 6$ magic number and associated parity inversion in exotic halo nuclei at the neutron drip line. And, thus, of crucial importance to find the missing link. This is the challenge we take up in the present Rapid Communication. And to do so, we use theoretical tools in which structure and reactions and, thus, bound states and continuum dynamics become unified.

Scope and outcome. In this Rapid Communication, we will show that the data of Cavallaro *et al.* [62] are consistent with the presence of $1/2^+$ strength at threshold—and, thus, with the findings of Refs. [21–42]—of similar magnitude as that of the $1/2^-$ resonance observed at 0.45 ± 0.03 MeV. But to observe it, one has to look into a rather different angular region than that used in the experiment (5.5°–16.5°) in which the s-strength $(2^-, 1^-)$ has a minimum, whereas the p-one $(1^+, 2^+)$ displays a maximum. Namely, conduct the search for angles $\theta_{\text{c.m.}} > 40^\circ$ and likely centered at backward angles, implying a different region of momentum transfer. Detailed predictions of the full angular distribution and of the absolute energy differential cross sections associated with the $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, and $d_{5/2}$ virtual and resonant states are given. The theoretical framework used provides, at the same time, a unified account of the experimental findings regarding the $N = 7$ isotones $^{13}_{6}$C, $^{12}_{5}$B, $^{11}_{4}$Be, and $^{10}_{3}$Li, and closes the issue concerning the missing s strength at threshold in ^{10}Li.

Below, the methods used and the results obtained in the description of the different virtual and resonant states are discussed, following the study reported in Ref. [63], extended to higher excitation energies so as to include a more significant fraction of the $5/2^+$ strength. Within this context, we note also Refs. [64,65].

Methods and results. In the calculation of $^{11}_{4}$Be [19], the four parameters characterizing the bare mean-field $U(r)$ [to be used in connection with an effective mass $m_k(r)(m_k = 0.7m(0.91m))$ for $r = 0$ $(r = \infty)$] were determined imposing the self-consistent condition that the dressed single-particle levels resulting from the coupling to the quadrupole vibration of the core ^{10}Be reproduce the experimental energies, in particular, those of the parity inverted $1/2^+$ and $1/2^-$ states, but also equally well that of the $5/2^+$ resonance. We have extended this approach to the normal sequence of $N = 7$ isotones $^{12}_{5}$B and $^{13}_{6}$C (for more details see Supplemental Material [66]). With this global potential and k

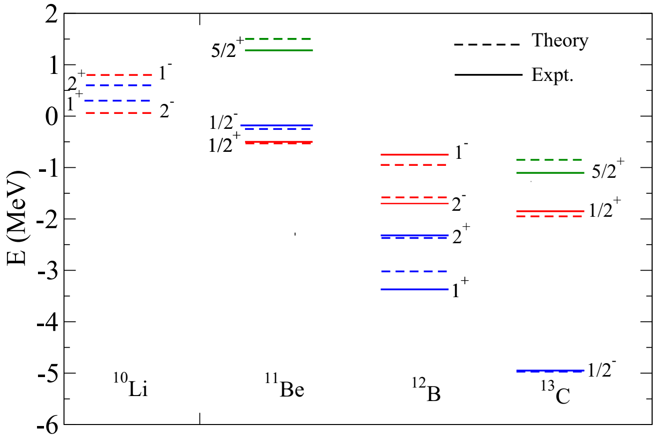

FIG. 1. The experimental energies of the low-lying states in the $N = 7$ isotones ^{11}Be, ^{12}B, and ^{13}C are shown by solid lines. The corresponding theoretical energies are displayed by dashed lines. Also reported are the predictions for ^{10}Li. States based on $1/2^-$, $1/2^+$, and $5/2^+$ neutron configurations are shown by blue, red, and green lines, respectively.

mass, together with the quadrupole vibration of ^{9}Li $(\hbar\omega_2 = 3.37$ MeV, $\beta_2 = 0.72)$, we have calculated the self-energy matrix $\Sigma^a_{ik}(E)$ for the single-particle levels i, k (50 MeV $> \epsilon_i, \epsilon_k > \epsilon_F$) of ^{10}Li in a box of radius 60 fm, carrying the quantum numbers $a = \{lj\}$. Renormalization processes associated with the coupling to the quadrupole vibrations of the core were considered,[1] in a similar manner to our previous work on ^{11}Be. The dressed $\widetilde{j^\pi}$ neutron states were determined by diagonalizing $\Sigma^a_{ik}(E)$, obtaining an accurate account of the experimental spectra (see Fig. 1). The effect of the neutron-proton interaction, leading to the observed doublet splitting in ^{12}B, is discussed below.

