PHYSICAL REVIEW C **100**, 054332 (2019)

Hybrid model for two-proton radioactivity

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(Received 6 March 2019; revised manuscript received 15 October 2019; published 26 November 2019)

A hybrid model is proposed which combines the nuclear structure part from configuration-interaction (CI) calculations and the emission dynamics from a three-body approach. By normalizing the partial half-lives from the three-body model with CI amplitudes, we can reasonably well reproduce all known experimental two-proton ($2p$) emission half-lives, except for ^{67}Kr. For the purpose of the present paper, the full body of experimental $2p$ radioactivity data for nuclei for which the half-life and the $2p$ emission branching ratio are experimentally known has been analyzed in our hybrid model.

DOI: 10.1103/PhysRevC.100.054332

I. INTRODUCTION

Two-proton ($2p$) radioactivity as the simultaneous emission of two protons from a nuclear ground state with a measurable half-life was first discovered about 15 years ago in the decay of ^{45}Fe [1,2]. It is the latest nuclear decay type observed experimentally. This decay mode was predicted more than 40 years earlier by Goldanskii [3] and others (see, e.g., Refs. [4,5]). Only the continued development of experimental techniques allowed finally its observation.

These experimental efforts were accompanied by theoretical developments aiming first to reliably predict the best candidates for this new decay mode. For this purpose, methods were developed [6–13] to predict the $2p$ decay energy, the Q_{2p} value. By comparison with the competing β^+-decay half-lives, one obtains the likelihood of the $2p$ decay taking place. These estimates for the decay energy and half-lives guided the choice of experiments to be carried out. These models were quite schematic and could only yield a rough lower limit of the $2p$ half-life. In particular, the assumption used in all these theoretical descriptions that the two protons form a quasibound ^{2}He nucleus which would decay only far outside the Coulomb barrier was a rather crude hypothesis.

More refined models yielding better agreement with the first experimental data were also developed, such as the R-matrix approach [14] or the shell model embedded in the continuum [15]. These theoretical descriptions yielded reasonable agreement with the experimental data (see, e.g., Ref. [16]). However, as we will not further use them in the present work, we refrain from a detailed description of these approaches.

The three-body model developed by Grigorenko and coworkers [17,18] and used in the present work will be described below. We will use this approach in combination with nuclear CI predictions to propose a hybrid model for two-proton radioactivity. The idea is to use the best model for the $2p$ emission dynamics (the three-body approach of Grigorenko and coworkers) together with the best theoretical description for predicting the different partial emission spectroscopic amplitudes (the CI model).

II. THE EXPERIMENTAL BODY OF DATA

Experiments on $2p$ radioactivity were first performed on ^{45}Fe [1,2]. These early experiments conducted with silicon detector telescopes were completed by a follow-up experiment with a similar set-up at Grand Accelerateur National D'Ions Lourds [19], an experiment with the Bordeaux time projection chamber (BxTPC) [20], and an experiment at Michigan State University with an optical time projection chamber (OTPC) from Warsaw [21]. Subsequently, measurements were also performed on ^{48}Ni with silicon detectors [19] and that same OPTC [22,23] as well as for ^{54}Zn again with silicon detectors [24] and the BxTPC [25]. Finally, the decay of ^{67}Kr was observed in an experiment at the Radioactive Ion Beam Facility of the Institute of Physical and Chemical Research (RIKEN) in Japan [26] with a silicon detector telescope.

Due to its short half-life, the decay of ^{19}Mg had to be studied in a different type of experiments. Using a particle tracking set-up directly behind the production target [27–29], its half-life could be determined from the vertex position and its branching ratio is taken as 100%.

The experimental data of interest for the present study are summarized in Table I. The theoretical data used for the comparison experiment-theory are also given there.

