α -decay half-lives of superheavy nuclei with Z=122-125

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For α -decay half-life calculations in this work, the Coulomb and proximity potential model with a new semiempirical formula for diffuseness parameter developed in previous work [Phys. Rev. C 100, 024601 (2019)] is used. The present model in this work is compared with the generalized liquid-drop model (GLDM), universal decay law (UDL), and experimental half-lives in the region Z=104–118. Next, the predicted half-lives of 51 superheavy nuclei (SHN) with Z=122–125 by the present model are compared with those of GLDM and UDL. The present model is revealed to be more accurate in reproducing experimental half-lives compared to GLDM and UDL. Moreover, it is found that the predictions of the present model and UDL are highly consistent while GLDM largely deviates from the other two. A study of the competition between α -decay and spontaneous fission (SF) shows that α decay is the dominant mode. Among the studied SHN with Z=122–125, $^{295-307}122$ and $^{314-320}125$ are identified as potential candidates whose half-lives are relatively long enough to be experimentally detected in the future through their α -decay chains. The identified candidates are in good agreement with other recent work.

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I. INTRODUCTION

Shortly after its discovery by Rutherford and Geiger [1], α decay was explained as a quantum tunneling process by Gamow in 1928 [2]. To this day, the topic of α decay remains an important one in nuclear physics, being the dominant mode of decay for superheavy nuclei (SHN) and a simple mode of decay compared to other modes (e.g., cluster decay and fission) [3,4]. Among many things, α decay reveals information about nuclear structure and stability and can help in identifying new superheavy elements [5–8]. Since the time of Gamow, many theoretical and empirical models with varying degrees of accuracy and sophistication have been developed for the calculation and prediction of α -decay half-life [9–18].

In previous work with Abdul-latif [19], the Coulomb and proximity potential model for the calculation of half-life was employed. A novel semiempirical formula for diffuseness was proposed and used to calculate and predict the halflives of 218 SHN [19]. By incorporating the formula for diffuseness in the calculations, the model in the past work was able to reproduce experimental half-lives pretty accurately and better than a lot of popularly used models and semiempirical formulas (e.g., deformed Woods-Saxon model, UNIV, SemFIS, Viola-Seaborg, and Royer10 formulas). This work extends the previous work in two aspects. First, the improved model is compared with two more models, namely, the generalized liquid-drop model (GLDM) and the universal decay law (UDL). Second, half-lives of 51 superheavy nuclei in the region Z = 122-125 are predicted. The outline of this paper is as follows: in Sec. II, the theoretical model that

II. THEORETICAL FRAMEWORK

The effective potential for α decay consists of three parts, namely nuclear, Coulomb, and angular parts:

$$V_{\text{eff}}(r) = V_N(r) + V_C(r) + V_I(r),$$
 (1)

where r is the separation distance between the center of the daughter and α nuclei. The angular part will be neglected in the present study. For the nuclear potential $V_N(r)$, the proximity potential previously proposed by Zhang *et al.* [12] is adopted:

$$V_N(s_0) = 4\pi b_{\text{eff}} \overline{R} \gamma \phi(s_0), \tag{2}$$

$$\phi(s_0) = \frac{p_1}{1 + \exp\left(\frac{s_0 + p_2}{p_3}\right)}.$$
 (3)

In the two-parameter Fermi distribution (2pF) of nuclear matter, b_{eff} and the effective diffuseness a_{eff} of the proximity

will be used in the calculations of half-lives is described. In Sec. III, the present model is compared with GLDM, UDL, and experimental half-lives in the region Z=104-118. Moreover, the present model is used to predict the half-lives of 51 SHN with Z=122-125 and compared with those of GLDM and UDL, and their relative consistency is studied. The competition between α decay and spontaneous fission (SF) for Z=122-125 is studied. Finally, potential SHN candidates that can be detected in future experiments through their α -decay chains are identified and compared with results from other work. In Sec. IV, the main conclusions and a summary of the work are presented.