$\widetilde{1/2^+}$ *and* $\widetilde{1/2^-}$ *waves*. In the following, we will discuss the results associated with the $1/2^+$ and the $1/2^-$ waves, which essentially determine the spectrum up to about 2 MeV. The $1/2^+$ scattering length is $\alpha = -\lim_{k\to 0} tg(\delta_{1/2^+})/k = -8$ fm,

[1]Within this context, it has been stated (see, e.g., Ref. [67]) that exotic (halo) nuclei being much less bound than nuclei lying along the stability valley, offer a unique framework to study mean-field properties without the complications of polarization from valence particles (separability issue). In particular, one could argue that valence neutrons of ^{210}Pb can exert a much larger polarization of the core ^{208}Pb than those of ^{11}Li regarding the ^{9}Li core, let alone in the case of ^{209}Pb as compared with the extreme case of ^{10}Li. Inverting the issue one could, e.g., posit that the dressing of the valence neutron in ^{10}Li due to the quadrupole vibration of the core is weaker than that of the valence neutron of ^{209}Pb due to a similar mechanism. Now, the total zero point amplitude associated with the lowest mode of ^{208}Pb is $\beta_2 \approx 0.06$ [see, e.g., Ref. [68] Eqs. (6–52) and (6–390)] whereas that of ^{9}Li is $\beta_2 \approx 0.66$, namely, one order of magnitude larger. Because the dressing (self-energy) of single-particle states is proportional to β_2^2, the assumption of separability, essentially not applicable in the case of ^{209}Pb, can hardly be used in treating ^{10}Li or, for that sake, in treating any of the $N = 7$ isotones considered in the present Rapid Communication (see also, Ref. [69])

corresponding to the energy $\epsilon_{\widetilde{1/2}^+} = \frac{\hbar^2\kappa^2}{2m} = 0.32$ MeV, where $\kappa = 1/\alpha$ [70,71]. The eigenfunction of a state lying close to $\epsilon_{\widetilde{1/2}^+}$ and, thus, representative of this virtual state is as follows:

$$|\widetilde{1/2}^+\rangle = \sqrt{0.98}|s_{1/2}\rangle + \sqrt{0.02}|(d_{5/2} \otimes 2^+)_{1/2^+}\rangle. \quad (1)$$

Similarly, the resonant $\widetilde{1/2^-}$ state can be written as

$$|\widetilde{1/2}^-\rangle = \sqrt{0.94}|p_{1/2}\rangle + \sqrt{0.07}|\left[\left(p_{1/2}, p_{3/2}^{-1}\right)_{2^+} \otimes 2^+\right]_{0^+}, p_{1/2}; 1/2^-\rangle. \quad (2)$$

The peak and the width of the resonance are $\epsilon_{\widetilde{1/2}^-} = 0.50$ MeV, and

$$\Gamma_{\widetilde{1/2^-}} = 2\left(\frac{d\delta_{1/2^-}}{dE}\bigg|_{\epsilon_{\widetilde{1/2^-}}}\right)^{-1} = 0.35 \text{ MeV},$$

respectively. The parallel between these results and those shown in Eqs. (1)–(3) of Ref. [19] for ^{11}Be, let alone with those displayed in Fig. 1 of Ref. [61] and in Eqs. (1)–(4) of Ref. [55] is apparent. The dressed neutron couples to the $1p_{3/2}^{-1}(\pi)$ proton hole leading to the doublets $(1^-, 2^-)$ $[\widetilde{1/2}^+ \otimes p_{3/2}^{-1}(\pi)]$ and $(1^+, 2^+)$ $[\widetilde{1/2}^- \otimes p_{3/2}^{-1}(\pi)]$, both in ^{10}Li and ^{12}B. In keeping with a well-established approach [72–74], we assume that the proton interacts with the odd neutron through a spin-dependent contact interaction $V_{np} = F_0\delta(\mathbf{r}_1 - \mathbf{r}_2)[1 - \alpha(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)]$. The two parameters F_0 and α were determined by fitting the experimental splittings in ^{12}B (see Fig. 1) and optimizing the agreement with the experimental data in ^{10}Li (see also the Supplemental Material [66]). The scattering lengths of the resulting 2^-, 1^- states are equal to -19 and -5 fm, respectively ($\epsilon_{2^-} \approx 0.05$, $\epsilon_{1^-} \approx 0.8$ MeV). For the positive-parity doublet, one finds $\epsilon_{1^+} \approx 0.3$ MeV, $\epsilon_{2^+} \approx 0.6$ MeV from the corresponding strength functions (Fig. 1).

