Possible Bose-Einstein condensation of α particles in the ground state of nuclear matter

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The phase diagram of isospin-symmetric chemically equilibrated mixture of α particles and nucleons (N) is studied in the mean-field approximation. Skyrme-like parametrization is used for the mean-field potentials as functions of partial densities n_α and n_N. We find that there is a threshold value a_* of the parameter $a_{N\alpha}$ which describes the attractive interaction between α particles and nucleons. At $a_{N\alpha} < a_*$ the ground state of nuclear matter at zero temperature consists only of interacting nucleons, whereas at $a_{N\alpha} > a_*$ the nuclear ground state includes also an admixture of α's. We demonstrate that the equation of state of such α-N system contains both the first-order liquid-gas phase transition and the Bose-Einstein condensation of α particles.

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I. INTRODUCTION

As commonly accepted, see, e.g., Refs. [1,2], the ground state (GS) of isospin-symmetric nuclear matter at zero temperature and pressure is characterized by the following parameters ($\hbar = c = k_B = 1$):

$$n_B = n_0 \simeq 0.15\ \mathrm{fm}^{-3},$$
$$W = \frac{\varepsilon}{n_B} - m_N = W_0 \simeq -15.9\ \mathrm{MeV}, \qquad (1)$$

where n_B is the baryonic density, W is the binding energy per baryon, ε is the energy density, and m_N is the nucleon mass. The Coulomb interaction effects are assumed to be switched off. In what follows we use $m_N \simeq 938.9$ MeV, neglecting a small difference between the proton and neutron masses.

The nuclear GS is usually considered in terms of interacting nucleons, i.e., neutrons and protons. On the other hand, it is well known that nuclear matter has a tendency for clusterization at subsaturation densities and moderate temperatures, as observed at intermediate and high collision energies. Especially clear this was demonstrated by nuclear fragmentation reactions which have been extensively studied, both experimentally [3–5] and theoretically [6–8]. Particularly, α particles (^{4}He nuclei) are abundantly produced in intermediate-energy heavy-ion collisions [9]. The α-decay of heavy nuclei is another indication that α-like correlations exist in cold nuclei.

In recent years many theoretical models have been used to describe nuclear systems with light clusters, see, e.g., Refs. [10–20]. The quartet-type nucleon correlations have been introduced in Ref. [10] and then studied in subsequent publications, see, e.g., Refs. [11,12]. The authors claim that at moderate excitation energies such correlations may give rise to the α-particle condensate, analogously to the famous Hoyle state in ^{12}C. Below we consider a similar type of correlations but represented by α particles coexisting with nucleons even at zero temperature.

The focus of our present study is on the role of interaction between α particles and nucleons. It is well known from scattering experiments (see, e.g., Ref. [21]) that the α-N interaction is attractive in the p-wave channel and the corresponding phase shifts show characteristic structures associated with the ^{5}He and ^{5}Li resonances. Such behavior is well reproduced by modern *ab initio* calculations [22]. In the present paper we include this feature by introducing an attractive α-N term in the Skyrme-like interaction potential.

In our previous paper [23] we have proposed a mean-field model with Skyrme-like interactions to describe the isospin-symmetric α-N matter under conditions of chemical equilibrium. It was assumed that the GS of such matter contains only nucleons and no α's. Below we demonstrate that this model allows also another possibility, when the GS contains both nucleons and α particles. This is controlled by the parameter $a_{N\alpha}$ which determines the strength of attractive αN interaction. For $a_{N\alpha} < a_*$, where $a_* \sim 2$ GeV fm^3 is a certain threshold value (see below), the GS of nuclear matter at temperature $T = 0$ contains only nucleons. In the present study we show that for sufficiently strong αN attraction, $a_{N\alpha} > a_*$, the nuclear GS contains a nonzero fraction of α particles. In this case we obtain a qualitatively different phase diagram of α-N matter which contains both the first-order liquid-gas phase transition and the Bose-Einstein condensate (BEC) of α particles.