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potential are related as [20]

$$b_{\rm eff} = \frac{\pi}{\sqrt{3}} a_{\rm eff}.\tag{4}$$

In many works, $b_{\rm eff}$ is taken as a constant equal to 0.99–1 fm (equivalently $a_{\rm eff}=0.54$ fm) [21–29]. In previous work, it was found that such an approximation yields unacceptable errors [19]. Hence, the semiempirical formula developed in the previous work will be employed since it was shown to be accurate in reproducing experimental half-lives [19]. The formula is given by

$$a_{\text{eff}} = -1.09535 + 0.012063Z + 0.0019759N.$$
 (5)

 γ is given by

$$\gamma = 0.9517 \left[1 - 1.7826 \left(\frac{N - Z}{A} \right)^2 \right] \text{MeVfm}^{-2}.$$
 (6)

 $\overline{R} = R_{\alpha}R_d/(R_{\alpha} + R_d)$ is the reduced radius of the α -daughter system, where the radius of each nucleus in terms of its mass number is given by

$$R = 1.28A^{1/3} + 0.8A^{-1/3} - 0.76. (7)$$

 $\phi(s_0)$ is the universal function expressed in terms of the reduced separation distance s_0 :

$$s_0 = \frac{r - R_\alpha - R_d}{b_{\text{eff}}}. (8)$$

The constants p_1 , p_2 , and p_3 appearing in Eq. (3) are given by -7.65, 1.02, and 0.89, respectively. The Coulomb part $V_C(r)$ is given by

$$V_C(r) = Z_{\alpha} Z_d e^2 \begin{cases} \frac{1}{r}, & r \geqslant R_C, \\ \frac{1}{2R_C} \left[3 - \left(\frac{r}{R_C} \right)^2 \right], & r < R_C, \end{cases} \tag{9}$$

where Z_{α} and Z_{d} are the α and daughter charge numbers and $R_{C} = R_{\alpha} + R_{d}$. Using the Wentzel-Kramers-Brillouin (WKB) approximation, the half-life is given by

$$T_{1/2} = \frac{\pi \hbar \ln 2}{P_r F_{rr}} [1 + \exp(K)], \tag{10}$$

where P_o , E_{ν} , and K are the preformation factor, zero-point vibration energy, and action integral, respectively. The action integral is given by

$$K = \frac{2}{\hbar} \int_{r_1}^{r_2} \sqrt{2\mu [V_{\text{eff}}(r) - Q_{\alpha}]} dr, \tag{11}$$

where Q_{α} , μ , r_1 and r_2 are the decay energy, reduced mass, and first and second turning points, respectively. A classical description of E_{ν} shall be used since it was previously found [30] that classical and quantum mechanical (e.g., modified harmonic oscillator) approaches yield similar results, hence

$$E_{\nu} = \frac{\hbar\omega}{2} = \frac{\hbar\pi}{2R_{p}} \sqrt{\frac{2E_{\alpha}}{m_{\alpha}}},\tag{12}$$

where R_p , E_α , and m_α are the radius of the parent nuclues and kinetic energy and mass of the α particle, respectively. The

TABLE I. Statistical comparison between models Prox, UDL, and GLDM. Half-life calculations for the three models are taken from Refs. [19,33].

Model	$\sqrt{\overline{\delta^2}}$	$\overline{ \delta }$	$\bar{\delta}$	$ \Delta _{ ext{max}}$
Prox	0.41	0.36	0.064	0.926
UDL	0.641	0.435	0.141	2.671
GLDM	0.714	0.47	-0.078	3.197

kinetic energy of the α particle is given by [31]

$$E_{\alpha} = \frac{A_d}{A_n} Q_{\alpha} - \left(6.53 Z_d^{7/5} - 8 Z_d^{2/5}\right) 10^{-5} \text{ MeV}, \tag{13}$$

where A_d and A_p are the mass numbers of the daughter and the parent nuclei, respectively. Finally, the preformation factor in the present work is given by [32]

$$\log_{10} P_o = a + b(Z - Z_1)(Z_2 - Z_1) + c(N - N_1)(N_2 - N) + dA + e(Z - Z_1)(N - N_1), \tag{14}$$

where a, b, c, d, and e are given by 34.90593, 0.003011, 0.003717, -0.151216, and 0.006681, respectively. Z_1 , Z_2 , N_1 , and N_2 are the magic numbers given by 82, 126, 152, and 184, respectively.

