Elastic transfer and parity dependence of the nucleus-nucleus optical potential

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Background: A recent coupled-reaction-channel (CRC) study shows that the enhanced oscillation of the elastic $^{16}O + {}^{12}C$ section at backward angles is due mainly to the elastic α transfer or the core exchange. Such a process gives rise to a parity-dependent term in the total elastic S matrix, an indication of the parity dependence of the $^{16}O + {}^{12}C$ optical potential (OP).
Purpose: To explicitly determine the core exchange potential (CEP) induced by the symmetric exchange of the two ^{12}C cores in the elastic $^{16}O + {}^{12}C$ scattering at $E_{lab} = 132$ and 300 MeV and explore its parity dependence.
Method: S matrix generated by CRC description of the elastic $^{16}O + {}^{12}C$ scattering is used as the input for the inversion calculation to obtain the effective local OP that contains both the Wigner and Majorana terms.
Results: The high-precision inversion results show a strong contribution by the complex Majorana term in the total OP of the $^{16}O + {}^{12}C$ system and thus provide for the first time a direct estimation of the parity-dependent CEP.
Conclusions: The elastic α transfer or exchange of the two ^{12}C cores in the $^{16}O + {}^{12}C$ system gives rise to a complex parity dependence of the total OP. This should be a general feature of the OP for the light heavy-ion systems that contain two identical cores.

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I. INTRODUCTION

It is well established that in the elastic scattering of light heavy-ion (HI) systems where the projectile and target differ only by one nucleon or a nuclear cluster, elastic nucleon, or cluster transfer processes can take place between the two identical cores [1]. Such pairs of nearly identical nuclei are known as "core-identical" nuclei. The widely used approach to describe the elastic transfer is to add coherently, in the distorted wave Born approximation (DWBA), the elastic transfer amplitude to that of the elastic scattering. The interference between these two amplitudes gives a rapidly oscillating cross section as observed in both the excitation function and elastic-scattering cross section at backward angles. A more consistent approach is to study the elastic transfer within the coupled-reaction-channels (CRC) formalism [2–4], which provides the most accurate physics description and a clear insight into this process [5].

At low energies, the enhanced oscillating cross section at backward angles known as anomalous large angle scattering (ALAS) has been observed in the elastic scattering of various light HI systems [6]. The elastic transfer is the main physical origin of the ALAS observed in the elastic scattering of core-identical systems such as $^{16}O + {}^{12}C$ at low energies [1,5]. Such transfer processes can be found not only in the elastic scattering but also in inelastic scattering [1,7] and fusion [8,9].

The ALAS pattern can be reproduced in the optical-model (OM) calculation using an explicitly parity-dependent optical potential (OP) [1,6]. Such a procedure was studied by Frahn [10] and results in a modified elastic S matrix that contains a parity-dependent component. The elastic transfer (or the core exchange) reaction has been used to study the cluster- or nucleon spectroscopic factors [1], molecular orbitals [7,11], pairing effect [12,13], and cluster correlations [5,14] in stable and exotic nuclei [15]. Given its peripheral nature, HI-induced elastic transfer process can also be used to extract the asymptotic normalization coefficient, an important ingredient in the nuclear astrophysics studies [16,17]. The relation among ALAS, elastic transfer, and parity dependence of the OP has been shown in a recent study of $\alpha + {}^{12}C$ scattering, where the elastic transfer is used to study the Hoyle state [18] while a parity-dependent potential is required to reproduce the important $^{12}C(\alpha,\gamma)^{16}O$ radiative capture process [19]. Last, the observation of nuclear rainbow scattering in the core-identical systems like $^{16}O + {}^{12}C$ or $^{13}C + {}^{12}C$ [20,21] requires a better understanding of the low-energy elastic transfer that deteriorates the rainbow pattern at large angles [5] and its link to a parity-dependent OP in the OM description of elastic-scattering data.

In a conventional single-channel OM calculation, it has been suggested that the elastic transfer process generates an additional term in the total OP [22–25], which we refer to hereafter as the core exchange potential (CEP). The CEP originates from the exchange of the two identical cores and should be, therefore, parity dependent (i.e., containing a Majorana

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term). Moreover, it has been suggested in Refs. [23,24] that the CEP is also complex. The existence of a parity-dependent potential due to the core exchange was already pointed out in the early studies using the microscopic Resonating Group Method (RGM) and Generator Coordinate Method (GCM) methods [26–29] that treat exactly the antisymmetrization implied by the Pauli principle. In general, the exchange of nucleons in a microscopic model or identical (structureless) cores in a macroscopic model leads readily to the parity dependence of the OP. Nevertheless, the explicit derivation of the CEP within the general Feshbach formalism [24,25] that is capable of reproducing the scattering data still remains a challenge. Given the description of the core exchange by different models of elastic transfer [1,22–25], many OM analyses of elastic-scattering data measured for the core-identical systems at low energies were done using the real CEP based either on the linear combination of nuclear orbitals (LCNO) [30–32] or phenomenological parity-dependent potentials [33,34]. Although the use of these parity-dependent potentials drastically reduces the complexity of calculations, their connections to the underlying core exchange process is still not yet fully understood. So far, the CEP has never been directly derived from the elastic transfer calculation using DWBA or CRC methods, and a better understanding of the physics origin of the CEP in an elastic-scattering process is of high interest.

The iterative-perturbative (IP) inversion of the scattering S matrix to the equivalent local OP has been proven to be accurate, especially when applied to the S matrix given by CRC calculations [35]. Therefore, it is of high interest to use this inversion method to explicitly determine the CEP in the OP of a typical core-identical system. A recent extensive CRC study of the elastic $^{16}\text{O} + {}^{12}\text{C}$ scattering with up to 10 reaction channels included [5] has clearly demonstrated a strong impact of the elastic α transfer on the elastic-scattering cross section at different energies. In the present work, we apply the IP inversion method to the complex S matrix given by the CRC calculation of elastic $^{16}\text{O} + {}^{12}\text{C}$ scattering at $E_{\text{lab}} = 132$ and 300 MeV [5] to deduce the radial strength of the local CEP that is directly generated by the elastic α transfer and explore the parity dependence of the OP.

