$16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions at near-Coulomb-barrier **energies from statistical model calculations**

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Fusion-evaporation reactions of carbon and oxygen isotopes at near-Coulomb-barrier energies are important subjects in nuclear-reaction studies. They are closely related to nucleosynthesis in astrophysics and may provide novel opportunities to study the α -clustering effect in light nuclei. In this work, the $16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions are studied by using the statistical model based on the Hauser-Feshbach formalism. It is found that the statistical model with the original Gilbert-Cameron level-density formula typically underestimates the α -emission cross sections. To overcome this problem, we introduce for the residual nuclei a new level-density formula with the correction term inspired by theoretical considerations on the α-clustering effect. Compared with the original Gilbert-Cameron level-density formula, our statistical-model calculations with the new level-density formula show significant improvements.

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I. INTRODUCTION

The compound nucleus (CN) plays an important role in nuclear physics. It lies behind many important nucleosynthesis reactions in astrophysics [\[1–4\]](#page-6-0) and the artificial synthesis of superheavy elements [\[5–8\]](#page-6-0). The evaporation process of the compound nucleus could be studied by the statistical model based on the Hauser-Feshbach (HF) formalism [\[9\]](#page-6-0), which has been applied successfully to study the evaporation process of various CNs and give theoretical results in good agreement with experimental data [\[10–14\]](#page-6-0). Like other successful models in nuclear physics, the statistical model needs to be improved in specific situations in order to explain the experimental data better. In Refs. [\[15–20\]](#page-6-0), the possible memory effect from the entrance channel is included in the statistical model, as well as the contributions from direct reactions. In Refs. [\[21–24\]](#page-6-0), some physically motivated modifications are made to the level-density formula and the optical model potentials to improve the theoretical results. References $[25,26]$ further suggest combining nuclear-structure models with the statistical model to give refined descriptions of the fusion-evaporation reactions.

In recent years, there were some attempts to study the α -clustering effect in fusion-evaporation reactions [\[27–31\]](#page-6-0), which could be valuable complements to studies based on nuclear structures and direct nuclear reactions [\[32–38\]](#page-6-0). Some works show that, in some fusion-evaporation reactions of light nuclei, the α -emission cross sections are significantly

underestimated by the statistical-model calculations [\[39–43\]](#page-6-0). This might be related to the α -clustering effect in the entrance channels. In Ref. [\[44\]](#page-6-0), we modify the level densities in the $16,18$ O + $12,13$ C fusion-evaporation reactions in a manner inspired by the α -clustering effect. With such a modification, the improved statistical model now reproduces successfully the α -emission cross sections and thus provides a possible resolution of the α -emission underestimation discrepancy of the original statistical-model calculations. It is interesting to generalize our previous work to other fusion-evaporation reactions of light nuclei.

In this work, we generalize our previous work further to the $16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions at near-Coulomb-barrier energies. Similar to the 16,18 O + 12,13 C fusion-evaporation reactions studied in Refs. [\[43,44\]](#page-6-0), the α -emission cross sections of these reactions are also found to be underestimated by the statistical-model calculations with the original Gilbert-Cameron level-density formula. We propose a new level-density formula motivated by the α -clustering effect to give better theoretical descriptions of not only their α -emission cross sections but also some other emission channels. In contrast to the statisticalmodel calculations with the original Gilbert-Cameron leveldensity formula, our calculations show significant improvements. Compared with the level-density formula given in Ref. [\[44\]](#page-6-0), the new level-density formula also has the advantage to have fewer adjustable parameters. Therefore, it improves the predictability of our model. The rest of this work is organized as follows: In Sec. [II,](#page-2-0) the theoretical formalism of our work is introduced. Section [III](#page-4-0) gives the numerical results on the $16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions at near-Coulomb-barrier energies. Section [IV](#page-5-0) ends this work with conclusions and remarks.

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FIG. 1. The level densities of the residual nuclei as a function of the excited energy *E*[∗] for the oxygen + oxygen system. The solid red lines are the modified Gilbert-Cameron level densities ρ_{MGC} and the dashed green lines are the original Gilbert-Cameron level densities ρ_{OGC} .

FIG. 2. The level densities of the residual nuclei as a function of the excited energy *E*[∗] for the oxygen + carbon system. The solid red lines are the modified Gilbert-Cameron level densities $\rho_{\rm MGC}$ and the dashed green lines are the original Gilbert-Cameron level densities $\rho_{\rm OGC}$.