III. RESULTS AND DISCUSSION

To assess the relative accuracy of the models, the half-life calculations of the present model (dubbed Prox), UDL, GLDM, and experimental half-lives in the region Z=104–118 were compared. For the present model, calculations of 68 SHN were available and taken from previous work [19]. For UDL and GLDM, half-life calculations of 40 SHN were available and taken from Ref. [33]. Using this data set, statistical parameters that measure the accuracy of the models were computed. The statistical parameters include root-mean-square (rms) deviation, mean deviation, mean, and magnitude of maximum error $|\Delta|_{\rm max}$. The rms deviation $\sqrt{\delta^2}$ of each model from experimental half-lives is given by

$$\sqrt{\overline{\delta^2}} = \sqrt{\frac{1}{M} \sum_{i}^{M} \Delta_i^2}.$$
 (15)

The mean deviation $|\delta|$ is computed by

$$\overline{|\delta|} = \frac{1}{M} \sum_{i=1}^{M} |\Delta_i|. \tag{16}$$

The mean is given by

$$\bar{\delta} = \frac{1}{M} \sum_{i}^{M} \Delta_{i},\tag{17}$$

where $\Delta = \log_{10} T_{\rm exp} - \log_{10} T_{\rm calc}$ is the deviation of the calculated logarithm of half-life from the experimental one and M is the number of SHN under study. The results of these calculations are shown in Table I. Looking at rms and mean deviations, one concludes that the present model is the most

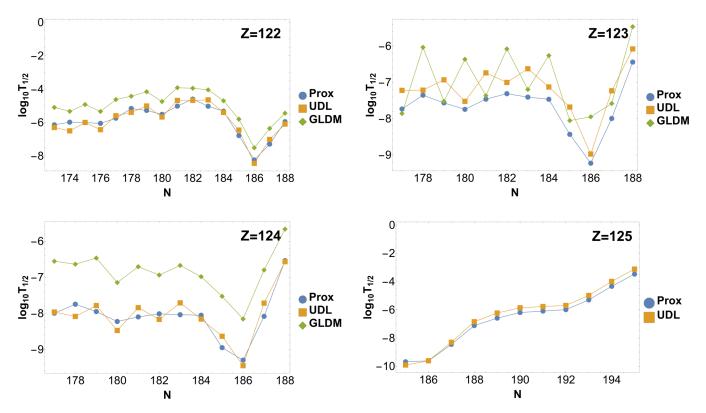


FIG. 1. Plot of the logarithm of half-life (seconds) for the models Prox, UDL, and GLDM vs parent neutron number N for Z = 122-125.

accurate in reproducing experimental half-lives, followed by UDL then GLDM. Looking at the maximum error, one sees that the deviation of GLDM from true half-life values can be as large as 3.19 (i.e., three orders of magnitude off) while the maximum error of UDL is 2.671 (i.e., off by a factor of 470) and the Prox model's maximum deviation from true half-life values does not exceed 0.926 (i.e., approximately off by a factor of 8). This statistical analysis concludes that the present model is the best one overall.

Next, the half-lives of 51 SHN with Z=122,123,124, and 125 were predicted using the present model and compared to those of UDL and GLDM. The values of Q_{α} and half-life predictions of UDL and GLDM were taken from Refs. [33,34]. The predicted half-lives using the three models are shown in Table II with the values used for Q_{α} . One notes that the calculations for Z=125 were not available for GLDM.

Figure 1 displays plots of the predicted logarithm of half-life for the Prox model, UDL, and GLDM vs parent neutron number N for the sake of visual comparison. From the figure, one makes two observations. The first observation to make is that the predictions of the Prox model and UDL are very similar to each other for all Z while GLDM deviates from the other two. Moreover, one finds the largest discrepancy between the three models for Z=123, where one also finds GLDM to exhibit a peculiar zig-zag pattern that the other two models do not show.

To measure the consistency of predicted half-lives and how close the predictions are of one model with another, the rms deviation of each model with respect to the other was computed. For instance, to measure consistency of the Prox model with UDL, one defines $\Delta = \log_{10} T_{\rm UDL} - \log_{10} T_{\rm prox}$ and,

using Eq. (15) and the data in Table II, one computes the rms deviation of the Prox model with respect to UDL. The same process is applied for the UDL/GLDM and GLDM/Prox cases. The results of these calculations are shown in Table III. It is interesting to find that Prox and UDL are highly consistent and similar in their half-life predictions, with an rms error of 0.342. However, one finds that the predictions of GLDM deviate significantly from the predictions made by UDL and Prox, with rms deviations of 0.969 and 1.006, respectively.

