Coexistence and 2*n* transfer among Mo nuclei

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 29 October 2019; published 30 December 2019)

I have analyzed experimental $2n$ transfer data for ground and first-excited 0^+ states in even $^{92-100}$Mo nuclei, using a generalized coexistence model. Output of the analysis is the mixing intensity for each nucleus in terms of a dimensionless parameter R, which is limited to a narrow range. Throughout the allowed range, the mixing is found to be monotonic in A, and is largest in ^{100}Mo.

DOI: 10.1103/PhysRevC.100.064322

I. INTRODUCTION AND MODEL

Carchidi and co-workers [1–3] developed a generalized coexistence model for use in analyzing $2n$ transfer ratios for 0^+ states in even nuclei. To take full advantage of the model required a minimum of four adjacent even isotopes. The model was most useful in a region in which the properties of the ground state (g.s.) were changing rapidly, and the g.s. and an excited 0^+ state could be understood in terms of linear combinations of two 0^+ basis states whose properties were changing smoothly with A.

Strengths for $2n$ transfer were denoted as (using Ge as an example)

$$T_A^2 = \sigma[^A\mathrm{Ge}(t,p)^{A+2}\mathrm{Ge}(\mathrm{exc.}0^+)]/\sigma[^A\mathrm{Ge}(t,p)^{A+2}\mathrm{Ge}(\mathrm{g.s.})],$$

and

$$P_A^2 = \sigma[^{A+2}\mathrm{Ge}(p,t)^A\mathrm{Ge}(\mathrm{exc.}\,0^+)]/\sigma[^{A+2}\mathrm{Ge}(p,t)^A\mathrm{Ge}(\mathrm{g.s.})].$$

Note that the subscript is the lighter of target and residual nucleus in both pickup and stripping.

Briefly, the model assumes that the ground state (g.s.) and the first excited 0^+ state are linear combinations of two basis states whose structures change slowly with A. Then,

$$|\mathrm{g.s.}(A)\rangle = a_A g_A + b_A e_A,\ |\mathrm{exc.}\,0^+(A)\rangle = -b_A g_A + a_A e_A.$$

Results were described in terms of a dimensionless parameter R of order unity, which represents the ratio of $2n$ transfer amplitudes between basis states:

$R = \langle e|2n$ transfer $|e\rangle/\langle g|2n$ transfer $|g\rangle$, which is taken to be independent of A. The situation is depicted schematically in Fig. 1. In the simple model limit (SML), R is 1.0 and r is zero. A useful experimental figure of merit is the quantity

$$K_A = (T_{A-2} - P_A)(P_{A-2} - T_A)/[(T_{A-2}T_A - P_AP_{A-2})^2 + (T_{A-2} + T_A - P_A - P_{A-2})^2],$$

which is a measure of the deviation from the SML, which has $K_A = -1/4$. The basis-state transfer ratios are related through the equation

$$r^2 = R + K_A(R+1)^2.$$

The model was initially applied to Ge and Zn nuclei, and more recently to Sm [4] and Zr [5]. Here, I apply it to Mo.

II. RESULTS AND ANALYSIS

For Mo nuclei, five adjacent even nuclei are stable, having $A = 92$–100. Energies of the first-excited 0^+ states in these nuclei (plus 102) are plotted in Fig. 2. In all but one case, the needed cross-section ratios are available [6–18]. The exception is P_{96}, the cross-section ratio for the reaction ^{98}Mo$(p,t)^{96}$Mo. The 0^+ state at 1.148 MeV is not observed in that reaction, partially because it has about the same Q value as the g.s. in the reaction ^{96}Mo$(p,t)^{94}$Mo. In one study [11], in which the ^{96}Mo content of the ^{98}Mo target was 0.6%, a limit of $\sigma < 1\,\mu$b/sr was placed on the excited state at an angle of 15°, where the g.s. cross section was 135 μb/sr. This limit provides an estimate of $|P_{96}| = 0.058(29)$. Coincidently, Carchidi had developed a procedure to estimate the value of a missing datum in order for all the data to agree with the general model [19]. In the present case, that requirement is $|P_{96}| \approx 0.07$, with some uncertainty.

