

Fine structure in the α decay of ^{219}U

M. M. Zhang^{1,2}, Y. L. Tian,^{1,2} Y. S. Wang,^{1,2,3} X. H. Zhou,^{1,2,*} Z. Y. Zhang,^{1,2} H. B. Yang,¹ M. H. Huang,^{1,2} L. Ma,¹ C. L. Yang,^{1,2} Z. G. Gan,^{1,2} J. G. Wang,^{1,2} H. B. Zhou,⁴ S. Huang,⁴ X. T. He,⁵ S. Y. Wang,⁶ W. Z. Xu,⁶ H. W. Li,⁶ X. X. Xu,⁷ L. M. Duan,^{1,2} Z. Z. Ren,⁸ S. G. Zhou,^{9,10} and H. S. Xu^{1,2}

¹CAS Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

²School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

³School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

⁴Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China

⁵College of Material Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

⁶Shandong Provincial Key Laboratory of Optical Astronomy and Solar-Terrestrial Environment, School of Space Science and Physics, Shandong University, Weihai 264209, China

⁷Department of Nuclear Physics, China Institute of Atomic Energy, Beijing 102413, China

⁸School of Physics Science and Engineering, Tongji University, Shanghai 200092, China

⁹CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

¹⁰Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China



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The heaviest $N = 127$ even-odd isotone ^{219}U was produced in a fusion evaporation reaction employing ^{40}Ar ions bombarding ^{183}W target. Fusion evaporation residues were separated in flight by the gas-filled recoil separator Spectrometer for Heavy Atoms and Nuclear Structure and subsequently identified using a recoil- α correlation method. The α -decay properties of ^{219}U were measured with improved precision, and two new α -decay lines were observed and assigned as the decays from the ground state of ^{219}U to the $(5/2^-)$ and $(3/2^-)$ states of ^{215}Th , respectively. The systematics for the α decay of the $N = 127$ even-odd isotones as well as the low-lying nuclear structure of their $N = 125$ daughter nuclei are discussed.

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I. INTRODUCTION

α -decay spectroscopy is a powerful tool to identify new superheavy elements and to investigate the nuclear structure of ground and excited states in neutron-deficient heavy nuclei [1–3]. In the framework of nuclear shell models [4,5], the valence neutron in the ground state of the $N = 127$ even-odd isotones is expected to be in the $\nu 2g_{9/2}$ orbital, resulting in a spin and parity of $9/2^+$. Meanwhile, the low-lying excited states of the $N = 125$ even-odd isotones are associated with the $\nu 3p_{1/2}$, $\nu 2f_{5/2}$, and $\nu 3p_{3/2}$ orbits successively. The ground-state α decays of the $N = 127$ even-odd isotones from Po to Th were studied in detail experimentally [6–9], and their decay schemes were well established [10]. For the next heavier isotone ^{219}U , only the α decay populating the ground state in ^{215}Th was observed due to the extremely low production cross section [11,12]. In addition, the 561-keV excited state in ^{215}Th was observed in the previous work [13], which was populated in an isomeric decay and proposed to be the $(5/2^-)$ state. Therefore, it is interesting to study the fine structure of the α decay of ^{219}U .

In this work, the α -decay properties of ^{219}U were measured with better statistics using a complete fusion reaction $^{40}\text{Ar} + ^{183}\text{W}$. Two new α -decay lines were observed and assigned as the decays from the ground state of ^{219}U to the low-lying excited states in the daughter nucleus ^{215}Th .