FIG. 3. The α -emission cross sections versus the center-of-mass incident energies $E_{\text{c.m.}}$ for the $16-18$ O + 16 O fusion-evaporation reactions. Solid points are the experimental data taken from Refs. [\[49,50\]](#page-6-0). The dashed red lines are the theoretical results given by the statistical-model calculations with the original Gilbert-Cameron level-density formula, while the solid green lines are theoretical results given by the statisticalmodel calculations with the new level-density formula inspired by the α -clustering effect.

II. THEORETICAL FRAMEWORK

In this work, we study the $16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions at near-Coulomb-barrier energies using the statistical model based on the HF formalism. In the statistical model, the evaporation width Γ of the particle *b* emitting from the compound nucleus *C* is given by

$$
\Gamma_b(E_C^*, J_C) = \frac{1}{2\pi \rho_C(E_C^*, J_C)}\n\times \int d\epsilon_b \sum_{S=|S_B - S_b|}^{S_B + S_b} \sum_{\ell=|J_C - S|}^{J_C + S} T_\ell(\epsilon_b) \rho_B(E_B^*, S_B).
$$
\n(1)

Here, E_C^* and J_C are the excitation energy and the spin of the compound nucleus *C*, while ϵ_b , S_b , and ℓ are the emission energy, the spin, and the orbital angular momentum of the particle *b*. After the emission of the particle *b*, a residual nucleus *B* is left, with the excitation energy E_B^* and the spin S_B . The quantum number *S* is the eigenvalue of the total spin in the exit channel, ranging from $|S_B - S_b|$ to $S_B + S_b$. For

the ¹⁶O + ¹⁶O reaction, the total angular momentum $\overrightarrow{J_C} = \overrightarrow{\ell} + \overrightarrow{S}$ takes only even values due to the indistinguishability of the two ¹⁶O nuclei in the entrance channel. $T_l(\epsilon_h)$ is the transmission coefficient, which can be obtained from opticalpotential calculations. $\rho_B(E_B^*, S_B)$ and $\rho_C(E_C^*, J_C)$ are the level densities of the residual and compound nuclei, which takes a form different from the original Gilbert-Cameron leveldensity formula [\[45\]](#page-6-0):

$$
\rho(E^*, J) = \rho_1(J) \rho_2(E^*), \tag{2}
$$

$$
\rho_1(J) = \frac{2J+1}{2\sigma^2} \exp\left[-\frac{J(J+1)}{2\sigma^2}\right],
$$
\n(3)

$$
\rho_2(E^*) = \frac{c}{T} \exp\left[\frac{E^* - E_0}{T}\right], \quad E^* < E_x,\tag{4}
$$
\n
$$
= \frac{c}{12(2\sigma^2)^{\frac{1}{2}} a^{\frac{1}{4}} (E^* - \Delta - \kappa(A)\delta(\alpha))^{\frac{5}{4}}}
$$
\n
$$
\times \exp\{2[a(E^* - \Delta - \kappa(A)\delta(\alpha))]^{\frac{1}{2}}\}, \quad E^* \ge E_x.\tag{5}
$$

FIG. 4. The cross sections of the residual nuclei from the exit channels without α emissions versus the center-of-mass incident energies $E_{\rm c.m.}$ for the ¹⁶O + ¹⁶O fusion-evaporation reaction. Solid points are the experimental data taken from Ref. [\[49\]](#page-6-0). The dashed red lines are the theoretical results given by the statistical-model calculations with the original Gilbert-Cameron level-density formula, while the solid green lines are theoretical results given by the statistical-model calculations with the new level-density formula inspired by the α -clustering effect.

In the above, E_x is the connection point between the constant temperature formula $(E^* < E_x)$ and the Fermi-gas-like formula ($E^* \ge E_x$). *T* and E_0 are determined from the smoothness condition. Δ and σ^2 are the pairing correction and the spin cutoff parameter. Following Ref. [\[46\]](#page-6-0), the level-density parameter *a* is taken to be energy dependent. Compared with the original Gilbert-Cameron level-density formula, here we introduced an additional correction term $\kappa(A)\delta(\alpha)$ and an overall scaling constant c as inspired by previous studies on α clustering. The correction term $\kappa(A)\delta(\alpha)$ is given explicitly by

$$
\kappa(A)\delta(\alpha) = (e^{k_1A} - k_2)\delta(\alpha),\tag{6}
$$

with $A = N + Z$ being the atomic number, and k_1 and k_2 being adjustable parameters to be determined later on. $\delta(\alpha)$ is taken from Ref. [\[47\]](#page-6-0) and gives a quantitative measure of the strength of the α -clustering effect,