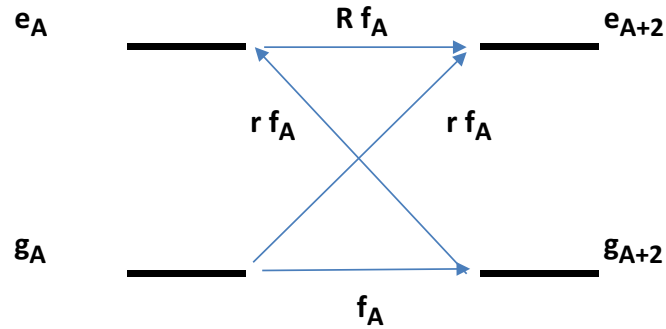

FIG. 1. Schematic depicting the 0^+ basis states and the $2n$ transfer amplitudes connecting them.

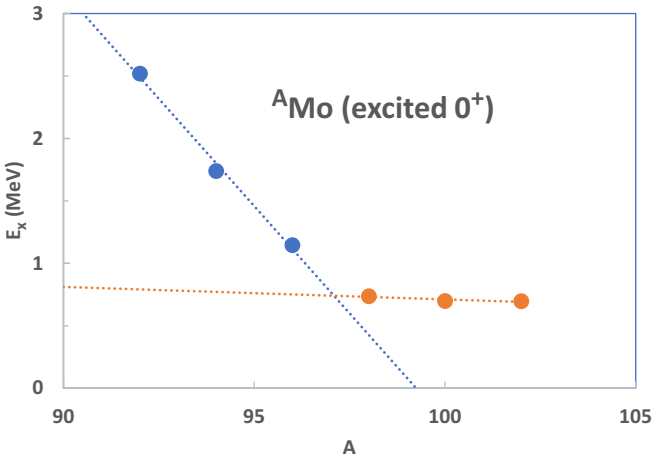

FIG. 2. Excitation energies of first excited 0^+ state in even Mo nuclei from $A = 92$ to 102.

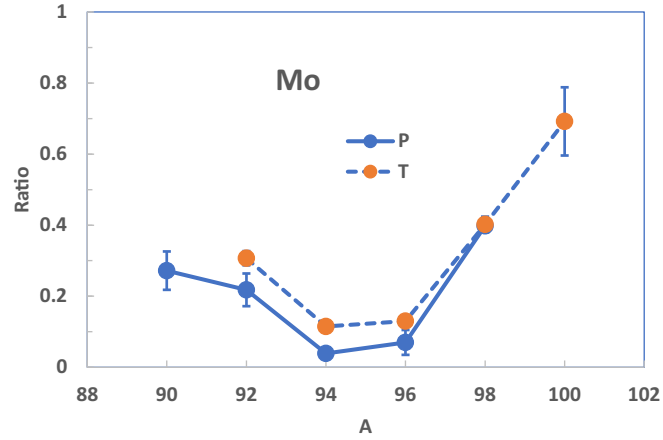

FIG. 3. Experimental ratios of $2n$ transfer amplitudes are plotted vs A.

TABLE I. Ratios of cross sections $\sigma(\text{exc.}\,0^+)/\sigma(\text{g.s.})$ in the reactions $^{A+2}\text{Mo}(p,t)^{A}\text{Mo}$ and $^{A}\text{Mo}(t,p)^{A+2}$.

	E_x	E_p	E_t	Cross-section ratio	
A	(MeV)	(MeV)	(MeV)	(p,t)	(t,p)
90	1.90	28		0.082(7) [7]	
		39		0.058(9) [13]	
92	2.52	31	17	0.034(9) [8]	0.094(12) [17]
		40		0.062(27) [10]	
94	1.389	40	17	0.0015(10) [11]	0.0133(13) [17]
96	1.295	40	17	<0.007 [11]	0.0168(27) [16]
98	1.594	19	15.8	0.158(14) [12]	0.172(19) [18]
			17		0.152(17) [17]
100	0.854		15		0.48(14) [15]
			15.8		0.459(46) [18]
			17		0.494(79) [16]

TABLE II. Amplitude ratios.

A	$\lvert P_A(\text{expt.})\rvert$	$P_A(\text{fit})$	$\lvert T_A(\text{expt.})\rvert$	$T_A(\text{fit})$
90	0.272(54)	±0.272		
92	0.218(46)	−0.206	0.307(20)	0.309
94	0.039(13)	−0.040	0.115(6)	0.115
96	0.058(29)	−0.070	0.130(10)	0.129
98	0.398(18)	−0.397	0.402(22)	0.404
100			0.692(96)	±0.692

Experimental cross-section ratios are listed in Table I. A slight dependence on Q value has been removed with standard distorted-wave calculations. Resulting amplitude ratios are listed in Table II and plotted in Fig. 3. One output of the model is a set of amplitudes that fit the model exactly and agree with the experimental ratios within their uncertainties. These fit amplitudes are then used to extract the wave-function mixing for each nucleus, in terms of a single parameter R. Alternately, the results can be considered to be a determination of the mixing in every nucleus in terms of the mixing in any one.