II. EXPERIMENTAL DETAILS

The $^{40}\text{Ar} + ^{183}\text{W}$ experiment was carried out at the gas-filled recoil separator Spectrometer for Heavy Atoms and Nuclear Structure (SHANS) [14]. The ^{40}Ar beam with an energy of 190 MeV and a typical intensity of 500 pnA was delivered by the Sector Focusing Cyclotron of the Heavy Ion Research Facility in Lanzhou (HIRFL), China. The ^{183}W target (enrichment of 97%) with an average thickness of $200 \mu\text{g}/\text{cm}^2$ was made by sputtering the material onto a $80 \mu\text{g}/\text{cm}^2$ thick carbon foil and then covered by a $10 \mu\text{g}/\text{cm}^2$ thick carbon layer. The effective beam energy at the center of target was about 188 MeV calculated by the program SRIM [15] and the total irradiation time was 75 h.

The separator was filled with helium gas at a pressure of 0.6 mbar and the magnets were set to guide the evaporation residues (ERs) to the center of the focal plane. ERs surviving during the flight were implanted into three 300- μm -thick position-sensitive strip detectors (PSSDs) installed side by

*Corresponding author: zxh@impcas.ac.cn

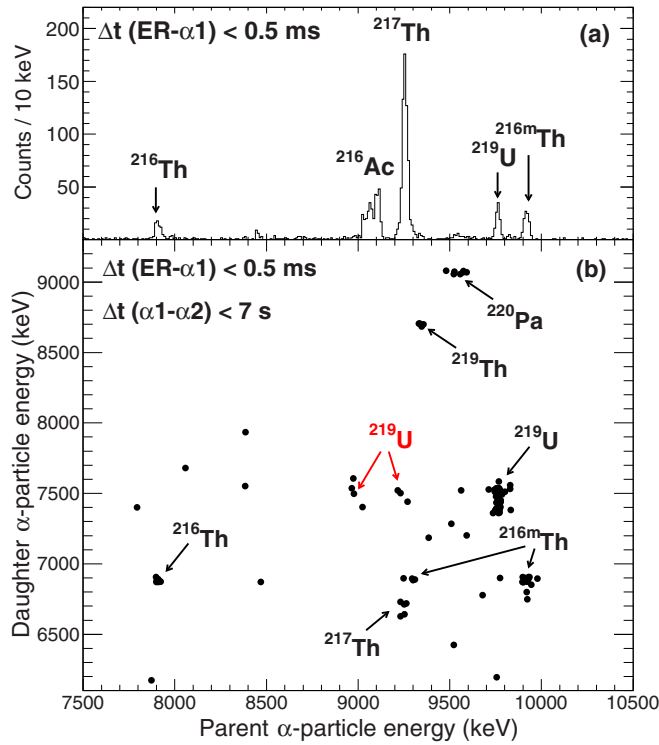


FIG. 1. (a) Energy spectrum for α particles following implanted residues within a time window of 0.5 ms. (b) Parent and daughter α -particle energies of the type ER- α 1- α 2 measured in the PSSDs. The searching time windows were 0.5 ms for the ER- α 1 pair and 7 s for the α 1- α 2 pair. The new α -decay transitions of ^{219}U are marked with red arrows.

side at the focal plane of the separator. Each PSSD with an active area of $50 \times 50 \text{ mm}^2$ was divided into 16 vertical strips. To detect the α particles escaped from the PSSDs, eight non-position-sensitive side silicon detectors (SSDs) of $50 \times 50 \text{ mm}^2$ size were mounted perpendicular to the

surface of the PSSDs. Behind the PSSDs, three punch-through detectors were placed to provide veto signals for energetic light particles passing through the PSSDs. All the silicon detectors were cooled down to gain a better energy resolution using an alcohol cooling system. In addition, two multiwire proportional counters (MWPCs) were mounted 15 and 25 cm upstream from the PSSDs to distinguish the decay events from the implantation events.

Signals from the preamplifiers of the detectors were recorded employing a digital data acquisition system, consisting of 16 waveform digitizers V1724 from CAEN S.p.A. [16]. In the offline data analysis, a complete waveform was obtained by summing up traces from both sides of the strip, taking into account the amplification factors. Two kinds of pulse-shape analysis procedures, trapezoidal filter and pulse fitting method, were used to extract the energies of the signals. Details of the system are given in Refs. [17, 18].