II. CORE-CORE SYMMETRY AND PARITY DEPENDENCE OF THE OP

We show here briefly that the parity dependence caused by the elastic transfer process is a natural consequence of the core-core symmetry that shows up in the exchange of the two identical cores. In the center-of-mass (c.m.) frame of a core-identical system like $^{16}\text{O} + {}^{12}\text{C}$, the elastic transfer of the valence nucleon or cluster is equivalent to the exchange of the two cores as illustrated in Fig. 1. Quantum mechanically, such an exchange process is possible by acting the core-exchange Majorana operator P_c on the scattering wave function in a manner similar to the projectile-target exchange in the elastic scattering of two identical nuclei. For this purpose, we consider the transition amplitude of the nucleus-nucleus scattering in the following general form [2,4]:

$$T = \langle\phi|V|\Psi_L\rangle, \tag{1}$$

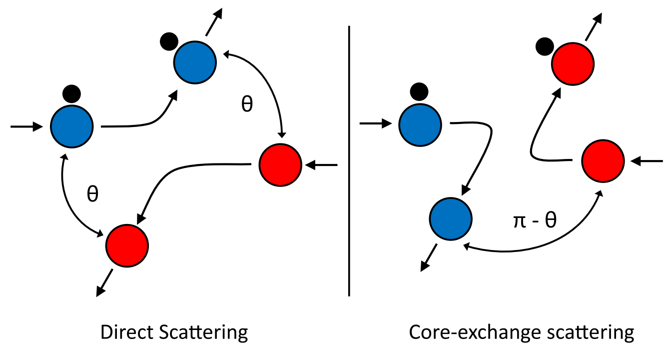

FIG. 1. Elastic transfer viewed as the exchange of the two identical cores. The red and blue spheres are the two cores in different initial states, and the black circle represents either the valence nucleon or cluster being transferred.

where Ψ and ϕ are the total scattering wave function and that of the entrance, respectively. Orbital momentum L of each partial wave is specified explicitly in Eq. (1) to trace the parity of the scattering wave function and the bra-ket notation is assumed, therefore, to contain also the summation over all partial waves. When both cores have the same nonzero spin I_c, the total wave function must be properly symmetrized to account for both the direct elastic scattering and exchange of the two identical cores,

$$\Psi_L(\boldsymbol{R}) \to \Psi_L(\boldsymbol{R}) + (-1)^{2I_c} P_c \Psi_L(\boldsymbol{R}), \tag{2}$$

where

$$P_c \Psi_L(\boldsymbol{R}) = X(R)\Psi_L(-\boldsymbol{R}) = (-1)^L X(R)\Psi_L(\boldsymbol{R}). \tag{3}$$

The function $X(R)$ originates, in general, from both the transfer form factor and spectroscopic factor of the valence nucleon or cluster and is radial dependent and complex [24,25]. The phase $(-1)^{2I_c}$ in Eq. (2) is implied by the spin statistics of the dinuclear wave function. In the $^{16}\text{O} + {}^{12}\text{C}$ case, the two ^{12}C cores are spinless and this phase can be dropped. Then

$$\begin{aligned} T &= \langle\phi|V|[1 + (-1)^L X(R)]\Psi_L(\boldsymbol{R})\rangle \\ &= \langle\phi|V[1 + (-1)^L X(R)]|\Psi_L(\boldsymbol{R})\rangle. \end{aligned} \tag{4}$$

Consequently, the formal expression of the nucleus-nucleus OP for the single-channel OM calculation is

$$V_{\text{OP}} = [1 + (-1)^L X(R)]V(\boldsymbol{R}), \tag{5}$$

where the second term is the CEP. The potential V_{OP} is, in fact, similar to the ones derived by Fuller and McVoy [24] and Frahn and Hussein [25]. For a direct reaction process, the parity-dependent CEP is closely associated with the transfer process that favors the transfer of a small number of nucleons. The earlier microscopic studies [26–29] have also suggested a strong (Majorana) core exchange term for the nucleus-nucleus systems with small mass difference. From the consideration leading to Eq. (5), we have assumed in the present study a local OP that contains the parity-independent potential referred to as the Wigner term (V_W) and parity-dependent one as the Majorana term (V_M),

$$V_{\text{OP}}(R) = V_W(R) + (-1)^L V_M(R). \tag{6}$$

It is obvious from the kinematic illustration of elastic transfer in Fig. 1 that the total elastic-scattering amplitude can be written as a coherent sum of the amplitudes of both the direct elastic scattering and elastic transfer [1,25],

$$f(\theta) = f_{\rm ES}(\theta) + f_{\rm ET}(\pi - \theta). \quad (7)$$

Given the standard expansion of the direct elastic-scattering (ES) amplitude into the partial wave series,

$$f_{\rm ES}(\theta) = f_R(\theta) + \frac{1}{2ik}\sum_L (2L+1)e^{2i\sigma_L}\left[S_L^{(\rm ES)} - 1\right]P_L(\cos\theta), \quad (8)$$

where $f_R(\theta)$ and σ_l are the Rutherford-scattering amplitude and Coulomb phase shift, respectively [2], the elastic transfer (ET) amplitude can be expressed in the same manner,

$$\begin{aligned} f_{\rm ET}(\theta) &= \frac{1}{2ik}\sum_L (2L+1)e^{2i\sigma_L}S_L^{(\rm ET)}P_L[\cos(\pi-\theta)] \\ &= \frac{1}{2ik}\sum_L (2L+1)e^{2i\sigma_L}S_L^{(\rm ET)}(-1)^L P_L(\cos\theta). \end{aligned} \quad (9)$$

The total elastic amplitude is then obtained as

$$f(\theta) = f_R(\theta) + \frac{1}{2ik}\sum_L (2L+1)e^{2i\sigma_L}(S_L - 1)P_L(\cos\theta), \quad (10)$$

where

$$S_L = S_L^{(\rm ES)} + (-1)^L S_L^{(\rm ET)}. \quad (11)$$

We have thus obtained the total elastic amplitude (10) in the same partial-wave expansion as Eq. (8), but with a parity-dependent contribution from elastic transfer added to that of elastic scattering. The interference between these two terms gives rise naturally to an oscillating elastic cross section at large angles, similarly to the Mott oscillation observed in elastic scattering of two identical nuclei like ^{12}C + ^{12}C or ^{16}O + ^{16}O [36]. Relation (11) is a simplified version of the formal expression derived by Frahn and Hussein [25] for elastic transfer, where the impact of the dynamic L-dependent coupling potential caused by elastic transfer was shown to be equivalent to that of a modified elastic S matrix that contains a parity-dependent component. Therefore, the assumption (6) for the local OP to be derived from the IP inversion of the elastic S matrix is well founded.