II. THE MODEL

Let us consider an isosymmetric system (with equal numbers of protons and neutrons) composed of nucleons and α particles with vacuum mass $m_\alpha \simeq 3727.3$ MeV. In the grand-canonical ensemble the system pressure $p(T,\mu)$ is a function of temperature T and baryon chemical potential μ. The latter is responsible for conservation of the baryon number. The chemical potentials of N and α satisfy the relations $\mu_N=\mu$, and $\mu_\alpha=4\mu$, which correspond to condition of the chemical equilibrium in the α-N system with respect to reactions $\alpha\leftrightarrow 4N$. The baryon number density $n_B(T,\mu)=n_N+4n_\alpha$, the entropy density $s(T,\mu)$, and the energy density $\varepsilon(T,\mu)$ can be calculated from $p(T,\mu)$ and its partial derivatives using the standard thermodynamic relations. Our consideration is restricted to temperatures $T\lesssim 20$ MeV to avoid complications due to production of mesons and hadronic resonances.

In our mean-field approach the pressure $p(T,\mu)$ of the α-N system is taken in the form

$$p=p_N^{\rm id}(T,\widetilde{\mu}_N)+p_\alpha^{\rm id}(T,\widetilde{\mu}_\alpha)+\Delta p(n_N,n_\alpha). \qquad (2)$$

Here $p_i^{\rm id}$ are the ideal-gas pressures of ith particles:

$$p_i^{\rm id}(T,\widetilde{\mu}_i)=\frac{g_i}{(2\pi)^3}\int d^3k\frac{k^2}{3E_i}\times\left[\exp\left(\frac{E_i-\widetilde{\mu}_i}{T}\right)\pm 1\right]^{-1},\quad (i=N,\alpha), \qquad (3)$$

where $E_i=\sqrt{m_i^2+k^2}$ and g_i is the spin-isospin degeneracy factor ($g_\alpha=1$, $g_N=4$). Upper and lower signs in Eq. (3) correspond to $i=N$ and $i=\alpha$, respectively. Particle interactions in the α-N mixture are described by introducing temperature-independent mean-field potentials which are parametrized in a Skyrme-like form. These potentials lead to the shifts of the chemical potentials with respect to their ideal gas values.

Following Ref. [23], we apply the parametrizations

$$\widetilde{\mu}_N=\mu+2a_Nn_N+2a_{N\alpha}n_\alpha-\frac{\gamma+2}{\gamma+1}b_N(n_N+\xi n_\alpha)^{\gamma+1}, \qquad (4)$$

$$\widetilde{\mu}_\alpha=4\mu+2a_{N\alpha}n_N+2a_\alpha n_\alpha-\frac{\gamma+2}{\gamma+1}b_N\xi(n_N+\xi n_\alpha)^{\gamma+1}, \qquad (5)$$

and

$$\Delta p(n_N,n_\alpha)=-a_Nn_N^2-2a_{N\alpha}n_Nn_\alpha-a_\alpha n_\alpha^2+b_N(n_N+\xi n_\alpha)^{\gamma+2}, \qquad (6)$$

where a_N, a_α, $a_{N\alpha}$, b_N, ξ, and γ are positive model parameters. Using Eqs. (2)–(6) one can show that the condition of thermodynamic consistency, $n_B=(\partial p/\partial\mu)_T$, holds for all T and μ. The BEC of α particles becomes possible when $\widetilde{\mu}_\alpha$ reaches its maximum value $\widetilde{\mu}_\alpha=m_\alpha$.

The terms with coefficients $a_N, a_\alpha, a_{N\alpha}$ in Eqs. (4)–(6) describe attractive forces, whereas the terms proportional to b_N are responsible for repulsive interactions. Similarly to Ref. [23] we take the parameters $\gamma=1/6$, $a_\alpha=3.83$ GeV fm^3, and $\xi=2.01$. These values were fixed by fitting the parameters $n_\alpha=0.036$ fm^{-3} and $W_\alpha=\varepsilon_\alpha/n_\alpha-m_N=-12$ MeV for the GS of pure α matter at $T=0$ [24,25]. The nucleon parameters $a_N=1.17$ GeV fm^3 and $b_N=1.48$ GeV fm$^{7/2}$ were obtained in Ref. [23] by fitting the GS characteristics (1) of the pure nucleon matter, i.e., assuming that it contains no α particles.