The energy calibrations of PSSDs and SSDs were performed using a three-peak (^{244}Cm , ^{241}Am , and ^{239}Pu) external α source as well as the known peaks from the nuclei produced in the test reaction $^{40}\text{Ar} + ^{175}\text{Lu}$. The typical energy resolutions with all the strips summed up were about 40 keV (FWHM) for 6.5- to 10.5-MeV α particles and 100–180 keV (FWHM) for the reconstructed α particles which escaped from the PSSDs and were detected by the SSDs. The vertical position of each event was determined by the resistive charge division method and the position resolution of each strip was better than 1.5 mm (FWHM).

III. RESULTS

The identification of nuclei produced in the present experiment was carried out by searching for the correlated α -decay chains. Figure 1(a) shows the energy spectrum of α particles, in which only α decays following 5- to 20-MeV implanted residues within a time window of 0.5 ms are included. The neutron-deficient isotopes ^{216}Ac , $^{216,216m,217}\text{Th}$, and ^{219}U produced in charged particle and/or neutron evaporation

TABLE I. The α -decay properties of neutron-deficient isotopes observed in the present work. The literature values are taken from Refs. [10, 13, 19].

Nuclide	E_α (keV)	$T_{1/2}$ Meas.	E_α (keV)	$T_{1/2}$ Lit.
	Meas.		Lit.	
^{216}Th	7919(15)	26.3(5) ms	7923(5)	26.0(2) ms
^{216m}Th	9918(15)(83.6 $^{+11.7}_{-11.7}\%$)	126(14) μs	9930(10)(74%)	135(4) μs
	9301(16)(11.5 $^{+6.0}_{-4.3}\%$)		9312(12)(13%)	
	7998(18)(4.9 $^{+4.8}_{-2.7}\%$)		7999(10)(13%)	
^{217}Th	9257(15)	249(11) μs	9261(4)	251(5) μs
^{219}Th	9330(20)	1.24 $^{+0.68}_{-0.32}$ μs	9340(20)	1.05(3) μs
^{220}Pa	9541(20)	0.98 $^{+0.40}_{-0.22}$ μs	9520(16)	0.90(13) μs
^{219}U	9763(15)(89.2 $^{+9.8}_{-9.8}\%$)	60(7) μs	9774(18)	42 $^{+34}_{-13}$ μs
	9246(17)(4.3 $^{+3.4}_{-2.1}\%$)		-	
	8975(17)(6.5 $^{+3.7}_{-2.6}\%$)		-	
^{215}Th	7510(15)	1.5(2) s	7522(4)(40%)	1.2(2) s
	7387(15)		7392(3)(52%)	
	-		7334(4)(8%)	
^{211}Ra	6900(15)	10(3) s	6909(4)	13(2) s

TABLE II. Observed α -decay chains attributed to new decays of ^{219}U . The implantation energies of ERs, the α -particle energies, and time intervals of chain members are given. All the events in each chain were observed within a position window of ± 1.5 mm (± 3.5 mm) for full energy (partial energy) deposited in the PSSDs.

Group	Chain	ERs E (MeV)	^{219}U		^{215}Th		^{211}Ra	
			$E_{\alpha 1}$ (keV)	$\Delta t_{\alpha 1}$ (μs)	$E_{\alpha 2}$ (keV)	$\Delta t_{\alpha 2}$ (s)	$E_{\alpha 3}$ (keV)	$\Delta t_{\alpha 3}$ (s)
A	1	10.00	9239	135	7503	0.4		
	2	16.14	9275	34	7443	2.2		
	3	13.89	9219	114	7519	0.2	6807 (602 + 6205) ^a	27.9
	4	15.21	9250	4	7383 (2666 + 4717) ^a	4.4		
B	5	13.04	8971	48	7505	6.1		
	6	12.78	8961	72	7539	0.8		
	7	9.10	9021	190	7403	4.1		
	8	12.23	8946	96	7475 (1207 + 6268) ^a	2.5	6876	19.6
	9	13.96	9027 (1184 + 7843) ^a	466	7441	2.1		
	10	9.95	8988 (822 + 8166) ^a	38	7477	5.7		