For the illustration, we have plotted in Figs. 2 and 3 the elastic S matrix given by the recent two-channel CRC calculation of elastic ^{16}O + ^{12}C scattering at $E_{\rm lab} = 132$ and 300 MeV [5] (with the direct elastic scattering and elastic α transfer channels explicitly taken into account) versus the S matrix given by the single-channel OM calculation. One can see that with the elastic transfer channel taken into account, the total elastic S matrix becomes parity dependent as formally shown by relation (11). This is also known as the odd-even staggering which is most pronounced around the grazing angular momenta ($L_g \approx 26$ and 37 for the 132- and 300-MeV cases, respectively) as shown in lower panels of Figs. 2 and 3. Similar staggering occurs in the argument of the S matrix which is related to the real potential. We note that the pattern of ΔS_L and a strong parity dependence of the elastic S matrix near the grazing angular momenta are similar to the results obtained using the LCNO potential [37] for the ^{16}O + ^{20}Ne system. This range of grazing angular momenta corresponds to the forward-angle scattering caused by the ^{16}O + ^{12}C interaction at the surface, where the α transfer process was shown [5] to be dominant. The simple reason why the elastic α transfer (or the exchange of two identical cores) shows up in the enhanced oscillation of the elastic cross section at backward angles is that the elastic α transfer amplitude at $(\pi - \theta)$ is coherently added to the elastic-scattering amplitude at θ, as implied by the relation (7).

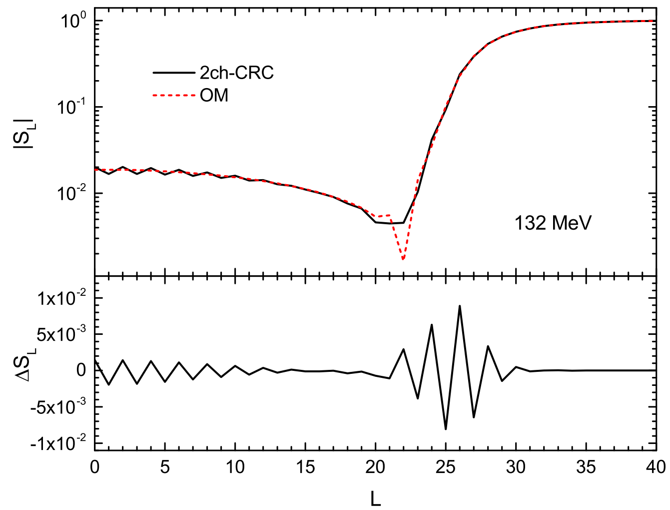

FIG. 2. Modulus of the elastic S matrix (upper panel) given by the single-channel OM (dashed line) and two-channel CRC (solid line) calculations of the elastic ^{16}O + ^{12}C scattering at $E_{\rm lab} =$ 132 MeV. The lower panel shows the difference $\Delta S_L = |S_L^{\rm CRC}| - |S_L^{\rm OM}|$.

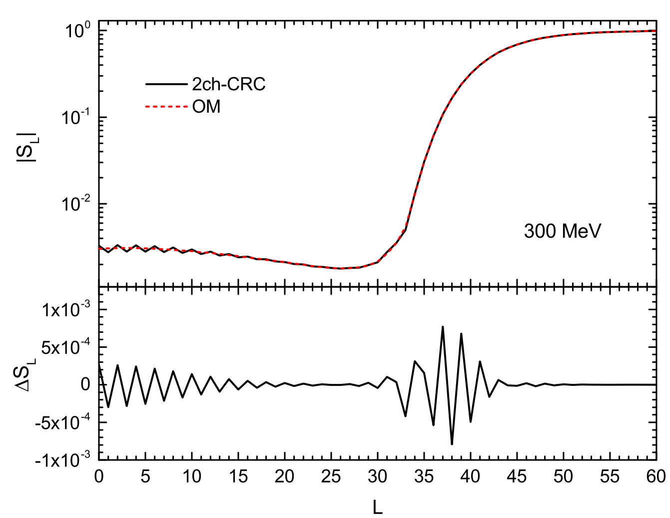

FIG. 3. The same as Fig. 1 but for the elastic ^{16}O + ^{12}C scattering at $E_{\rm lab} = 300$ MeV.

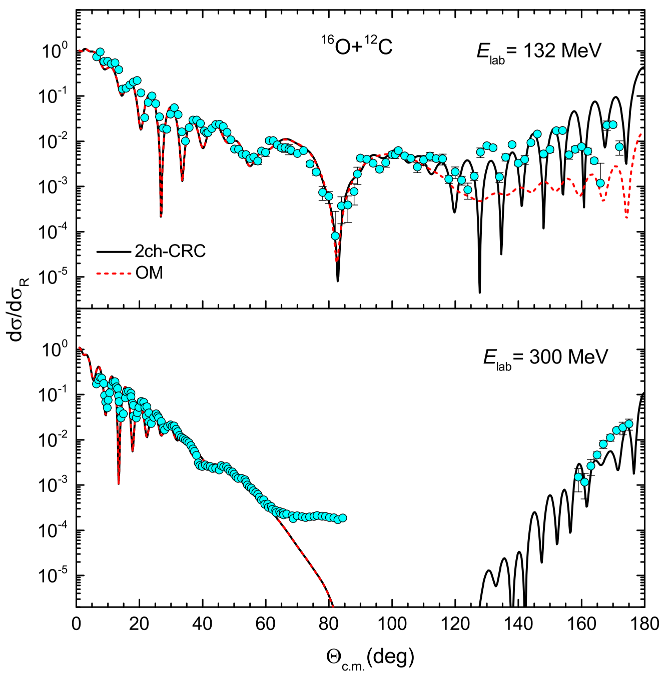

FIG. 4. Elastic $^{16}O + {}^{12}C$ scattering data measured at $E_{lab} =$ 132 MeV [38,39] and 300 MeV [40] in comparison with the results given by the single-channel OM (dashed line) and two-channel CRC (solid line) calculations [5].