Thus, only one unknown parameter is left in the parametrization (4)–(6), namely the cross-term coefficient $a_{N\alpha}$ which determines the αN attraction strength. It will be demonstrated below that this parameter plays a crucial role in thermodynamics of α-N matter. In Ref. [23] we have shown that the results are qualitatively different for $a_{N\alpha}$ smaller or larger than the threshold value

$$a_*=-\frac{2}{n_0}(W_0+B_\alpha)+\frac{1}{2}\frac{\gamma+2}{\gamma+1}b_N\,\xi\,n_0^\gamma\simeq 2.12\ \text{GeV fm}^3, \qquad (7)$$

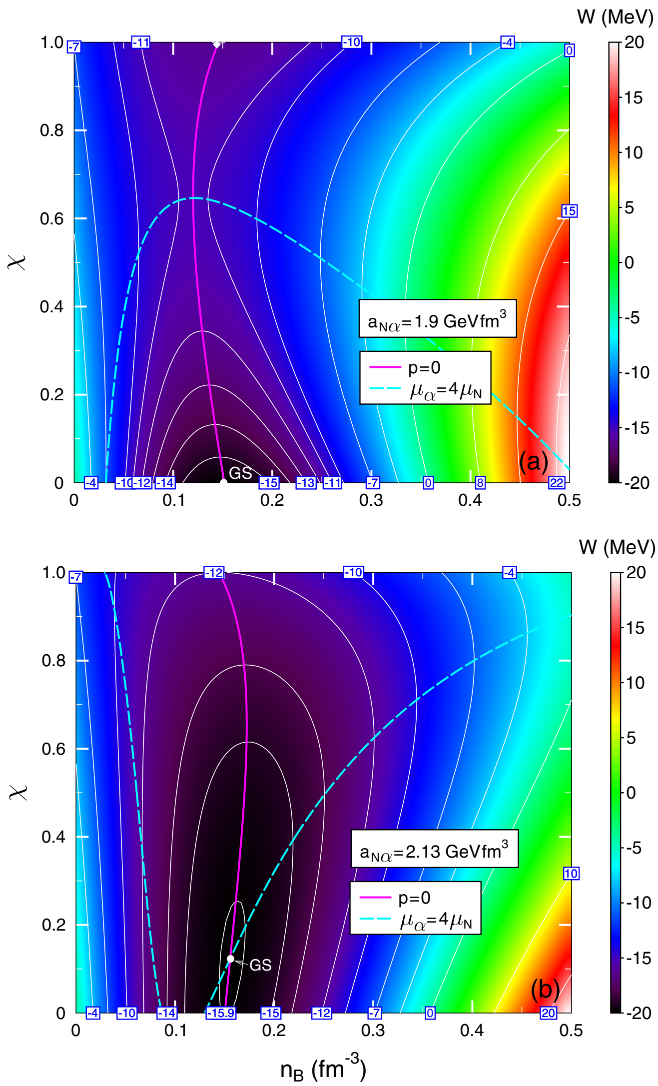

FIG. 1. The binding energy per baryon W for cold α-N matter on the (n_B,χ) plane. White contours correspond to constant W values (given in MeV inside white boxes). Panel (a) shows the results for $a_{N\alpha}=1.9$ GeV fm^3 $<a_*$. The GS corresponds to a pure nucleon matter ($n_\alpha=0$) with parameters given in Eq. (1). The metastable state of a pure α matter is shown by the white diamond at the $\chi=1$ axis. The results in (b) are obtained for $a_{N\alpha}=2.13$ GeV fm^3 $>a_*$. In this case the system has only one minimum of W and the GS contains both nucleons and α's.