The phase shifts associated with these states in comparison with those corresponding to the bare neutron states are shown in Figs. 2(a) and 2(b). From these phase shifts, one can derive the spectral functions $\bar{\rho}_{j^\pi}(\omega) = -\frac{(2j+1)}{\pi}\text{Im}[G_{j^\pi}(\omega + i0^+) - G_{0,j^\pi}(\omega + i0^+)] = \frac{(2j+1)}{\pi}\frac{d\delta_{j^\pi}}{d\omega}$ where G and G_0 denote the Green's functions of the renormalized and of the bare particles, respectively [75–77] (see also, Ref. [78], p. 226). The differences between the spectral functions associated with the 2^-, 1^- and 2^+, 1^+ states are shown in Figs. 2(c) and 2(d).

The slopes of phase shifts associated with the 2^-, 1^- states change sign in going from bare to renormalized states, resulting in a conspicuous modification of the strength functions [Fig. 2(c)]. The positive-parity states, which are bound in the bare potential, are moved into the continuum (note the corresponding change in the value of the phase shift from π to 0 for $E_x \approx 0$) and acquire a resonant character [Figs. 2(b) and 2(d); the values of δ_{1^+} and δ_{2^+} being equal to $\pi/2$ for $E_x \approx 0.3$ and 0.7 MeV]. Summing up, the importance of many-body effects is apparent.

Based on the nonlocal self-energy matrices $\Sigma^a(r, r'; E)$, whose configuration space representation corresponds to $\Sigma^a_{ik}(E)$, in conjunction with the optical parameters of Ref. [79], we have calculated the absolute double differential

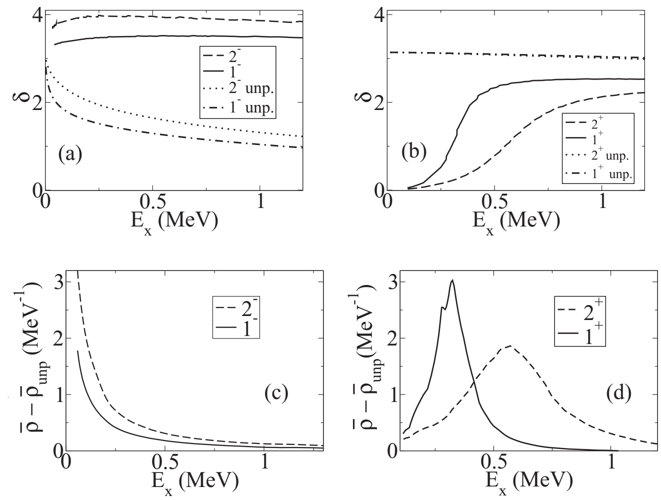

FIG. 2. Upper panels: Phase shifts associated with the renormalized and the unperturbed low-lying single-particle states of (a) negative and (b) positive parities in ^{10}Li. Lower panels: differences between the spectral functions of the renormalized (c) negative- and (d) positive-parity states and the spectral functions of the corresponding unperturbed single-particle states.

cross-section $d^2\sigma/dE\,d\Omega$ within the framework of induced-surrogate reaction formalism (Refs. [80,81] and references therein). The associated absolute single differential cross-sections $(d\sigma/dE)_{5.5^\circ–16.5^\circ}$ and $(d\sigma/d\Omega)_{0.2–1\text{ MeV}}$ were obtained by integrating $d^2\sigma/dE\,d\Omega$ in the angular and energy ranges within which data were recorded [62]. In keeping with the experimental energy resolution, the theoretical results were folded with Lorentzian functions of FWHM of 170 keV. As seen from Figs. 3(a) and 3(b), the theory provides a quantitative account of the experimental findings, confirming the (apparent) lack of any relevant contribution associated with $s_{1/2}$ strength.