$$
\delta(\alpha) = (-)^{Z+N+1} \frac{1}{2} [S_n(Z-1,N) - 2S_n(Z,N) + S_n(Z+1,N)], \tag{7}
$$

$$
S_n(Z, N) = B(Z, N) - B(Z, N - 1),
$$
\n(8)

where $B(Z, N)$ is the binding energy for the nucleus with Z protons and *N* neutrons. By summing all the angular momenta *J*, the Gilbert-Cameron formula as a function of *E*[∗] can be

written as

$$
\rho(E^*) = \frac{1}{T} \exp\left[\frac{E^* - E_0}{T}\right], \quad E^* < E_x,\tag{9}
$$

$$
= \frac{\exp\{2[a(E^* - \Delta)]^{\frac{1}{2}}\}}{12\sigma(2)^{\frac{1}{2}}a^{\frac{1}{4}}(E^* - \Delta)^{\frac{5}{4}}}, \quad E^* \geqslant E_x. \tag{10}
$$

In Figs. [1](#page-1-0) and [2,](#page-1-0) the modified Gilbert-Cameron level density ρ_{MGC} is compared with the original Gilbert-Cameron level density ρ_{OGC} for the oxygen + oxygen system and the oxygen + carbon system with the constant $c = 1$. For many residual nuclei, ρ_{MGC} and ρ_{OGC} almost coincide with each other. Important differences could be found in some selfconjugate nuclei with $Z = N$ and their neighbors. For these nuclei, the α -clustering effect could play an important role.

The introduction of $\kappa(A)\delta(\alpha)$ could be understood heuristically as follows. In Eq. (5) , three important physical effects are considered. First, the Fermi gas model is introduced as the foundation of the level-density formula, which captures the independent motion of the nucleons and acts as the basis to introduce additional corrections. Then the pairing corrections are introduced to give rise to the original Gilbert-Cameron level-density formula, which capture the pairing correlations of the two like nucleons. Finally, we introduce the correction

term $\kappa(A)\delta(\alpha)$ to try to capture the proton-neutron corrections and α -like correlations, which have been shown to be important in many light nuclei. In Ref. [\[47\]](#page-6-0), one of the authors introduces the factor $\delta(\alpha)$ to study α clustering in the ground states of heavy nuclei. Considering the complexity of the α clustering in compound nuclei, it is reasonable to introduce further corrections to $\delta(\alpha)$ to achieve better agreements with the experimental data. After much trial and error, we find that $k(A)$ in Eq. [\(6\)](#page-3-0) gives the best performance. Compared with our previous work in Ref. [\[44\]](#page-6-0), the new level-density formula given above also reduces significantly the number of adjustable parameters, therefore strengthening the predictability of our new model. For the overall scaling constant, please see Ref. [\[44\]](#page-6-0) for detailed discussions.

With the above inputs, the cross section from the entrance channel α to the exit channel β with an emitting particle *b* and a residual nucleus *B* is given by

$$
\sigma_{\alpha\beta}(J_C, S_B, S_b) = \sigma_{\alpha}(J_C) \frac{\sum_{\ell, S} T_{\ell}(\epsilon_b) \rho_B(E_B^*, S_B)}{\sum_{\beta', \ell', S'} T_{\ell'}(\epsilon_{\beta'}) \rho_{B'}(E_B^*, S_{B'})}.
$$
 (11)

Here, $\sum_{\beta',\ell',S'} T_{\ell'}(\epsilon_{b'})\rho_{B'}(E^*_{B'}, S_{B'})$ means summing all possible exit channels $\{\beta', \ell', S'\}$, and $\sigma_\alpha(J_C)$ is the partial cross section of the compound nucleus C in spin J_C from the entrance channel α . In our calculations, the experimental values of the fusion cross section are used directly to reproduce the partial cross section $\sigma_{\alpha}(J_C)$. For the ¹⁶O + ¹⁶O system only even angular momenta (J_C) are allowed in the calculations. All the other parameters remain to the same as in our previous work in Ref. [\[44\]](#page-6-0).