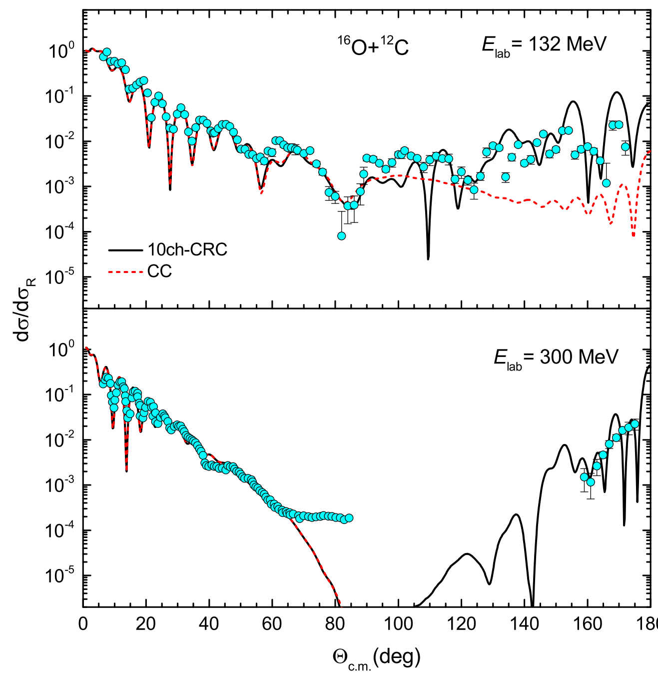

FIG. 5. The same as Fig. 4 but in comparison with the results given by the coupled-channels (dashed line) and 10-channel CRC (solid line) calculations [5].

III. CRC DESCRIPTION OF ELASTIC α TRANSFER

In the present work we aim to derive explicitly the CEP generated by elastic α transfer based on the CRC description of the elastic $^{16}O + {}^{12}C$ data measured at $E_{lab} = 132$ MeV [38,39] and 300 MeV [40]. The recent CRC study [5] has shown that elastic α transfer between the two identical ^{12}C cores is the main physics origin of the enhanced oscillation of the elastic $^{16}O + {}^{12}C$ cross section observed at backward angles at the two considered energies. The CRC results for the elastic $^{16}O + {}^{12}C$ scattering including explicitly up to 10 reaction channels for both the direct and indirect (multistep) α transfer account well for the measured data over the whole angular range, using the α spectroscopic factor S_α obtained from the large-scale shell-model calculation by Volya and Tchuvilsky [41,42].

We briefly discuss here the two coupling schemes for elastic α transfer used in our recent CRC analysis [5] of elastic $^{16}O + {}^{12}C$ scattering using the code FRESCO [3]. The first scenario is the two-channel CRC calculation that includes only the true elastic scattering and direct elastic α transfer as considered in Eqs. (7)–(11). From results shown in Fig. 4 one can see very clearly the contribution of the elastic α transfer or core-core exchange showing up at backward angles. Such an approximation requires the minimum model space in the CRC calculation and explicitly generates the direct (one-step) exchange of the two identical ^{12}C cores in elastic $^{16}O + {}^{12}C$ scattering. Although the best-fit α spectroscopic factor obtained in the two-channel CRC analysis [5] is larger than that predicted by the structure studies, the strong effect of elastic α transfer revealed in this calculation, especially at the energy of 300 MeV, provides an important test ground for our approach to determine explicitly the CEP in the local OP (6) for the one-channel OM description of these data.

The second scenario is the 10-channel CRC description where the elastic-scattering channel is coupled with the inelastic-scattering channels for the 2_1^+ (4.44 MeV) state of ^{12}C, and 0_2^+ (6.05 MeV), 3_1^- (6.13 MeV), and 2_1^+ (6.92 MeV) states of ^{16}O, and the direct and indirect α transfer channels through the considered excited states. The inelastic-scattering form factors for the CRC calculation were calculated in the generalized double-folding model [43] using the CDM3Y3 density-dependent interaction [36] and nuclear transition densities from the RGM by Kamimura [44] for ^{12}C and Orthogonality Condition Model by Okabe [45] for ^{16}O. As a result, the model space of the 10-channel CRC configuration is quite large, and the total elastic cross section (see Fig. 5) is not a simple interference pattern (7) of the two amplitudes but a superposition of all direct and indirect scattering and transfer amplitudes under consideration. The spurious deep minima between 120° and 180° in Fig. 4 have been eliminated (see Fig. 5) through an interference of a large number of direct and indirect transfer amplitudes. With the measured elastic data well reproduced by the CRC calculation using the α spectroscopic factors given by the large-scale shell-model calculation [41,42], the 10-channel CRC results shown in Fig. 5 are deemed to be more realistic. Due to very small α spectroscopic factors predicted for the unbound excited states of ^{12}C and ^{16}O [41,42], the breakup effect to the ALAS should be negligible, and we did not include the breakup channel into the model space of the CRC calculation of elastic $^{16}O + {}^{12}C$ scattering at low energies [5].

IV. OPTICAL POTENTIAL INVERTED FROM THE ELASTIC *S* MATRIX

To determine the equivalent CEP generated by elastic α transfer or core-core exchange, we have performed the inversion of the elastic-scattering S matrix given by the CRC calculation to a local, equivalent OP (6). The CEP is obtained as the Majorana potential $V_M(R)$ by substracting the Wigner potential $V_W(R)$ from the inverted OP. The IP inversion procedure [46–49] implemented in the code IMAGO [50] delivers a local complex OP that reproduces with very high precision the elastic-scattering S matrix given by the CRC calculation (referred to as the target S matrix, S_L^t). The accuracy of the inversion procedure is given by the quantity σ defined as

$$\sigma^2 = \sum_L \left|S_L^t - S_L^{\rm i}\right|^2, \qquad (12)$$

where $S_L^{\rm i}$ is the S matrix for the potential found by the inversion process. The IP procedure can yield separate potentials for the even-L partial waves and the odd-L partial waves, $V_{\rm even} = V_W + V_M$ and $V_{\rm odd} = V_W - V_M$, where $V_W(R)$ and $V_M(R)$ are, respectively, the Wigner and Majorana components defined in Eq. (6).

The inversion procedure begins the iterative process with a starting reference potential (SRP), which is usually the OP used in the original OM or coupled-channels calculation. It has been found that inversion with the IP method leads to potentials that are generally independent of the SRP [48,49], as can be tested in particular cases. The IP inversion method was used earlier to investigate the parity dependence of the nucleus-nucleus OP for some core-identical systems like ^{3}He$/t+\alpha$ [51] or ^{16}O$+^{20}$Ne [37] at low energies. These studies have used, however, the S matrices given by the models that are quite different from the CRC formalism. While S_L^t used in Ref. [51] was taken from the RGM calculation, the one used in Ref. [37] was given by the LCNO method that already includes a phenomenological parity-dependent potential into the real OP. The present work is the first attempt to determine the CEP for a light HI system at higher energies ($E > 5$ MeV/nucleon) exclusively from the coupling between the elastic-scattering channel and different inelastic-scattering and transfer reaction channels by inverting the elastic S matrix given by the CRC calculation of elastic ^{16}O$+^{12}$C scattering.