It is worth investigating the reasons behind the deviations of predictions of GLDM from that of the other two models in the region Z=122–125 since it is a somewhat unexpected result given that the three models are relatively consistent with each other and with the experimental half-lives in the region Z=104–118. A potential cause for this behavior was identified. This has to do with the fact that, in GLDM, the diffuseness parameter $b_{\rm eff}$ is set to a constant equal to 0.99 fm for all SHN [33,35–37]. The GLDM potential is the sum of volume, surface, Coulomb, and proximity potentials. The diffuseness parameter $b_{\rm eff}$ enters half-life calculations in GLDM through the proximity energy $V_N(r)$ given by Eq. (2).

It was shown in previous work with Abdul-latif and in the work of Dehghani *et al.* that the half-life of a given nucleus decreases with increasing diffuseness parameter (given that all other parameters like decay energy are held fixed) [19,38]. When the proximity potential is used, it was found that half-life depends more strongly on $b_{\rm eff}$ for larger values of $b_{\rm eff}$ (e.g., a change of 0.1 fm around $b_{\rm eff}=1.1$ fm will induce more change in half-life compared to a change of 0.1 fm around $b_{\rm eff}=0.4$ fm). Moreover, it was demonstrated that the common approximation $b_{\rm eff}=0.99$ fm for all SHN is not

TABLE II. Models Prox $\log_{10} T_{\text{prox}}$ vs UDL $\log_{10} T_{\text{UDL}}$ vs GLDM $\log_{10} T_{\text{GLDM}}$. Values for Q_{α} , $\log_{10} T_{\text{UDL}}$, and $\log_{10} T_{\text{GLDM}}$ are from Refs. [33,34].

Z	A	Q_{α}	$\log_{10} T_{\mathrm{prox}}$	$\log_{10} T_{\mathrm{UDL}}$	$\log_{10} T_{\rm GLDM}$
122	295	13.844	-6.13846	-6.298	-5.095
	296	13.705	-5.98077	-6.502	-5.334
	297	13.67	-6.00675	-5.997	-4.923
	298	13.65	-6.05305	-6.43	-5.339
	299	13.446	-5.7426	-5.594	-4.626
	300	13.115	-5.16303	-5.393	-4.429
	301	13.142	-5.27817	-5.013	-4.16
	302	13.234	-5.51409	-5.671	-4.746
	303	12.964	-5.01958	-4.678	-3.92
	304	12.742	-4.59973	-4.676	-3.949
	305	12.933	-5.02622	-4.647	-4.047
	306	13.068	-5.32347	-5.398	-4.699
	307	13.82	-6.78193	-6.455	-5.801
	308	14.653	-8.24236	-8.445	-7.516
	309	14.1	-7.29515	-7.014	-6.356
	310	13.377	-5.95135	-6.094	-5.444
123	300	14.703	-7.73571	-7.222	-7.86
	301	14.434	-7.3549	-7.216	-6.036
	302	14.519	-7.5682	-6.932	-7.524
	303	14.588	-7.74547	-7.525	-6.366
	304	14.393	-7.46473	-6.74	-7.369
	305	14.28	-7.31324	-7.004	-6.081
	306	14.313	-7.40856	-6.627	-7.198
	307	14.33	-7.46828	-7.128	-6.263
	308	14.89	-8.43457	-7.682	-8.055
	309	15.382	-9.23025	-8.977	-7.951
	310	14.613	-7.99778	-7.233	-7.586
	311	13.736	-6.44294	-6.08	-5.468
124	301	15.012	-7.99617	-7.963	-6.557
	302	14.811	-7.74724	-8.085	-6.638
	303	14.889	-7.95095	-7.784	-6.469
	304	15.016	-8.22584	-8.475	-7.146
	305	14.902	-8.09863	-7.842	-6.708
	306	14.819	-8.01316	-8.168	-6.939
	307	14.807	-8.03753	-7.709	-6.673
	308	14.794	-8.05269	-8.158	-6.984
	309	15.331	-8.95	-8.635	-7.528
	310	15.54	-9.29557	-9.449	-8.155
	311	14.773	-8.08294	-7.717	-6.799
	312	13.891	-6.53971	-6.568	-5.667
125	310	15.938	-9.6879	-9.916	
	311	15.873	-9.61948	-9.613	
	312	15.11	-8.45279	-8.309	
	313	14.319	-7.12356	-6.846	
	314	14.023	-6.59493	-6.234	
	315	13.811	-6.19899	-5.862	
	316	13.761	-6.09571	-5.774	
	317	13.717	-5.9958	-5.698	
	318	13.374	-5.30447	-4.989	
	319	12.919	-4.34303	-3.996	
	320	12.535	-3.4766	-3.1211	

TABLE III. rms deviations to measure consistency of one model with respect to the other.