Mixing intensities b^2 for the five stable even Mo nuclei are plotted vs R in Fig. 4. Inspection of Fig. 3 reveals that the pattern is close to that expected in the SML, in which $T_A{}^2 = P_A{}^2$ for each A. This is not surprising, because the Mo nuclei have $K_A = -0.2492(3)$, which is close to the SML of -0.25.

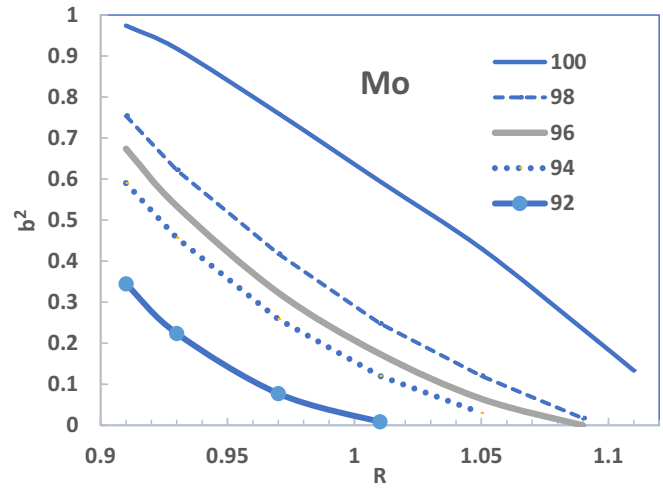

FIG. 4. Mixing intensities are plotted vs R for even Mo nuclei.

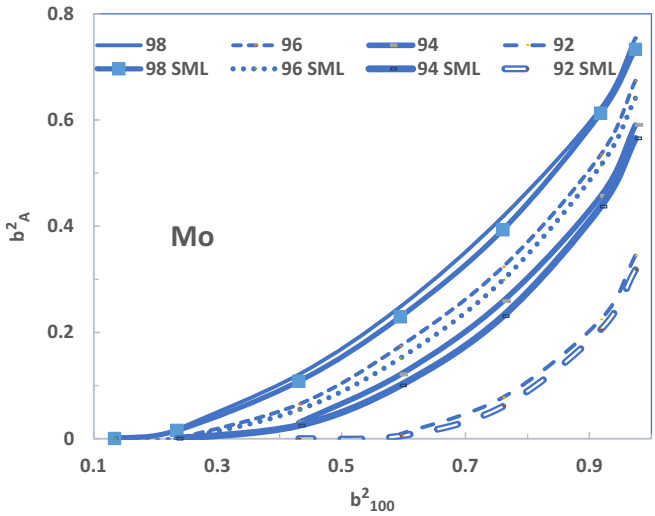

FIG. 5. As Fig. 4, but plotted vs mixing for ^{100}Mo. Results from the general model and the SML are compared.

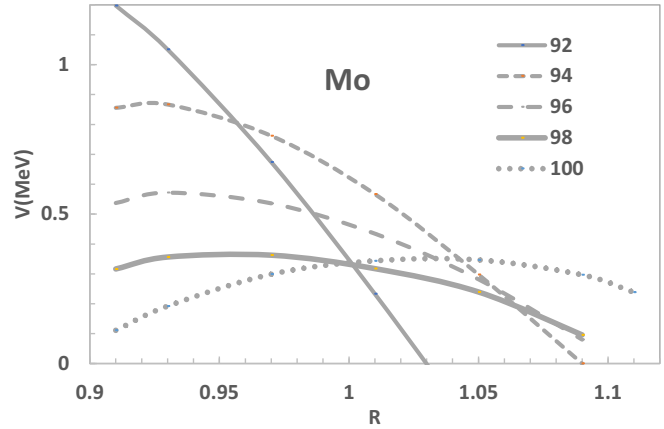

FIG. 6. Mixing matrix elements are plotted vs R.

I have compared the general results with those of the SML in Fig. 5, in which the mixing in each A is plotted vs the mixing in ^{100}Mo. Note that the SML and general results are similar, and that in all cases, the SML mixing is slightly less than that from the general model.