The reliability of the IP inversion method was tested first with S_L^t given by the single-channel OM calculation. In this case, the inverted OP is almost identical with the original OP and gives the elastic cross section that is graphically indistinguishable from that given by the OM calculation (dashed line in Fig. 4). We discuss now the inversion results obtained with the target S matrix given by the two-channel CRC calculation that includes only the true elastic scattering and direct elastic α transfer (solid line in Fig. 4). The necessity of a parity-dependent term in the total OP can be well illustrated by imposing a shape of the inverted OP in Eq. (6) that contains only the Wigner term. From the results shown in Figs. 6 and 7, one can see that both the real and imaginary parts of the inverted OP (obtained with a high precision of $\sigma = 2.3\times10^{-3}$) are strongly undulatory at the surface ($R \approx 3-6$ fm), especially at the energy of 300 MeV.

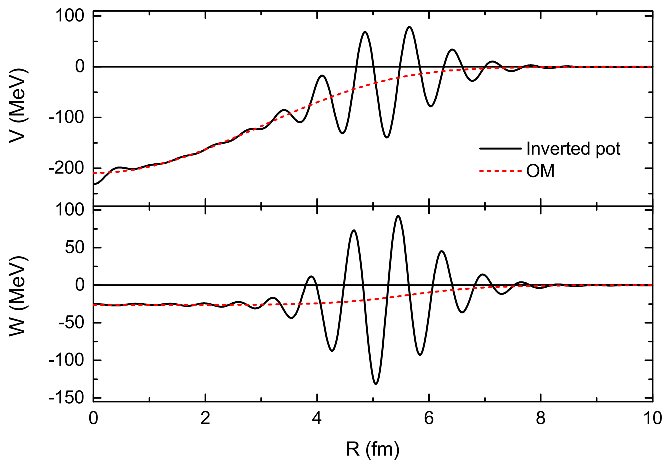

FIG. 6. OP assumed to contain only the Wigner term (solid line) inverted from the S matrix given by the two-channel CRC calculation of elastic ^{16}O$+^{12}$C scattering at $E_{\rm lab} = 300$ MeV. The original OP or SRP is shown as a dashed line.

This indicates clearly the lack of a parity-dependent term in the total OP [52], which is expected to peak in the surface region where the α transfer is dominant [5]. At 300 MeV, the distinctive "V-shape" cross section shown in lower panel of Fig. 4 and a strongly localized oscillation of the inverted OP suggest that the parity dependence of the OP caused by the elastic α transfer or core-core exchange is more pronounced at this energy, where the data points at the most backward angles are entirely due to the α transfer and cannot be reproduced by a single-channel OM calculation [5]. At lower energies, the elastic α transfer and elastic-scattering amplitudes are mixed at medium and large angles, and the enhanced oscillation of the elastic cross section can be reproduced in the single-channel OM calculation using a very small diffuseness of the imaginary Woods-Saxon (WS) potential [53]. Figure 7 shows that at the lower energy of 132 MeV the inverted OP is undulatory like that obtained in Ref. [54] for the ^{16}O$+^{12}$C system at $E_{\rm lab} = 116$ MeV, where S_L^t was given by the OP with quite a small diffuseness of the imaginary WS potential.

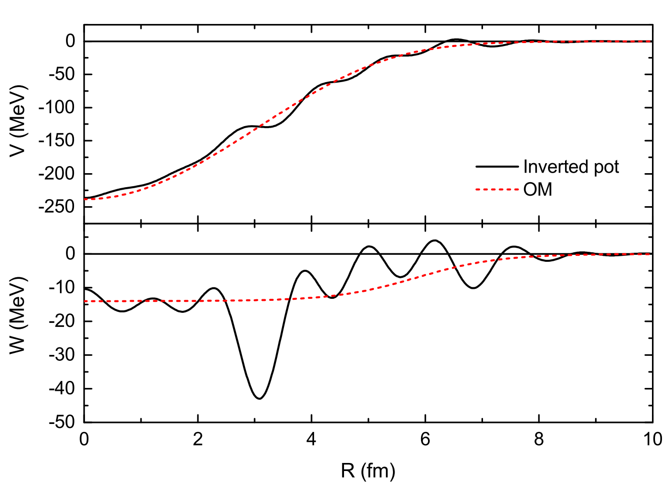

FIG. 7. The same as Fig. 6 but for $E_{\rm lab} = 132$ MeV.

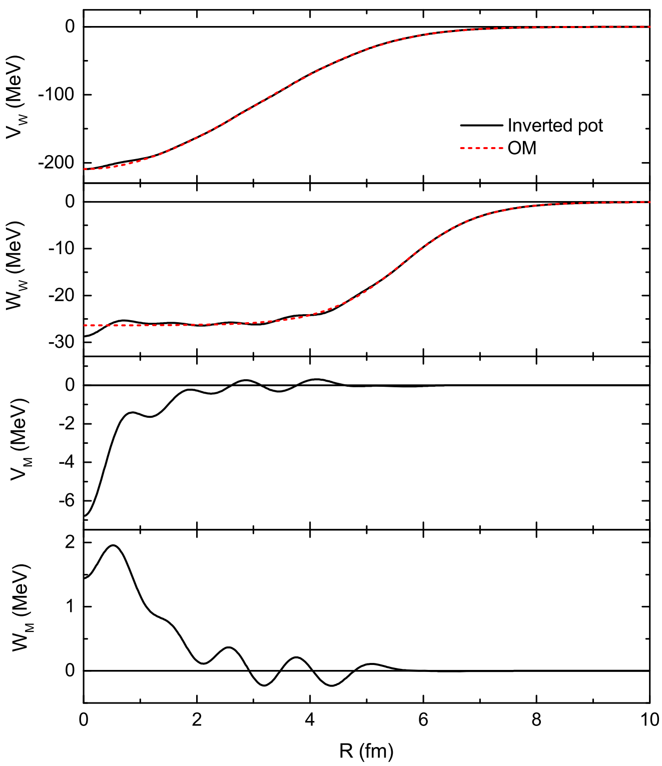

FIG. 8. OP assumed to contain both the Wigner and Majorana terms (solid line), inverted from the S matrix given by the two-channel CRC calculation of elastic $^{16}O + ^{12}C$ scattering at $E_{lab} =$ 300 MeV. The original OP or SRP is shown as dashed line.