Model	$\sqrt{\overline{\delta^2}}$
Prox/UDL	0.347
GLDM/UDL	0.969
Prox/GLDM	1.006

acceptable, especially in the region $b_{\rm eff} \geqslant 0.99$ fm, since an error on the order of 0.1 fm can yield an error in the logarithm of half-life as large as 0.7 [19]. Note that according to the formula for diffuseness, Eq. (5), $1.06 \leqslant b_{\rm eff} \leqslant 1.17$ fm for SHN in the region Z=122–125. Therefore, the approximation $b_{\rm eff}=0.99$ fm employed in GLDM always underestimates the true value of diffuseness, which results in its half-life predictions in Z=122–125 being systematically higher than the other two models (see Fig. 1) since decreasing diffuseness increases half-life. The remedy to this problem is to use an accurate formula for diffuseness that is dependent on Z and N, like the one used in the present work.

 α -decay half-life is among the quantities that are measured experimentally, and hence its accurate theoretical prediction is extremely important for discovering new SHN. Another crucial signature that identifies a nucleus is its α -decay chain when it undergoes several α decays followed by SF. Therefore, the competition between α decay and SF in the region Z=122-125 shall be studied next. SHN which have relatively short α -decay half-life compared to SF half-life will survive fission and hence can be synthesized in the laboratory and be detected by their α -decay chains. Therefore, SHN in Z=122-125 that survive fission and whose α -decay half-lives are long enough ($\geqslant 100\,\mathrm{ns}$) to be detected in future experiments shall be identified.

To this end, one defines the branching ratio b of α decay with respect to spontaneous fission as

$$b = \frac{\lambda_{\alpha}}{\lambda_{\rm SF}} = \frac{T_{\rm SF}}{T_{\alpha}},\tag{18}$$

where λ_{α} and T_{α} denote the α -decay constant and half-life while $\lambda_{\rm SF}$ and $T_{\rm SF}$ denote those of SF. $\log_{10} b > 0$ implies that α decay is dominant and the particular SHN will survive fission and decay through α -decay chains while SHN with $\log_{10} b < 0$ will not survive fission.

With the aid of the predicted α -decay half-lives of the present model and that of UDL and the data for SF half-life $T_{\rm SF}$ taken from Refs. [33,34], Table IV was compiled where $\log_{10}T_{\rm SF}$, $\log_{10}b_{\rm prox}$, and $\log_{10}b_{\rm UDL}$ are displayed. One finds that α decay is the dominant mode, as evidenced by the fact that $\log_{10}b_{\rm prox} \leq 0$ for only four SHN out of 51. UDL gives the same result with $\log_{10}b_{\rm UDL} \leq 0$ for the same four SHN. Therefore, one concludes that almost all of SHN in Z=122-125 will survive fission. The four SHN that will not survive fission are $^{309-310}122$, $^{311}123$, and $^{312}124$. One further notices that all SHN with Z=125 will survive fission. SHN with comparable SF and α -decay half-lives are $^{308-309}122$ and $^{310-311}124$.

Using the calculations displayed in Table II and IV, a search was carried out for SHN which both survive fission

TABLE IV. Logarithm of SF half-life $\log_{10} T_{\rm SF}$ taken from Refs. [33,34] with $\log_{10} b_{\rm prox}$ and $\log_{10} b_{\rm UDL}$ denoting the logarithm of branching ratio where predicted α -decay half-lives of models Prox and UDL were used.