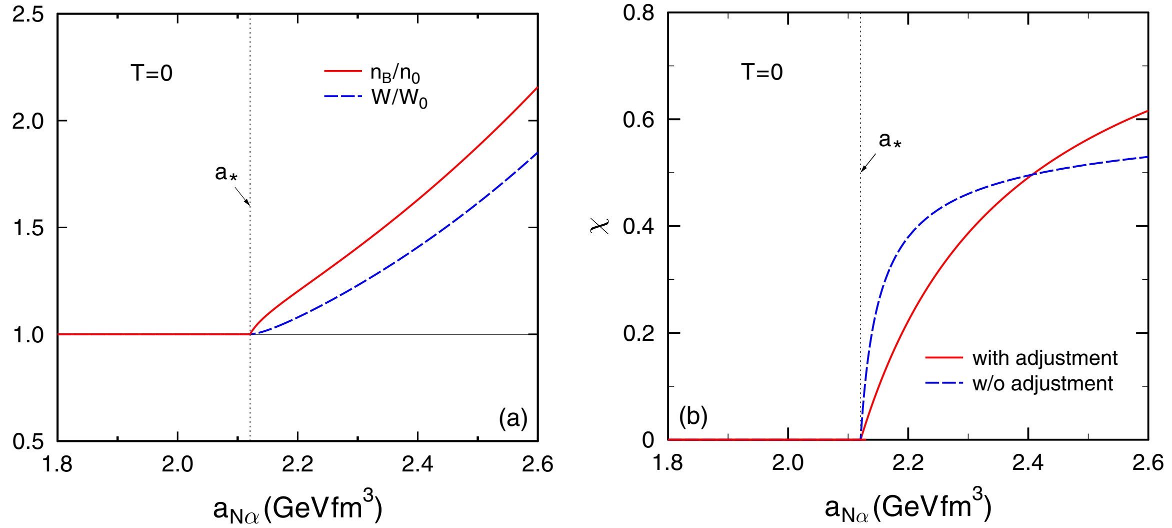

FIG. 2. (a) Relative change of baryon density (the solid line) and binding energy (the dashed curve) in the GS of α-N matter as a function of $a_{N\alpha}$. (b) Fraction of α's in the GS of α-N matter at different $a_{N\alpha}$. The solid (dashed) line is obtained with (without) readjusting a_N, b_N.

where $B_\alpha = m_N - m_\alpha/4 \simeq 7.1$ MeV is the binding energy per baryon of the α nucleus.[1]

To constrain the parameter $a_{N\alpha}$, we have compared our results with those obtained within the virial expansion approach [14] for the isosymmetric α-N matter. The latter is justified in the domain of small densities n_N and n_α where one can calculate thermodynamic properties of nuclear matter by using empirical phase shifts of NN, $N\alpha$, and $\alpha\alpha$ scattering. In this analysis, $a_{N\alpha}$ has been varied in the interval[2] from 1 to 2.5 GeV fm^3. The best agreement with the virial approach at $T \sim 2$ MeV has been achieved for $a_{N\alpha}$ close to a_* but with a significant uncertainty of about $\pm 10\%$. From this analysis we cannot conclude which option, $a_{N\alpha} > a_*$ or $a_{N\alpha} < a_*$, is more realistic and, therefore, consider both of them.

III. RESULTS FOR $T = 0$

At $a_{N\alpha} < a_*$ the GS of α-N mixture corresponds to a pure nucleonic matter ($n_\alpha = 0$) which satisfies the GS properties (1). This option has been studied earlier in Ref. [23]. The results for $a_{N\alpha} = 1.9$ GeV fm^3 are presented in Fig. 1(a) where contours of W are shown in the (n_B, χ) plane. Here $\chi \equiv 4n_\alpha/n_B$ is the fraction of nucleons carried by α particles. The red solid curve is the line of zero pressure $p = 0$. Besides the GS (1), there is another state with a local minimum of the energy per baryon at $\chi = 1$ and $n_N = 0$, which corresponds to the pure α matter, considered earlier in Ref. [24]. This state is metastable because it has a smaller binding energy $|W|$ as compared to the pure nucleonic matter. In the (n_B, χ) plane the two minima are separated by a potential barrier [see the dashed line in Fig. 1(a)].

In the present paper we consider a new possibility which arises at $a_{N\alpha} > a_*$. In this case, the energy per baryon has only one minimum in the (n_B, χ) plane at $\chi > 0$, and the GS contains both the Fermi distribution of nucleons as well as the Bose condensate of α particles. An example of such a system is shown in Fig. 1(b) for $a_{N\alpha} = 2.13$ GeV fm^3. Indeed, one can see that now the GS minimum of W is shifted from the $\chi = 0$ axis to $\chi \simeq 0.12$. In the considered case both pure nucleonic- and pure α matter appear as unstable states.