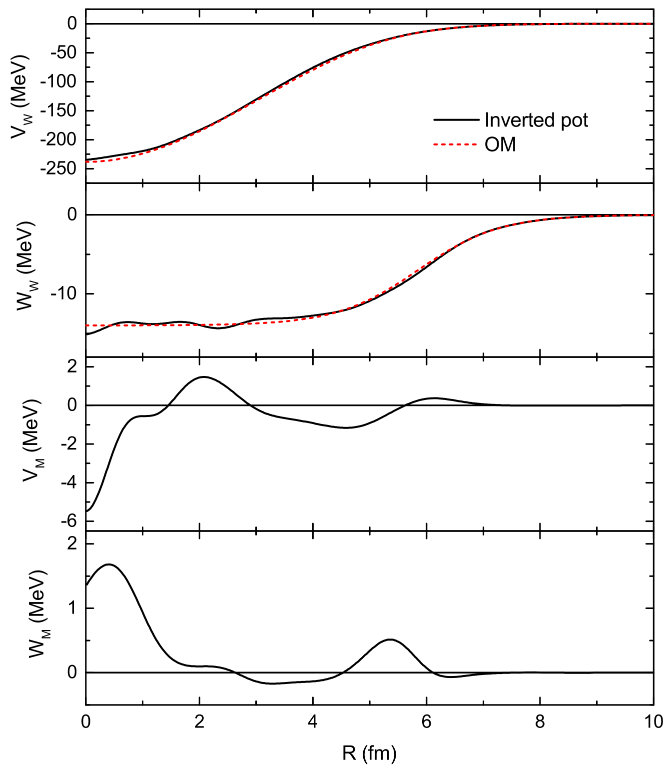

FIG. 9. The same as Fig. 8 but for $E_{lab} = 132$ MeV.

In order to investigate the parity dependence of the OP resulting from the core exchange process, we assume the shape of the local OP to contain both the Wigner and Majorana terms as in Eq. (6). The IP inversion with this prescription gives the inverted OP shown in Fig. 8 ($\sigma = 3.3 \times 10^{-4}$) and Fig. 9 ($\sigma = 1.7 \times 10^{-3}$) for $E_{lab} = 300$ and 132 MeV, respectively. The inclusion of the *parity-dependent* Majorana term into the OP significantly improves the accuracy of the inversion procedure, and the elastic S matrix and -scattering cross section given by the inverted OP in the single-channel OM calculation are nearly identical to those given by the two-channel CRC calculation. This similarity between the bare potential and the Wigner term of the inverted one is the direct evidence of the parity dependence of the CEP. One can see that both the real and imaginary parts of the Wigner potential become quite smooth when a complex Majorana term is included to account for the elastic α transfer. This means that the core exchange contribution to the elastic $^{16}O + ^{12}C$ cross section is well accounted for by the parity-dependent (Majorana) component of the OP.

The results shown in Figs. 8 and 9 are the direct representation of a complex parity-dependent CEP in the OP inverted for the single-channel OM calculation. The inverted real and imaginary Wigner parts are almost the same as those of the original OP. The elastic-scattering cross section given by the single-channel OM calculation using the inverted OP is graphically indistinguishable from that given by the two-channel CRC calculation (solid line in Fig. 4). We found a small oscillation of the Majorana potential which should be associated with the elastic transfer form factor that by itself is undulatory [22,24,25]. The range of the Majorana term was found to be slightly shorter than that of the Wigner term. The short range and weak strength of the CEP are expected features as suggested by the earlier microscopic study [28] and systematic parity-dependent analysis [31].

It is not surprising that the Majorana terms have their largest magnitude at small dinuclear distances, as found in the earlier RGM calculations where the multinucleon exchange is included explicitly in the nucleon-nucleus [55] and nucleus-nucleus scattering [29,51]. It is reasonable that the parity-dependent term of the OP has the same behavior when the core exchange process is included explicitly in the CRC calculation. We have also done a test of cutting off the strength of the Majorana terms at small radii and found that the elastic $^{16}O + ^{12}C$ scattering cross section at the considered energies is sensitive to the Majorana potential mainly in the subsurface region, with $R \gtrsim 3$ fm.

The two-channel CRC calculation reproduces nicely the measured elastic data, but it requires an "effective" S_α that is much larger than that predicted by the structure studies. The main reason is that the two-channel CRC model space does not explicitly take into account the excitation of the two colliding nuclei. To have a more realistic estimate for the CEP given by the multistep process through different inelastic-scattering and transfer channels, we have further performed the IP inversion of the S matrix given by the 10-channel CRC calculation [5] (see Fig. 5). The inversion results are presented in Fig. 10 ($\sigma = 1.5 \times 10^{-5}$) and Fig. 11 ($\sigma = 1.4 \times 10^{-4}$)

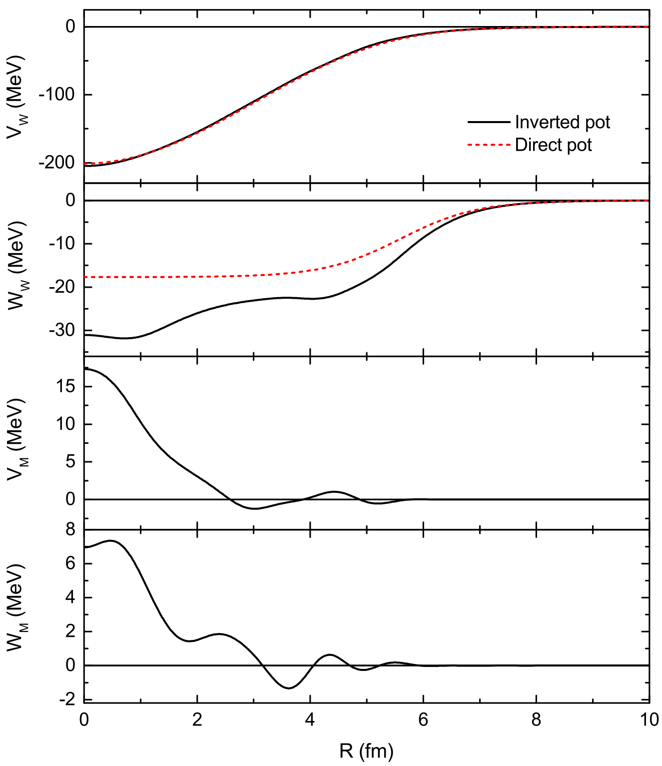

FIG. 10. OP assumed to contain both the Wigner and Majorana terms (solid line), inverted from the S matrix given by the 10-channel CRC calculation of elastic $^{16}O + ^{12}C$ scattering at $E_{lab} = 300$ MeV. The original OP or SRP is shown as dashed line.