Z	A	$\log_{10} T_{\rm SF}$	$\log_{10} b_{\mathrm{prox}}$	$\log_{10} b_{\mathrm{UDL}}$
122	295	7.301	13.43946	13.599
	296	4.016	9.99677	10.518
	297	6.772	12.77875	12.769
	298	3.677	9.73005	10.107
	299	6.587	12.3296	12.181
	300	5.302	10.46503	10.695
	301	8.806	14.08417	13.819
	302	4.937	10.45109	10.608
	303	5.992	11.01158	10.67
	304	1.195	5.79473	5.871
	305	2.875	7.90122	7.522
	306	-2.21	3.11347	3.188
	307	-3.116	3.66593	3.339
	308	-8.084	0.15836	0.361
	309	-7.892	-0.59685	-0.878
	310	-13.149	-7.19765	-7.055
123	300	10.421	18.15671	17.643
	301	9.448	16.8029	16.664
	302	13.333	20.9012	20.265
	303	11.063	18.80847	18.588
	304	8.852	16.31673	15.592
	305	4.034	11.34724	11.038
	306	5.661	13.06956	12.288
	307	0.388	7.85628	7.516
	308	-0.4	8.03457	7.282
	309	-5.249	3.98125	3.728
	310	-4.498	3.49978	2.735
	311	-9.296	-2.85306	-3.216
124	301	7.676	15.67217	15.639
	302	8.836	16.58324	16.921
	303	12.76	20.71095	20.544
	304	10.464	18.68984	18.939
	305	14.209	22.30763	22.051
	306	0.176	8.18916	8.344
	307	1.746	9.78353	9.455
	308	-3.784	4.26869	4.374
	309	-3.748	5.202	4.887
	310	-8.684	0.61157	0.765
	311	-7.242	0.84094	0.475
	312	-11.922	-5.38229	-5.354
125	310	22.64246	32.33036	32.55846
	311	21.40278	31.02226	31.01578
	312	20.48101	28.9338	28.79001
	313	19.11059	26.23415	25.95659
	314	17.99047	24.5854	24.22447
	315	16.39863	22.59762	22.26063
	316	14.89048	20.98619	20.66448
	317	12.95885	18.95465	18.65685
	318	11.14114	16.44561	16.13014
	319	5.625929	9.968959	9.621929
	320	3.8892	7.3658	7.021727

and whose predicted half-lives are relatively long enough ($\geqslant 100$ ns) to be detected in future experiments. By stipulating these requirements, it was found that $^{295-307}122$ and $^{314-320}125$ are potential candidates to be experimentally detected and identified via their decay chains. All other SHN either do not survive fission or have very short half-lives to be detected. Note that the isotopes under study in this paper are $295 \leqslant A \leqslant 310$ for Z=122, $300 \leqslant A \leqslant 311$ for Z=123, $301 \leqslant A \leqslant 312$ for Z=124, and $310 \leqslant A \leqslant 320$ for Z=125. Therefore, there could be other potential candidates that are experimentally detectable for Z=122-125 for larger or smaller A which were not investigated here.

Compared to a series of recent work done by Santhosh et al. [34,39-41], one finds that there is a good agreement with the experimentally detectable SHN for Z = 122, where they concluded that ^{298–307}122 are potential candidates. There is also a good agreement for $\hat{Z} = 125$, where they predict 310-320 125 as potential candidates, thus agreeing with the present work on the detectability of ^{314–320}125 and disagreeing on ^{310–313}125. Again, there is a relatively good agreement for Z = 124, where the present work excludes all SHN with $301 \leqslant A \leqslant 312$, while their model also excludes all SHN except for the four SHN 305-308124. The biggest disagreement is for Z = 123, where this work excludes all SHN with $300 \le A \le 311$, while the other work posits that ^{300–307}123 are detectable. All in all, the level of agreement between the present paper and the others is satisfactory given the different decay energies, SF half-lives, and theoretical models employed to determine possible candidates.

IV. SUMMARY AND CONCLUSIONS

In this work, the Coulomb and proximity potential model with a new semiempirical formula for diffuseness parameter was used for half-life calculations. The half-life calculations of the present model, UDL, GLDM and experimental halflives in the region Z = 104-118 were compared. Various statistical parameters show that the present model is the most accurate in reproducing experimental half-lives, followed by UDL then GLDM. The predicted half-lives of 51 SHN in the Z = 122-125 region by the present model were compared with those of UDL and GLDM. The predictions of the present model and UDL are highly consistent while GLDM largely deviates from the two. The deviation of GLDM in the region Z = 122-125 is potentially caused by the approximation $b_{\rm eff} = 0.99$ fm which leads to systematic deviation and overestimation of true half-life values. The study of competition between α decay and SF for Z=122–125 concludes that α decay is the dominant mode. SHN that will not survive fission and those with comparable SF and α -decay half-lives were also identified. Moreover, using the present calculations, a systematic search was carried for SHN that survive fission and whose predicted half-lives are within experimental limits so that they can be detected and identified via their α -decay chains. SHN ^{295–307}122 and ^{314–320}125 were identified as candidates to be detected. The identified candidates are in good agreement with other recent work. One hopes that the present half-life calculations and identified candidates will guide future experiments in detecting new SHN.

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