^aReconstructed events given as $x + y$ in parentheses, which deposited x keV in the PSSDs and y keV in the SSDs.

channels are identified. The α -decay properties of those isotopes (except ^{216}Ac for its broad decay energy) measured in the present work are summarized in Table I with the comparison of the literature values, indicating a good agreement with the known decay properties.

In Fig. 1(b), a two-dimensional scatter plot showing the correlation between the parent and daughter α -particle energies is presented. The searching time windows were 0.5 ms for the ER- $\alpha 1$ pair and 7 s for the $\alpha 1$ - $\alpha 2$ pair. The α -decay correlations originated from known U, Pa, and Th isotopes are clearly identified based on their known decay properties [10]. In the region where the parent α -particle energy is around 9.0 and 9.2 MeV and the daughter α -particle energy is around 7.5 MeV, the correlations are assigned to the new α decays of ^{219}U .

A search for decay chains with three consecutive α decays (ER- $\alpha 1$ - $\alpha 2$ - $\alpha 3$) was performed so as to reliably identify the α decays of ^{219}U . Eighty-three decay chains were assigned to the ground-state to ground-state (g.s. to g.s.) transition of ^{219}U , and the α -particle energy was deduced to be 9763(15) keV, which is consistent with the value of 9774(18) keV measured previously [10]. Importantly, 10 new correlated α -decay chains were attributed to ^{219}U , and the details for these chains are summarized in Table II. The upper panels of Fig. 2 show the α -particle energy distributions of ^{219}U and its corresponding descendants ^{215}Th and ^{211}Ra . As shown in Fig. 2, the ten α particles from ^{219}U decay form two groups according to their energies. For the group labeled as A, there are four parent α decays with energies of 9239, 9275, 9219, and 9250 keV, respectively. The average α -particle energy is 9246(17) keV and the half-life of 50^{+50}_{-17} μs was deduced using the maximum likelihood method [20]. The parent radioactivity was followed by a second α decay with $E_{\alpha} = 7503, 7443, 7519,$ or 7383 keV, and in one of the chains by a third decay with $E_{\alpha} = 6807$ keV (reconstructed events). The second decay in the chains is associated with ^{215}Th , for which the decay properties of $E_{\alpha} = 7522(4), 7392(3),$ and $7334(4)$ keV with $T_{1/2} = 1.2(2)$ s were reported in Ref. [10]. It should be noted that the slightly larger α -particle energy of 7443 keV measured in present work compared with the

7392-keV literature value might be due to the small number of events and/or the summing effect between the α particles and the converted electrons, as the conversion coefficient of $\alpha_{\text{tot}} = 2.5$ for the 134-keV state of ^{211}Ra populated by the 7392-keV α decay of ^{215}Th was deduced [13]. The third decay might be attributed to ^{211}Ra [10]. The group B includes six decay chains as shown in Table II. The average α -particle energy from ^{219}U decay was deduced to be 8975(17) keV using the four full-energy deposit events, and the half-life was estimated to be 105^{+73}_{-30} μs . One of the second decays was succeeded by a third decay with $E_{\alpha} = 6876$ keV. As discussed above, these subsequent decays can be associated with ^{215}Th and ^{211}Ra decays, respectively. Therefore, we assigned the transitions