According to our calculation, at $a_{N\alpha} > a_*$ the GS parameters n_B and $|W|$ increase monotonously with $a_{N\alpha}$ as shown in Fig. 2(a). Therefore, in this case the GS of the α-N mixture does not satisfy the conditions (1) since $n_B > n_0$ and $|W| > |W_0|$. To fulfill the empirical constraints (1) we readjust the nucleon interaction parameters a_N and b_N.[3] The results of such calculation are shown in Fig. 2(b). For example, at $a_{N\alpha} = 2.13$ GeV fm^3 the readjusted coefficients a_N and b_N are increased by about 1% as compared to their values at $a_{N\alpha} < a_*$, but the fraction of α's, χ, dropped significantly, from 0.12 to 0.04.

IV. PHASE DIAGRAM OF α-N MATTER

In Figs. 3(a) and 4 we present the phase diagram of α-N matter for $a_{N\alpha} = 2.13$ GeV fm$^3 > a_*$ on the (μ, T) and (n_B, T) planes, respectively. Note that the squares show positions of the GS in both diagrams. The model predicts the first-order phase transition of the liquid-gas type with following parameters of critical point (CP),

$$T_{\rm CP} \simeq 14.7 \text{ MeV}, \quad \mu_{\rm CP} \simeq 907 \text{ MeV},$$
$$n_{B\rm CP} \simeq 0.048 \text{ fm}^{-3}, \quad \chi_{\rm CP} \simeq 0.19. \qquad (8)$$

[1]Note that $a_{N\alpha} = a_*$ satisfies approximately the "mixing rule" $a_{N\alpha} \simeq \sqrt{a_N a_\alpha}$, found experimentally [26] for attractive mean-field interactions in binary mixtures of molecular liquids.

[2]A similar analysis has been made in Ref. [23] for $a_{N\alpha} = 1$ and 1.9 GeV fm^3.

[3]We find that readjusted parameters a_N and b_N grow almost linearly with $a_{N\alpha}$. The increase factors are, respectively, 1.25 and 1.4 when $a_{N\alpha}$ changes from a_* to $1.1\,a_*$.

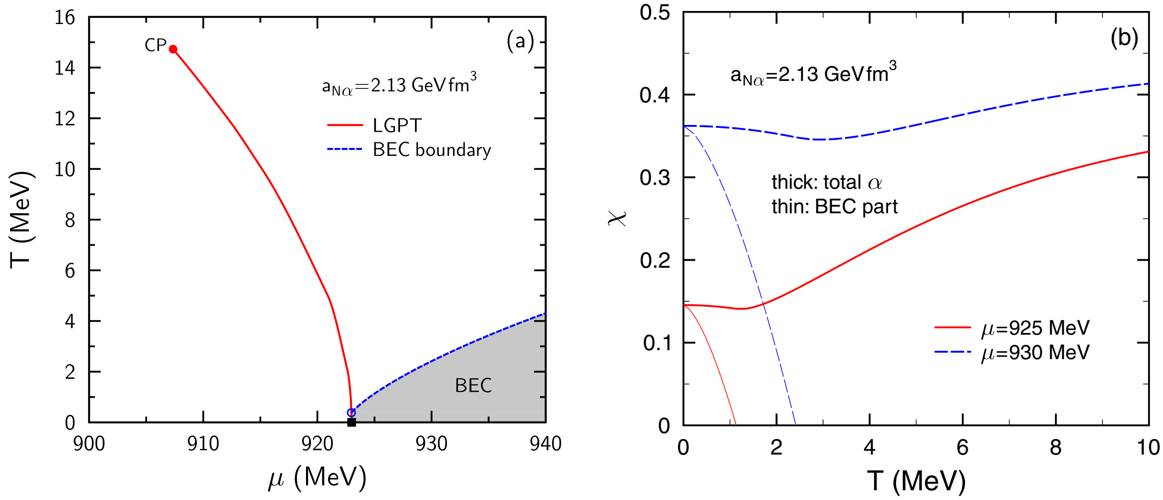

FIG. 3. (a) Phase diagram of α-N matter for $a_{N\alpha} = 2.13$ GeV fm^3 on the (μ, T) plane. The full square shows the GS position, whereas the full and open circles corresponds to the critical and triple points, respectively. Shading marks states with BEC of α particles. (b) Fractions of α particles in α-N matter as functions of T for $a_{N\alpha} = 2.13$ GeV fm^3. The solid and dashed lines correspond to $\mu = 225$ and 230 MeV, respectively. The thick lines show fractions of all α's, while the thin ones give the contribution of the condensate.