III. THEORETICAL MODELS

The theoretical models developed in the past have all a number of important deficiencies. Either they neglect largely the emission dynamics (e.g., the diproton model, the R-matrix

TABLE I. Experimental and theoretical data used in the present paper. The top part gives the experimental quantities, whereas the lower part is devoted to theoretical quantities. The experimental data for ^{19}Mg [29], ^{45}Fe [1,2,19–21], ^{48}Ni [19,22,23], ^{54}Zn [24,25], and ^{67}Kr [26] are from the literature. The three-body half-lives were taken from the papers of Grigorenko and coworkers for ^{19}Mg [30] as well as for ^{45}Fe, ^{48}Ni, ^{54}Zn, and ^{67}Kr [31]. The three-body s^2 values are discussed in the text.

	^{19}Mg	^{45}Fe	^{48}Ni	^{54}Zn	^{67}Kr
Expt. half-life	4.0(15) ps	$2.5^{+0.2}_{-0.2}$ ms	$2.1^{+1.2}_{-0.5}$ ms	$1.78^{+0.66}_{-0.37}$ ms	7.4(30) ms
Expt. branching	100%	68.1(35)%	52^{+24}_{-20}%	$90.3^{+5.1}_{-10.3}$%	37(14)%
Expt. partial $2p$ half-life	4.0(15) ps	$3.62^{+0.37}_{-0.33}$ ms	$4.1^{+2.9}_{-1.8}$ ms	$1.94^{+0.74}_{-0.47}$ ms	20(11) ms
Expt. Q_{2p} value	750(50) keV	1156(14) keV	1340(20) keV	1480(20) keV	1690(17) keV
Three-body $T_{1/2}(s^2)$	$2.0^{+3.7}_{-0.5} \times 10^{-13} s$	0.24(12) ms	0.04(3) ms	0.12(6) ms	0.038(18) s
Three-body $T_{1/2}(1p^2)$	–	1.8(7) ms	0.3(2) ms	0.91(42) ms	0.28(13) s
Three-body $T_{1/2}(0d^2)$	$2.5^{+6.5}_{-0.5} \times 10^{-12} s$	–	–	–	–
Three-body $T_{1/2}(0f^2)$	–	99(40) ms	10(5) ms	45(27) ms	13.5(42) s

model, the CI model) or the nuclear structure is taken into account only at a rather low level (three-body models that assume a residual nucleus together with two protons in one shell-model orbital). The purpose of the present article is to describe a model which uses the best of the models for the two aspects, the emission dynamics and the nuclear structure. We will therefore use the three-body half-lives for the emission of the two protons from the different orbitals active in the parent nuclei and combine them with the two-nucleon spectroscopic factors as determined with the CI model.

A. The three-body model

The three-body model developed by Grigorenko and coworkers [17,18,32] uses the hyperspherical harmonics method to describe the three-body decay. This approach reduces the Jacobi coordinates for three particles to the hyperradius ρ and the hyperangle θ. Two different Jacobi systems can be defined: the “T” and the “Y” models. The “T” model is a system where the proton-proton cluster is coupled to the heavier core, whereas in the “Y” model the proton-core system is coupled to the remaining proton. The Schrödinger equation is solved in a narrow box with outgoing wave boundary conditions. A special treatment is also used for the Coulomb potential which has a ρ^{-1} dependence.

In the model, the proton-proton final-state interaction is tuned to the proton-proton phase shift. The proton-core channel is described by a Woods-Saxon potential. The structure of the decaying parent nucleus is adjusted by tuning parameters of the proton-core interaction potential for different ℓ values. By solving the decay kinematics in hyperspherical variables, the model allows one to study angular and energy correlations between decay products.

The largest deficiency of this model is probably the poor treatment of nuclear structure which is basically only present via the angular-momentum quantum number ℓ^2. For the nuclei of interest in the present paper, the model gives a too-fast decay for the smaller ℓ^2 (s^2 and p^2) values, whereas the larger ℓ^2 (d^2 and in particular f^2) values give too-long half-lives (see, e.g., Ref. [24]).