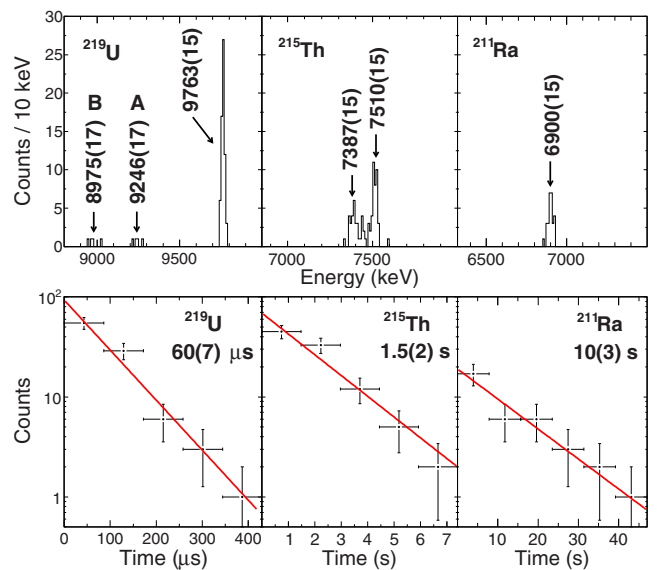


FIG. 2. The upper panels show the α -particle energy spectra of ^{219}U and its descendants ^{215}Th and ^{211}Ra , and the unit is in keV. The reconstructed events were not used to deduce the α -particle energies due to their poor energy resolution. In the lower panels, the decay curves of ^{219}U , ^{215}Th , and ^{211}Ra are displayed, and half-lives were deduced using all events.

with $E_\alpha = 9246(17)$ and $8975(17)$ keV as the decays from the ground state of ^{219}U to the excited states of ^{215}Th . The half-lives of ^{219}U , ^{215}Th , and ^{211}Ra were deduced and shown in the lower panels of Fig. 2 and Table I using all events observed in the present work, which are consistent with the literature data within experimental uncertainties [10].

Using a transmission efficiency of 14% of SHANS, the production cross section for ^{219}U at the center-of-target energy of 188 MeV was estimated to be $2.5(4)$ nb, in which the error only represents the statistical error.

IV. DISCUSSION

The neutron-deficient nucleus ^{219}U was first synthesized by Andreyev *et al.* [11]. The identification of the α -decaying isotope was based on six correlated α -decay chains. The α -decay energy and half-life were determined to be $9680(40)$ keV and 42_{-13}^{+34} μs , respectively. Later, other four α -decay events of ^{219}U were identified in Ref. [12], giving an α -decay energy and a half-life of $9774(18)$ keV and 80_{-30}^{+100} μs , respectively. Because of the similar values of the α -decay reduced widths for the $N = 127$ even-odd isotones, these decays were assigned as from the $(9/2^+)$ ground state in ^{219}U to the $(1/2^-)$ ground state of daughter nucleus ^{215}Th . Although the half-lives measured in the two experiments are compatible, the α -decay energy remains in conformity with the margin of experimental uncertainty. In the present work, we have obtained the improved values of $E_\alpha = 9763(15)$ keV and $T_{1/2} = 60(7)$ μs for this decay based on better statistics.

For the two new α decays of ^{219}U observed in the present work, we assigned that both of them decay from the $(9/2^+)$ ground state to the different excited states in ^{215}Th due to their comparable half-lives with the g.s. to g.s. transition. The branching ratios were determined to be $89.2_{-9.8}^{+9.8}\%$, $4.3_{-2.1}^{+3.4}\%$, and $6.5_{-2.6}^{+3.7}\%$ for the $E_\alpha = 9763$, $E_\alpha = 9246$, and $E_\alpha = 8975$ keV lines, respectively. The energy of the first excited state in ^{215}Th was determined to be $527(23)$ keV, which might be the 561-keV state determined previously using ER- γ - α coincidence measurement and assigned to be the $(5/2^-)$ state [13]. Therefore, the 9246-keV α line was assigned as the decay from the ground state of ^{219}U to the $(5/2^-)$ excited state in ^{215}Th . The 8975-keV α line is proposed tentatively to originate from the ground state to the $(3/2^-)$ excited state based on the systematics of α -particle energies and relative intensities observed in the $N = 127$ even-odd isotones [10]. The $3/2^-$ state energy in ^{215}Th was determined to be $803(23)$ keV in the present work for the first time, which is in good agreement with the level systematics as shown in Fig. 3.