The picture changes radically when looking at a quite different angular region, this time centered around more backward angles ($\theta_{\text{c.m.}} \geqslant 40^\circ$) and, thus, corresponding to larger momentum transfer as testified by the absolute cross-sections $(d\sigma/dE)$ integrated in the angular range of 50°–180° [Fig. 3(c)] as well as by the absolute differential cross section at angles $\theta_{\text{c.m.}} > 40^o$ [Fig. 3(d)]. These results unarguably demonstrate the presence of a virtual $s_{1/2}$ state, which appears absent (nonobservable) from $(d\sigma/dE)_{5.5^\circ–16.5^\circ}$.

To make such a definite statement, it is required that one is able to predict absolute one-particle transfer cross sections within experimental errors. To fulfill such requirements, one has to able to calculate continuum self-energy processes. That is, the dressing of a particle state through the coupling of the quadrupole vibration of the core ^{9}Li, renormalizing in the process energies, single-particle spectroscopic amplitudes, and wave functions (form factors) of virtual and resonant states.

Similar calculations to the ones discussed above were carried out, but, this time, for the reaction ^{9}Li(d, p) investigated at 2.36 MeV/A at the REX-ISOLDE facility (see Ref. [26] where the presence of $1s_{1/2}$ is apparent), making use of the optical potentials of this reference (see also, Ref. [82]). The outcome of such calculations provides an overall quantitative

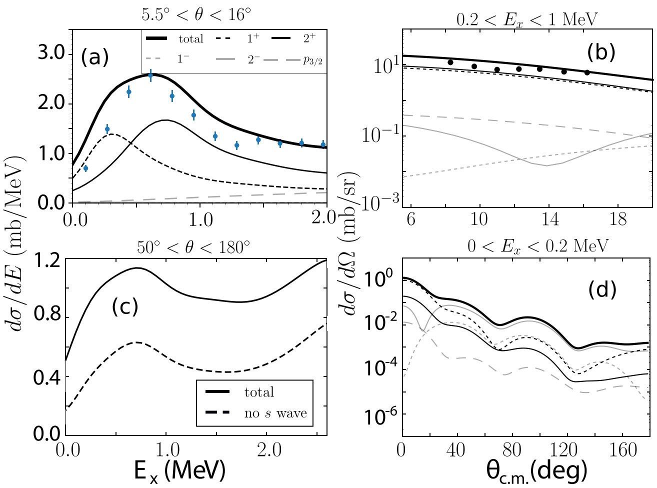

FIG. 3. (a) Theoretical prediction (continuous thick solid curve) of the ^{10}Li strength function for the $d(^{9}\mathrm{Li}, p)$ ^{10}Li reaction at 100-MeV incident energy and $\theta_{\mathrm{c.m.}} = [5.5^\circ, 16.5^\circ]$ in comparison with the experimental data (solid dots with errors) [62]. The contributions associated with states of different angular momenta are also shown; (b) corresponding angular distributions associated with the states in the energy interval 0.2–1 MeV in comparison with the experimental data; (c) predicted strength function integrated in the angular range of $\theta_{\mathrm{c.m.}} = [50^\circ, 180^\circ]$, compared to the result obtained neglecting the contributions from the s-wave (1^- and 2^- states); (d) predicted angular distributions integrated in the energy interval of 0–0.2 MeV.

description of the experimental findings (Fig. 4). Summing up, the results shown in Figs. 3 and 4 dissipate the possible doubts concerning the presence of a virtual $s_{1/2}$ state in the low-energy continuum spectrum of ^{10}Li and confirm the soundness of the picture at the basis of the description of ^{11}Li provided in Refs. [55,61] (see also, Ref. [42]). Within this scenario, Figs. 3(c) and 3(d) constitute the absolute strength function and differential cross-section predictions with an estimated error of 10%.