The three-body half-lives from Refs. [30,31] depend on the orbital angular momentum ℓ and the two-proton decay Q_{2p} value. They can be converted into single-orbital two-proton decay widths

$$\Gamma_s(Q_{2p}, \ell^2) = \frac{\hbar \ln(2)}{T_{1/2}(Q_{2p}, \ell^2)} \tag{1}$$

and two-proton decay amplitudes

$$A_s(Q_{2p}, \ell^2) = \sqrt{\Gamma_s(Q_{2p}, \ell^2)}. \tag{2}$$

B. Two-nucleon emission amplitudes for the shell model

In the CI model, we calculate two-nucleon decay amplitudes (TNA) for the removal of two protons from the initial state $|n\omega' J'\rangle$ leaving it in the final state $\langle (n-2)\omega J \mid$ given by the reduced matrix element

$$\mathrm{TNA}(k_a, k_b) = \frac{\langle (n-2)\omega J || [\tilde{a}_{k_a} \otimes \tilde{a}_{k_b}]^{J_o} || n\omega' J'\rangle}{\sqrt{(1+\delta_{k_a k_b})(2J+1)}}, \tag{3}$$

where $\tilde{a}_k$ is an operator that destroys a proton in the orbital k, k standing for the (n, ℓ, j) quantum numbers of the transferred proton. We apply this to the case with $J' = J = J_o = 0$, and $k_a = k_b = k$. For a pure k^2 two-nucleon initial state, TNA $= 1$.

The problem is how to combine the single-particle decay information contained in $A_s(Q_{2p}, \ell^2)$ with the many-body transfer information contained in the TNA. Ultimately, this must be contained in a consistent formalism that combines both. For this paper we assume two extreme approximations. The first is that the decay for each orbital k is uncorrelated with the others. Thus we obtain a many-body decay width from the incoherent sum:

$$\Gamma_i(Q_{2p}) = \sum_k \Gamma_s(Q_{2p}, \ell^2)\ [\mathrm{TNA}(k^2)]^2. \tag{4}$$

The second is that all of the amplitudes combine coherently as they do in two-nucleon transfer reactions. This gives

$$A_c(Q_{2p}) = \sum_k A_s(Q_{2p}, \ell^2)\ \mathrm{TNA}(k^2) \tag{5}$$

and

$$\Gamma_c(Q_{2p}) = [A_c(Q_{2p})]^2. \tag{6}$$

The phase is taken as positive for all terms since this is what is expected from the pairing part of the Hamiltonian.

TABLE II. Two-nucleon removal amplitudes for the different $2p$ emitters. The model spaces used for these calculations are given in the text.

k	^{19}Mg $1/2^-$	k	^{45}Fe $3/2^+$	^{48}Ni 0^+	^{54}Zn 0^+	^{67}Kr $3/2^-$	^{67}Kr $1/2^-$
$0d_{5/2}$	1.032	$0f_{7/2}$	1.062	1.000	0.316	0.156	0.175
$0d_{3/2}$	0.336	$0f_{5/2}$	0.223	0.111	0.312	0.820	0.719
$1s_{1/2}$	0.423	$1p_{3/2}$	0.235	0.159	0.654	0.419	0.432
		$1p_{1/2}$	0.116	0.061	0.263	0.371	0.315

C. Shell-model results for the two-nucleon transfer amplitudes

Initial and final wave functions for ^{48}Ni, ^{54}Zn, and ^{67}Kr were obtained in the $1p$-$0f$ model space with the GPFX1A Hamiltonian [33–35]. For ^{67}Kr, $J^{\pi} = 3/2^-$ is the ground-state spin-parity of the mirror nucleus ^{67}Ga. ^{67}Ga has a low-lying $1/2^-$ excited state at 0.167 MeV. It is possible that $1/2^-$ is the ground-state spin-parity of ^{67}Kr if there is some Thomas-Ehrman shift. Thus, we give results for both of these states. Their decay properties turn out to be similar. The resulting two-nucleon amplitudes are given in Table II.

For ^{45}Fe, we used the SDPFMU Hamiltonian [36] in a model space of four protons in the $1p$-$0f$ shells and one neutron hole in the $1s$-$0d$ shells. For ^{19}Mg, we used the WBP Hamiltonian [37] in a model space of four protons in the $1s$-$0d$ shells and one neutron hole in the $0p$ shell. The resulting two-nucleon amplitudes are given in Table II.