The results deliver additional information on systematics in the α decay of the $N = 127$ even-odd isotones and on the nuclear structure of their $N = 125$ daughter nuclei. For the low-lying excited states of the $N = 125$ even-odd isotones, our new data on ^{215}Th fit the systematics well. The similarity between the levels of ^{207}Pb and its heavier even-odd isotones suggests that the lowest states in the $N = 125$ isotones up to ^{215}Th retain a significant single-particle character. The excitation energies of the $3/2^-$ states become lower and lower as

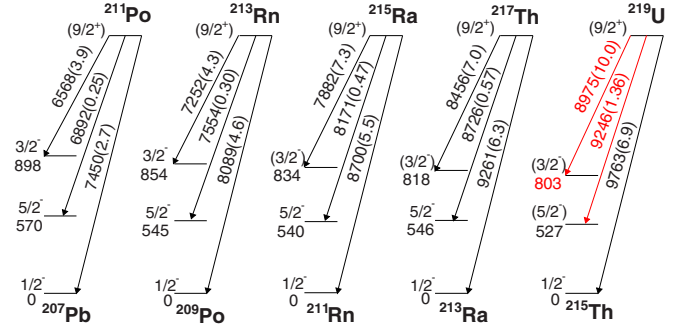


FIG. 3. α -decay schemes of the $N = 127$ even-odd isotones. Values are taken from Ref. [10] and the present work. The α -decay reduced widths δ^2 (in keV), calculated according to Ref. [21], are given in brackets behind the α -particle energies (in keV). New α decays from the ground state of ^{219}U to the $(3/2^-)$ and $(5/2^-)$ states of ^{215}Th as well as the excitation energy of the $(3/2^-)$ state obtained in the present work are marked with red color.

adding more protons to ^{207}Pb , which might be a consequence of core polarization [8].

The α -decay reduced widths δ^2 , calculated according to the formalism of Rasmussen [21], are given in brackets behind the α -particle energies as shown in Fig. 3. In these calculations, we used $\Delta l = 5$ for the $9/2^+ \rightarrow 1/2^-$ transitions and $\Delta l = 3$ for the $9/2^+ \rightarrow 5/2^-$ and $9/2^+ \rightarrow 3/2^-$ transitions, respectively. For the α decay of the $N = 127$ even-odd isotones, the $9/2^+ \rightarrow 3/2^-$ transitions have the largest δ^2 values compared with the other two transitions, indicating that they are favored by the nuclear structure. The g.s. to g.s. $9/2^+ \rightarrow 1/2^-$ transitions, which hold the maximum intensity, are favored by the Q value although their δ^2 values are lower than the $9/2^+ \rightarrow 3/2^-$ transitions. The δ^2 values for the $9/2^+ \rightarrow 5/2^-$ transitions are minimum, indicating a structural hindrance. Additionally, the δ^2 values for corresponding transitions decrease gradually from ^{219}U to ^{211}Po , which exhibit an enhanced structural hindrance approaching the closed proton shell at $Z = 82$.

V. CONCLUSION

In conclusion, the heaviest $N = 127$ even-odd isotone ^{219}U was produced using the $^{40}\text{Ar} + ^{183}\text{W}$ complete fusion reaction. The α -decay properties of ^{219}U were measured with improved precision. Two new α decays from the $(9/2^+)$ ground state to the $(5/2^-)$ and $(3/2^-)$ states in ^{215}Th were observed. The excitation energy of the $(3/2^-)$ state in ^{215}Th was determined to be 803 keV for the first time. The systematics in the α decay of the $N = 127$ even-odd isotones as well as the low-lying nuclear structure of their $N = 125$ daughter nuclei are discussed.

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