III. NUMERICAL RESULTS

The numerical calculations are carried out by using the EVAPOR implementation of the statistical model [\[48\]](#page-6-0), along with the new level-density formula introduced in Sec. [II.](#page-2-0) For the k_1 and k_2 parameters, we take $k_1 = 0.0765$ and $k_2 = 9.0$ for the ^{16−18}O + ¹⁶O reactions, and $k_1 = 0.0600$ and $k_2 = 5.7$ for the 16,18 O + 12,13 12,13 12,13 C reactions. Figure 3 shows the results for the 18,17,16 O + 16 O fusion-evaporation reactions. Here, we do the theoretical calculations by using both the original Gilbert-Cameron level-density formula (labeled "Original EVAPOR" in the figures) and our new level-density formula (labeled "Modified EVAPOR" in the figures), along with the experimental data for the sake of making a comparison. It is straightforward to see that the statistical-model calculations with the original Gilbert-Cameron level-density formula underestimate the α -emission cross sections significantly for these three reactions, while our calculations with the new level-density formula inspired by the α -clustering effects agree much better with the experimental data. Furthermore, the cross sections for the residual nuclei are also calculated in detail for the ${}^{16}O + {}^{16}O$ fusion-evaporation reaction and are shown in Figs. [4](#page-3-0) and 5. One can see that, for both the exit channels with α emissions and those without α emissions, our statistical-model calculations with the new level-density formula give theoretical results in better agreement with the experimental data. However, there are still some differences

FIG. 5. The cross sections of the residual nuclei from the exit channels with α emissions versus the center-of-mass incident energies $E_{\text{c.m.}}$ for the ¹⁶O + ¹⁶O fusion-evaporation reaction. Solid points are the experimental data taken from Ref. [\[49\]](#page-6-0). The dashed red lines are the theoretical results given by the statistical-model calculations with the original Gilbert-Cameron level-density formula, while the solid green lines are theoretical results given by the statistical-model calculations with the new level-density formula inspired by the α clustering effect.

between the experimental data and the theoretical calculations in the exclusive cross sections, such as the one-proton

FIG. 6. The α -emission cross sections versus the center-of-mass incident energies $E_{c,m}$ for the ^{16,18}O + ^{12,13}C fusion-evaporation reactions. Solid points are the experimental data taken from Refs. [\[40,43,51,52\]](#page-6-0). The dashed red lines are the theoretical results given by the statisticalmodel calculations with the original Gilbert-Cameron level-density formula, while the solid green lines are theoretical results given by the statistical-model calculations with the new level-density formula inspired by the α -clustering effect.

emission channel ${}^{31}P(p)$ and the two-proton emission channel $30\text{Si}(2p)$ [see Figs. [4\(b\)](#page-3-0) and [4\(c\)\]](#page-3-0). These differences may result from complicated correlations in the proton emissions that are not captured efficiently by the modified Gilbert-Cameron level-density formula. It shows that more works are needed to continue improving the level-density formula in the future. Calculations of the inclusive α cross section have also been done for the 18,16 O + 13,12 C fusion-evaporation reactions. The theoretical results can be found in Fig. 6. Compared with the original Gilbert-Cameron level-density formula, the improvements given by our new level-density formula can be seen straightforwardly. These results demonstrate preliminarily the robustness of our new level-density formula in describing fusion-evaporation reactions of carbon and oxygen isotopes and suggest that the α -clustering effects might appear in these reactions.

IV. CONCLUSIONS

In this work, we generalize our previous work on the $16,18$ O + $12,13$ C fusion-evaporation reactions further to the $16-18$ O + 16 O and $16,18$ O + $12,13$ C fusion-evaporation reactions at near-Coulomb-barrier energies using the statistical

model based on the HF formalism. It is found that the statistical-model calculations with the original Gilbert-Cameron level-density formula underestimate the α -emission cross sections for these reactions. Inspired by theoretical considerations on the α -clustering effect, we propose a new leveldensity formula which improves the original Gilbert-Cameron level-density formula by taking into account extra correlations related to α clustering. Compared with the level-density formula given by Ref. [\[44\]](#page-6-0), the new level-density formula has fewer adjustable parameters. This improves the predictability of our model. With explicitly statistical-model calculations, we obtain the α -emission cross sections for ^{16−18}O + ¹⁶O and 16,18 O + 12,13 C fusion-evaporation reactions, as well as the cross sections of various residual nuclei with and without α emissions in the 16 O + 16 O fusion-evaporation reaction. It is straightforward to see that significant improvements can be achieved by using the new level-density formula and the theoretical results are in better agreement with the experimental data. Along with our previous work on the $16,18$ O + $12,13$ C fusion-evaporation reactions [\[44\]](#page-6-0), these studies could be helpful for developing better theoretical models of fusion-evaporation reactions in the light-mass region and pursuing a deeper understanding of the α -clustering effect.

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