In our calculations we use the Gibbs conditions of the phase equilibrium [27]. We also predict the BEC phase of α particle. The regions of phase diagrams containing states with α condensate are shown in Figs. 3(a) and 4 by shading.

In the present scenario, the BEC states are thermodynamically stable. This is different from the case considered in our previous work [23] where the BEC states of α's appeared as a metastable phase. One can see that these states are located to the right-hand side from the phase transition line started from the triple point in the (μ, T) plane. We find a rather low temperature of the triple point, $T_{\rm TP} \simeq 0.38$ MeV.

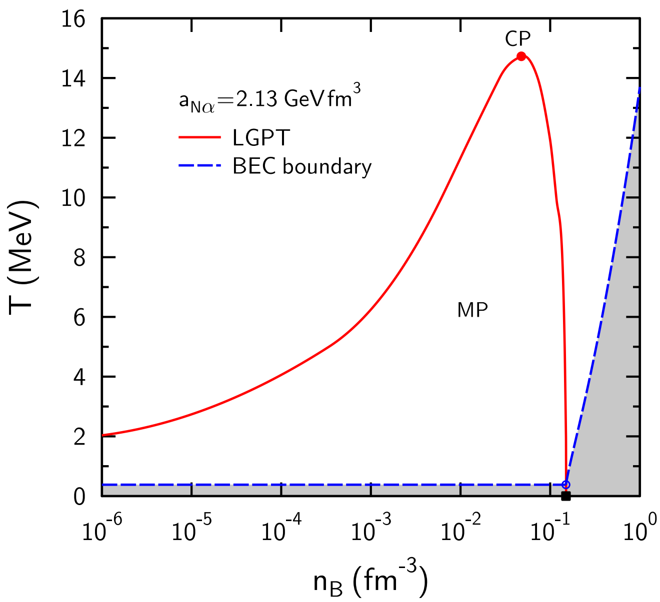

FIG. 4. Same as Fig. 3(a) but for the (n_B, T) plane. MP denotes the mixed-phase region.

Qualitatively similar results are found for other values of $a_{N\alpha} > a_*$.

As one can see from Fig. 4 the BEC states exist even in the two-phase coexisting region at $n_B < n_0$. Here the system splits into domains of higher (liquid) and smaller (gas) densities. In this case the α condensate is localized in liquid domains (droplets) which have baryon density close to n_0 irrespective of the baryon density n_B in the MP. On the other hand, the condensate does not appear in the pure gas phase located on the left from the MP region.

It is interesting to estimate the fraction of α particles in the BEC phase as a function of temperature. According to our present model, at $T \to 0$ and $\mu > m_N + W_0 \simeq 923$ MeV all α's are in the condensate, but at nonzero temperature they partly go to the noncondensed phase. A more detailed information is given in Fig. 3(b) where we show the temperature dependence of χ for two fixed values of μ. The thick lines represent the total fraction of α's and the thin ones give the fraction of α's in the BEC phase. At considered values of μ this fraction decreases with T and vanishes at the BEC boundary shown by the dashed line in Fig. 3(a).

To study sensitivity of the results to the coefficient of αN attraction, we calculated characteristics of the CP and TP at different $a_{N\alpha}$.[4] As one can see in Fig. 5(a), the critical temperatures $T_{\rm CP}$ at small and large $a_{N\alpha}$ are close to those for a pure nucleonic ($T_{\rm CP} \simeq 15.3$ MeV) and pure α ($T_{\rm CP} \simeq 10.2$ MeV) matter, respectively. The values of $T_{\rm CP}$ and $\chi_{\rm CP}$ show a nonmonotonous behavior at $a_{N\alpha} \sim a_*$. Both these quantities have local maxima at $a_{N\alpha} = a_*$. On the other hand, $T_{\rm TP}$ and $\chi_{\rm TP}$ are increasing functions of $a_{N\alpha}$. One can see that $\chi_{\rm TP}$ rapidly raises with $a_{N\alpha}$ above the threshold $a_{N\alpha} = a_*$.