The potential matrix elements responsible for the mixing in each nucleus can be computed from the mixing amplitudes and the observed excitation energy of the excited 0^+ state: $V_A = a_A b_A E_A$. These are plotted vs R in Fig. 6. It is not clear what the A dependence of V_A should be. Note that at $R \approx 1$, three of the curves (for $A = 92$, 98, and 100) intersect, and likewise for $A = 94$, 96, and 98 near $R = 1.065$. At the extreme lower limit of R, the potentials are monotonic in A, as evidenced in Fig. 7.

III. SUMMARY

With a generalized coexistence model, I have analyzed experimental 2n transfer data for ground and first-excited 0^+ states in even Mo nuclei. These ratios are quite close (but not equal) to those expected in the simple model limit. Output of the analysis is the mixing intensity for each nucleus in terms of a dimensionless parameter R, which is limited to a narrow range. Throughout the allowed range, the mixing is monotonic in A, and is largest in ^{100}Mo.

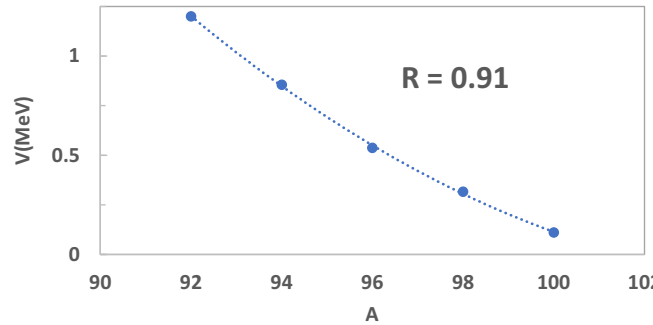

FIG. 7. Mixing matrix elements are plotted vs A for $R = 0.91$.

[1] M. Carchidi, H. T. Fortune, G. S. F. Stephans, and L. C. Bland, Phys. Rev. C **30**, 1293 (1984).
[2] M. Carchidi and H. T. Fortune, Phys. Rev. C **37**, 556 (1988).
[3] M. Carchidi and H. T. Fortune, J. Math. Phys. **27**, 633 (1986).
[4] H. T. Fortune, Nucl. Phys. A **984**, 1 (2019).
[5] H. T. Fortune, Phys. Rev. C **100**, 034303 (2019).
[6] J. B. Ball and J. S. Larsen, Phys. Rev. Lett. **29**, 1014 (1972).
[7] E. J. Kaptein, H. P. Blok, L. Hulstman, and J. Blok, Nucl. Phys. A **260**, 141 (1976).
[8] J. S. Larsen, J. B. Ball, and C. B. Fulmer, Phys. Rev. C **7**, 751 (1973).
[9] A. Moalem, M. A. Moinester, J. Alster, Y. Dupont, and M. Chabre, Phys. Lett. B **34**, 392 (1971).
[10] A. Moalem, M. A. Moinester, N. Auerbach, J. Alster, and Y. Dupont, Nucl. Phys. A **177**, 145 (1971).
[11] A. Moalem, M. A. Moinester, J. Alster, and Y. Dupont, Nucl. Phys. A **196**, 605 (1972).
[12] H. L. Sharma, R. Seltz, and N. M. Hintz, Phys. Rev. C **7**, 2567 (1973).
[13] D. H. Youngblood and R. L. Kozub, Nucl. Phys. A **192**, 442 (1972).
[14] H. Taketani, M. Adachi, M. Ogawa, K. Ashibe, and T. Hattori, Phys. Rev. Lett. **27**, 520 (1971); **27**, 1406(E) (1971).
[15] R. F. Casten, E. R. Flynn, Ole Hansen, and T. J. Mulligan, Nucl. Phys. A **184**, 357 (1972).
[16] E. R. Flynn, Ronald E. Brown, J. A. Cizewski, J. W. Sunier, W. P. Alford, E. Sugarbaker, and D. Ardouin, Phys. Rev. C **22**, 43 (1980).
[17] E. R. Flynn, F. Ajzenberg-Selove, Ronald E. Brown, J. A. Cizewski, and J. W. Sunier, Phys. Rev. C **24**, 2475 (1981); **25**, 2850(E) (1982).
[18] S. Takeda, S. Yamaji, K. Matsuda, I. Kohno, N. Nakanishi, Y. Awaya, and S. Kusuno, J. Phys. Soc. Jpn. **34**, 1115 (1973).
[19] M. Carchidi, Ph.D. dissertation, University of Pennsylvania, 1985.