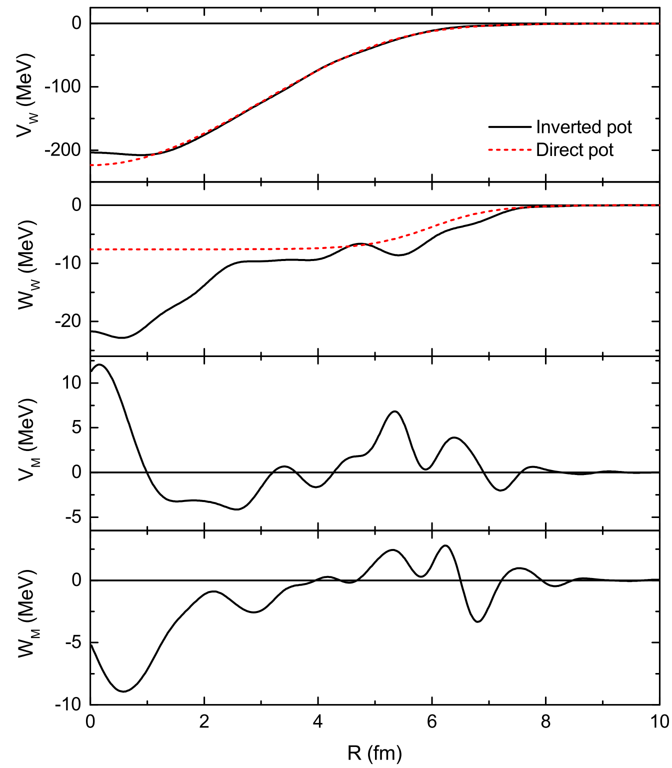

FIG. 11. The same as Fig. 10 but for $E_{lab} = 132$ MeV.

for $E_{lab} = 300$ and 132 MeV, respectively. Like the results obtained with the S matrix given by the 2-channel CRC calculation, the Wigner potential is quite smooth, with its real part being close to that of the original OP. On the other hand, the imaginary Wigner potential is deeper in the center compared with the original OP. Such a difference in the imaginary OP is due to the dynamic polarization potential (DPP) arising from the coupling to the inelastic-scattering channels. This coupling also increases the strength of the Majorana term and makes its structure more complicated compared to that obtained with the S matrix given by the 2-channel CRC calculation, especially, at the lower energy of 132 MeV (see Fig. 11).

This work also shows how the inclusion of an explicit parity dependence makes it possible to identify the contribution of the collective excitations to the OP. The inversion of the S matrix to the OP for the one-channel ON calculation has been used [46,49] to determine the contribution of inelastic channels or reaction channels to the nuclear OP. The core exchange process would make it impossible without allowing explicit parity dependence in the inversion. This is evident from the strong undularity seen in Figs. 6 and 7. The present work shows that the inclusion of a parity dependence into the inverted OP clearly reveals the contribution of the collective excitations to the OP (i.e., the DPP). For the Wigner term in Fig. 10, the difference between the solid and dashed lines is a direct measure of the DPP due to the coupling to the inelastic-scattering channels. Such a coupling leads to a very large absorptive term in the DPP, an increase of more than 50% of the imaginary term at the origin. The contribution to the real part is small on the scale of the figure but consistently repulsive by a few percentages, except at the origin, where it is slightly attractive. By contrast, Fig. 8 shows that without the inelastic coupling there is effectively no enhanced absorption. Figure 11 shows that similar conclusions can be drawn at 132 MeV. Thus, our results show that the inclusion of the parity dependence enables the determination of DPP by the inversion in the presence of strong core exchange effects.

V. SUMMARY

The elastic α transfer or core exchange process in elastic $^{16}O + ^{12}C$ scattering was shown to result in a parity-dependent CEP in the effective OP that gives (in a single-channel OM calculation) the same description of elastic data over the whole angular range as that given by the CRC calculation that takes into account the core exchange explicitly. The high-precision IP inversion of the S matrix given by the multichannel CRC calculation of elastic $^{16}O + ^{12}C$ scattering [5] gives readily a complex, parity-dependent Majorana potential that accounts for the core exchange in the $^{16}O + ^{12}C$ system. From a simple analytical derivation of the core exchange process, the parity dependence found in the OP inverted for a core-identical system is naturally explained. The inclusion of an explicit parity dependence also makes it possible to determine the DPP caused by the coupling to the collective excitations in the presence of elastic α transfer.

The complex structure of the obtained Majorana potential is likely associated with the properties of the elastic transfer or core exchange process, such as spectroscopic factors and transfer form factors. Therefore, the standard practice of using a simple prescription $[1 + \alpha(-1)^L]V(R)$ in some phenomenological OM studies cannot realistically represent the core exchange in elastic scattering of a core-identical system.

We found that the L dependence of the Majorana potential may be due partially also to the dynamic polarization of the OP by the coupling to different inelastic-scattering channels. A more detailed study of the parity dependence of the nucleus-nucleus OP is, therefore, necessary and this will be the subject of our further research on this interesting and fundamental topic.