$\widetilde{5/2^+}$ *and* $\widetilde{3/2^-}$ *waves*. Theory predicts the existence of a resonant many-body $\widetilde{5/2^+}$ state with a centroid at ≈3.5 MeV

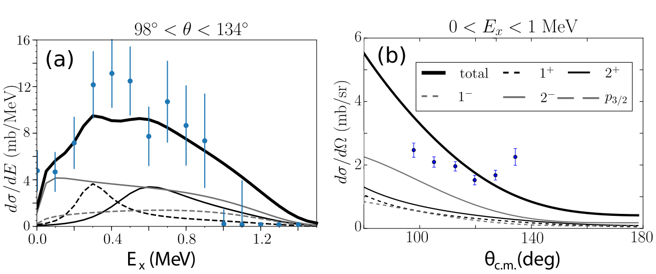

FIG. 4. (a) Theoretical prediction (continuous solid curve) of the ^{10}Li strength function for the $d(^{9}\mathrm{Li}, p)$ ^{10}Li reaction at 21.4-MeV incident energy and $\theta_{\mathrm{c.m.}} = [98^\circ, 134^\circ]$ in comparison with the experimental data (solid dots with errors) [26]. The data expressed as counts/MeV in Ref. [26] have been converted into mb/MeV using the appropriate acceptance function [84]. (b) Corresponding angular distributions associated with the states in the energy interval 0 to 1 MeV in comparison with the experimental data.

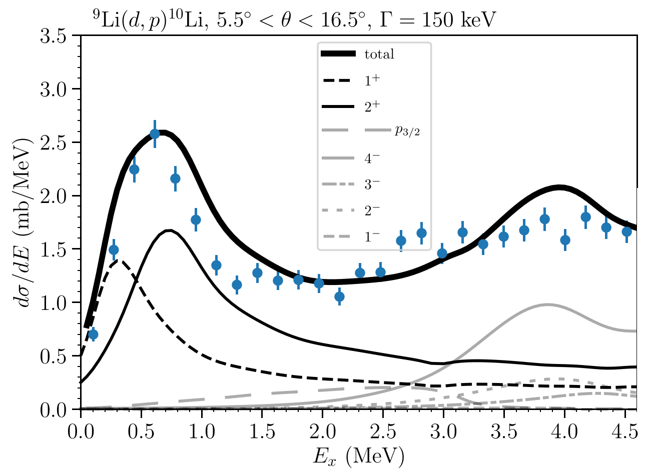

FIG. 5. Theoretical prediction (continuous solid curve) of the ^{10}Li strength function for the $d(^{9}\mathrm{Li}, p)$ ^{10}Li reaction at 100-MeV incident energy and $\theta_{\mathrm{c.m.}} = [5.5^\circ, 16.5^\circ]$ in comparison with the experimental data (solid dots with errors) [62] [see also, Fig. 3(a)].

which splits into four states $[\widetilde{5/2}^+ \otimes 1p_{3/2}(\pi)]_{1^- - 4^-}$ spanning the energy interval of 2–6 MeV. In the measured energy interval, the main contribution originates from the 4^- state. The calculation gives an overall account of the strength function as shown in Fig. 5. The $\widetilde{5/2^+}$ resonance state has a pronounced many-body character, similar to the virtual $\widetilde{1/2^+}$ and resonant $\widetilde{1/2^-}$ states discussed above. In particular, configurations including two quadrupole phonons and associated anharmonic effects play an important role in the renormalization of all three states but especially in the $\widetilde{5/2^+}$ case and have been calculated as outlined in Ref. [19] (see also the Supplemental Material [66] and Ref. [83]). These anharmonicities have been found to be more important in the present case in connection with the $\widetilde{5/2^+}$ and, in turn, for the $\widetilde{1/2^+}$ and $\widetilde{1/2^-}$ states than in the case of ^{11}Be. This is because the energy of the $\widetilde{5/2^+}$ resonance in ^{10}Li lies closer to the $\widetilde{1/2^+} \otimes 2^+$ configuration than in ^{11}Be. Theory also predicts the presence of a $3/2^-$ component, which splits into four states $[\widetilde{3/2}^- \otimes 1p_{3/2}(\pi)]_{0^+ - 3^+}$ with energies within the range of 3–6 MeV. This component, however, only produces a small and smooth background, included in Figs. 3 and 5. Another $3/2^-$ contribution, not included in our calculation, is expected at ≈5.4 MeV, based on state $|(p_{3/2}^{-1} \otimes 0_a^+)_{3/2^-}\rangle$ where $|0_a^+\rangle = |\mathrm{gs}(^{11}\mathrm{Li})\rangle$ is the pair addition mode of the core ^{9}Li, that is, the ground state of ^{11}Li.