We add a comment on Ref. [28] where the authors consider the α condensation at low temperatures ($T \lesssim 1$ MeV) and

[4] Some results for $a_{N\alpha} < a_*$ have been presented earlier in Ref. [23].

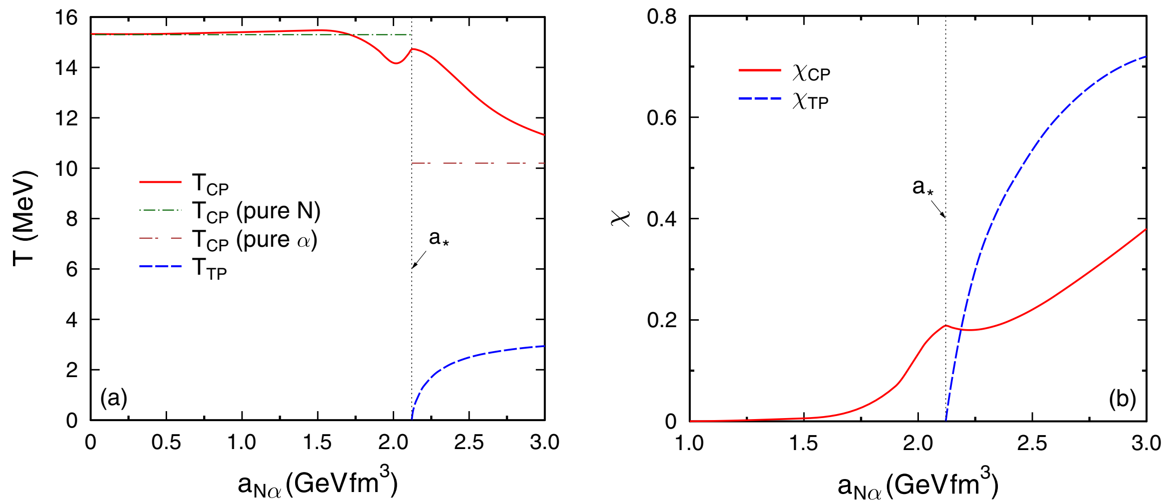

FIG. 5. (a) The temperatures at the CP (the solid line) and TP (the dashed curve) for α-N matter as functions of $a_{N\alpha}$. The horizontal upper and lower dash-dotted lines correspond to the critical temperatures of the pure nucleonic- and α matter, respectively. (b) Same as (a) but for relative fractions of α's.

densities ($n_B \lesssim 0.1\,n_0$) but assuming that the α-N system is homogeneous. However, our calculation has shown, that under such conditions the nuclear matter exists in a highly inhomogeneous liquid-gas phase. As seen in our Fig. 4, a homogeneous gaslike phase at $T \lesssim 1$ MeV appears only at extremely small densities $n_B \ll n_0$. According to our model, the BEC does not occur in this low-density domain.

V. CONCLUSIONS

We have considered a possibility that the Bose-Einstein condensate of α particles may coexist with nucleons in the ground state of cold isosymmetric nuclear matter. In our Skyrme-like mean-field model this possibility arises when attractive αN interaction is strong enough. We have investigated the phase diagram of α-N matter at finite temperatures in a broad interval of baryon densities. It turns out that such a system has both the liquid-gas phase transition and the BEC of α particles. It is interesting that the BEC phase appears also in the liquid-gas coexistence region at temperatures below the triple point. This picture differs significantly from that considered in Ref. [23], where smaller α-N couplings have been used and the α condensate appeared only as a metastable phase.

In this paper we have considered an idealized system consisting of nucleons and α particles. Of course, other light clusters and heavier fragments can play an important role in intermediate-energy heavy-ion collisions and in astrophysical processes, such as supernova explosions and neutron-star merges. We plan to include such clusters in future calculations.

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