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[1] W. von Oertzen and H. G. Bohlen, Phys. Rep. **19** 1 (1975).
[2] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon, Oxford, 1983).
[3] I. J. Thompson, Comput. Phys. Rep. **7**, 167 (1988).
[4] I. J. Thompson and F. M. Nunes, *Nuclear Reactions for Astrophysics* (Cambridge University Press, Cambridge, UK, 2009).
[5] Nguyen Tri Toan Phuc, N. H. Phuc, and D. T. Khoa, Phys. Rev. C **98**, 024613 (2018).
[6] P. Braun-Munziger and J. Barette, Phys. Rep. **87**, 209 (1982).
[7] B. Imanishi and W. von Oertzen, Phys. Rep. **155**, 29 (1987).
[8] J. A. Christley, M. A. Nagarajan, and A. Vitturi, Nucl. Phys. A **591**, 341 (1995).
[9] N. Rowley and K. Hagino, Phys. Rev. C **91**, 044617 (2015).
[10] W. E. Frahn, in *Treaties on Heavy-Ion Science*, edited by D. A. Bromley (Plenum Press, New York, 1984), Vol. 1, p. 135.
[11] W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. **432**, 43 (2006).
[12] W. von Oertzen and A. Vitturi, Rep. Prog. Phys. **64**, 1247 (2001).
[13] M. J. Ermamatov *et al.*, Phys. Rev. C **94**, 024610 (2016).
[14] A. Lépine-Szily, M. S. Hussein, R. Lichtenthäler, J. Cseh, and G. Lévai, Phys. Rev. Lett. **82**, 3972 (1999).
[15] N. Keeley, N. Alamanos, K. W. Kemper, and K. Rusek, Prog. Part. Nucl. Phys. **63**, 396 (2009).
[16] A. M. Mukhamedzhanov *et al.*, Phys. Rev. C **56**, 1302 (1997).
[17] D. Howell, B. Davids, J. P. Greene, R. Kanungo, S. Mythili, C. Ruiz, G. Ruprecht, and I. J. Thompson, Phys. Rev. C **88**, 025804 (2013).
[18] T. L. Belyaeva, A. N. Danilov, A. S. Demyanova, S. A. Goncharov, A. A. Ogloblin, and R. Perez-Torres, Phys. Rev. C **82**, 054618 (2010).
[19] M. Katsuma, Phys. Rev. C **78**, 034606 (2008).
[20] M. E. Brandan and G. R. Satchler, Phys. Rep. **285**, 143 (1997).
[21] D. T. Khoa, W. von Oertzen, H. G. Bohlen, and S. Ohkubo, J. Phys. G **34**, R111 (2007).
[22] W. von Oertzen, Nucl. Phys. A **148**, 529 (1970).
[23] W. von Oertzen and W. Nörenberg, Nucl. Phys. A **207**, 113 (1973).
[24] R. C. Fuller and K. W. McVoy, Phys. Lett. B **41**, 257 (1972).
[25] W. E. Frahn and M. S. Hussein, Nucl. Phys. A **346**, 237 (1980).
[26] M. Lemere, D. J. Stubeda, H. Horiuchi, and Y. C. Tang, Nucl. Phys. A **320**, 449 (1979).
[27] D. Baye, J. Deenen, and Y. Salmon, Nucl. Phys. A **289**, 511 (1977).
[28] D. Baye, Nucl. Phys. A **460**, 581 (1986).
[29] K. Aoki and H. Horiuchi, Prog. Theor. Phys. **69**, 857 (1983).
[30] A. Barbadoro, D. Consolaro, F. Pellegrini, L. Taffara, D. Trivisonno, M. Bruno, and I. Gabrielli, Nuovo Cim. A **95**, 197 (1986).
[31] J. L. Ferrero, J. A. Ruiz, B. Bilwes, and R. Bilwes, Nucl. Phys. A **510**, 360 (1990).
[32] C. Gao and G. He, Phys. Lett. B **282**, 16 (1992).
[33] Y. Kondō, B. A. Robson, R. Smith, and H. H. Wolter, Phys. Lett. B **162**, 39 (1985).
[34] A. Vitturi and C. H. Dasso, Nucl. Phys. A **458**, 157 (1986).
[35] R. S. Mackintosh, Eur. Phys. J. A **55**, 147 (2019).
[36] D. T. Khoa, N. H. Phuc, D. T. Loan, and B. M. Loc, Phys. Rev. C **94**, 034612 (2016).
[37] S. Ait-Tahar, R. S. Mackintosh, and S. G. Cooper, Nucl. Phys. A **561**, 285 (1993).
[38] A. A. Ogloblin, D. T. Khoa, Y. Kondō, Yu. A. Glukhov, A. S. Dem'yanova, M. V. Rozhkov, G. R. Satchler, and S. A. Goncharov, Phys. Rev. C **57**, 1797 (1998).
[39] A. A. Ogloblin, Y. A. Glukhov, V. Trzaska, A. S. Demyanova, S. A. Goncharov, R. Julin, S. V. Klebnikov, M. Mutterer, M. V. Rozhkov, V. P. Rudakov, G. P. Tiorin, D. T. Khoa, and G. R. Satchler, Phys. Rev. C **62**, 044601 (2000).
[40] M. E. Brandan, A. Menchaca-Rocha, L. Trache, H. L. Clark, A. Azhari, C. A. Gagliardi, Y.-W. Lui, R. E. Tribble, R. L. Varner, J. R. Beene, and G. R. Satchler, Nucl. Phys. A **688**, 659 (2001).
[41] A. Volya and Y. M. Tchuvilsky, Phys. Rev. C **91**, 044319 (2015).
[42] A. Volya (private communication).
[43] D. T. Khoa and G. R. Satchler, Nucl. Phys. A **668**, 3 (2000).
[44] M. Kamimura, Nucl. Phys. A **351**, 456 (1981).
[45] S. Okabe, in *Tours Symposium on Nuclear Physics II*, edited by H. Utsunomiya *et al.* (World Scientific, Singapore, 1995), p. 112 (and private communications).
[46] R. S. Mackintosh and A. M. Kobos, Phys. Lett. B **116**, 95 (1982).
[47] S. G. Cooper and R. S. Mackintosh, Inverse Probl. **5**, 707 (1989).
[48] V. I. Kukulin and R. S. Mackintosh, J. Phys. G **30**, R1 (2004).
[49] R. S. Mackintosh, Scholarpedia **7**, 12032 (2012).
[50] S. G. Cooper, *Notes for Imago Users*, Open University report (1999) (unpublished).
[51] R. S. Mackintosh and S. G. Cooper, Nucl. Phys. A **589**, 377 (1995).
[52] S. G. Cooper and R. S. Mackintosh, Z. Phys. A **337**, 357 (1990).
[53] M. P. Nicoli, F. Haas, R. M. Freeman, S. Szilner, Z. Basrak, A. Morsad, G. R. Satchler, and M. E. Brandan, Phys. Rev. C **61**, 034609 (2000).
[54] R. S. Mackintosh, Phys. Rev. C **94**, 034602 (2016).
[55] S. G. Cooper and R. S. Mackintosh, Nucl. Phys. A **592**, 338 (1995).