We estimate the coupling between the $3/2^-$ resonances to occur mainly through the $(p_{1/2} \otimes 2^+)_{3/2^-}$ configuration and to be weak. Within this context, we recall a similar situation, this time for bound states, concerning the two $3/2^+$ states found in connection with the study of the septuplet of states $|h_{9/2} \otimes 3^-(^{208}\mathrm{Pb}); I\rangle$ $(I = 3/2^+, 5/2^+, \ldots, 15/2^+)$ of ^{209}Bi, the second $3/2^+$ being connected with the $2p - 1h$ state $|d_{3/2}^{-1} \otimes \mathrm{gs}(^{210}\mathrm{Po}); 3/2^+\rangle$ (see Ref. [85] and references therein). In this case, the mixing between the two states is much larger due mainly to the fact that the unperturbed energies of the two $3/2^+$ states are almost degenerate.

Conclusions. Structure and reactions, in particular, when referred to a specific elementary mode of excitation (e.g., single-particle motion) and its specific probe (one-nucleon transfer), are two aspects of the same physics. Renormalized energies and wave functions (effective Q values, spectroscopic amplitudes, and form factors) are the observables, the meeting point between theory and experiment being absolute differential cross sections.

For normal nuclei, structure essentially refers to bound states, reactions to continuum asymptotic waves. A distinction which becomes blurred in the case of exotic light bound halo nuclei, such as ^{11}Be and ^{11}Li. Just think, in the $5/2^+$ resonance in the first case (centroid $E_x = 1.28$ MeV, width $\Gamma = 100$ keV) and in the soft-dipole mode in the second ($E_x \lesssim 1$, $\Gamma = 0.5$ MeV), let alone on the virtual and resonant states in the case of ^{10}Li and of its specific probe ^{9}Li(d, p) ^{10}Li. Making use of the renormalized nuclear field theory of structure and reactions, we find it similarly possible (trying) to provide a complete description of the structure and reaction process associated with ^{11}Be and ^{11}Li than with ^{10}Li, in which case, one is referring exclusively to continuum spectroscopy (structure) and reactions. The parameters used to calculate the bare single-particle levels of ^{10}Li were obtained by extrapolating those determined following the protocol presented in Ref. [19] in connection with the calculation of the ^{11}Be spectrum applied also to the isotones $^{13}_{6}C_7$ and $^{12}_{5}B_7$ which display the Mayer-Jensen sequence. The apparent puzzle—(hieroglyphic-) like position (see Ref. [62] and Refs [7,8,10,11] therein, i.e., [30,32–34] of the present Rapid Communication) of the continuous structure and reaction aspects of ^{10}Li within the $N = 6$ closed shell, parity inverted scenario becomes readily understandable as the consequence of the choice of a restricted angular window associated with low linear momentum transfer.

Because of the strong mixing of the $s_{1/2}$ and $d_{5/2}$ virtual and resonant states through the quadrupole vibrations of the core ^{9}Li, either one reproduces both $d\sigma/dE(\widetilde{1/2^+})$ and $d\sigma/dE(\widetilde{5/2^+})$ or likely none of them. To be remembered, furthermore, is the fact that these couplings renormalize single-particle content, energies, and widths (spectral functions), and equally importantly for what concerns $d\sigma/dE$, the radial dependence of the wave functions (form factors), all elements which have to be calculated self-consistently. The role played by such requirements is likely emphasized in the case of continuous spectroscopy in which structure and reactions are just two aspects of the same physics. From here, the requirement to treat them both on a common basis and in a unified fashion.

Within the unified view adopted in the present Rapid Communication, ^{10}Li, ^{11}Be, and ^{11}Li can be viewed as the top, middle, and bottom texts of a rosettalike stone dealing with parity-inverted halo nuclei poised to acquire a permanent dipole moment. The uniqueness of the apparently different phenomena (texts) is due to the fact that they all emerge from the same underlying physics, namely: (a) a quantal phase transition [86] close to the crossing point (parity inversion); (b) spontaneous symmetry-breaking phenomenon (dipole instability) [87] and of their interplay. Another example of the relation existing among physical correct collective variables, emergent properties, transferability, and effective lower dimensionality of many-body systems (Refs. [88–90] and references therein).

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