In Ref. [20] the amplitude of the $1s^2$ TNA for the decay of ^{45}Fe was estimated in perturbation theory to be 0.140. That corresponds to only 2% in the transferred two-particle wave-function percentage, but it gave about a factor of three reduction in the half-life. s^2 is important because Γ_s is large due to the lack of a centrifugal barrier.

For this paper, we calculate the s^2 contributions by expanding the $1p$-$0f$ model space to include two proton holes in the $1s_{1/2}$ orbital and two proton particles in the $2s_{1/2}$ orbital. These orbitals are connected to the $1p$-$0f$ model space by the off-diagonal pairing two-body matrix elements of the form $\langle (1p-0f)^2, J=0, T=1|V|s^2, J=0, T=1\rangle$. We use the M3Y potential for these matrix elements [38,39]. The proton-hole energy for the $1s_{1/2}$ orbital was adjusted to reproduce the excitation energy of the lowest $1/2^+$ state in ^{47}Ca (for ^{45}Fe, ^{48}Ni, and ^{54}Zn) and in ^{55}Ni (for ^{67}Kr). The proton single-particle energy for the $2s_{1/2}$ orbital was adjusted to be at 10 MeV excitation energy in ^{57}Cu. In order to make this expansion, the $1p$-$0f$ part of the basis was truncated. The orbital occupation restrictions in this truncated space are given in Table III. The results are given in Table IV. In spite of the rather severe restrictions, the $1p$-$0f$ TNA values in Table IV are similar to those in the full $1p$-$0f$ model space calculations shown in Table II.

D. The shell-model-corrected three-body half-lives

We now combine the single-particle decay results from Table I to the two-nucleon amplitudes according to the incoherent and coherent combinations given above. The results for these "shell-model corrected three-body half-lives" are given in Table V. For the $1p$-$0f$ shell nuclei we give the results with and without the s^2 contributions. The errors on the calculated half-lives come from the errors in $T_{1/2}(Q_{2p}, \ell^2)$ due to the error in Q_{2p}.

The three-body s^2 values for ^{45}Fe, ^{48}Ni, ^{54}Zn, and ^{67}Kr are not available. For ^{45}Fe, the ratio of $[T_{1/2}(1p^2)/T_{1/2}(1s^2)]$ was estimated to be 7.4 in our previous paper [20]. We used the same ratio for the other nuclei in the pf shell. For the $2s^2$ contribution, we used also this ratio. The half-life results including the s^2 contribution given in Table V were obtained by adding the $s_{1/2}$ TNA from Table IV to the $1p$-$0f$ TNA from Table II. Even though the TNA amplitudes from the s^2 orbitals are relatively small, this contribution is important since the single-particle half-life is about an order of magnitude shorter than that for the p^2 contribution. For ^{45}Fe and ^{48}Ni, the s^2 contribution reduces the half-life by a factor of three.

As can be seen from the table, the range of calculated values is in reasonable agreement with the experimental partial $2p$ emission half-lives, except for the case of ^{67}Kr, for which all of the calculated half-lives are much longer than the experimental value. Overall for ^{19}Mg, ^{45}Fe, ^{48}Ni, ^{54}Zn, the incoherent sum with the s^2 contribution is in best agreement with experiment within the error bars.

Our conclusion from this comparison is that our model, although a hybrid model taking inputs from two conceptually different models, allows us to qualitatively describe the "true $2p$ emission" process in a reasonable manner. In the case of ^{67}Kr, a different approach is most likely needed. Indeed, the decay of this nucleus was studied recently in detail in two different theoretical papers.

TABLE III. Occupation restrictions used for calculations that include s^2 contributions. The numbers given are the minimum-maximum number of nucleons allowed in each orbital.

	k	^{45}Fe	^{48}Ni	^{54}Zn	^{67}Kr
Proton	$1s_{1/2}$	0-2	0-2	0-2	0-2
	$0d_{3/2}$	3-3	4-4	4-4	4-4
	$0f_{7/2}$	0-8	0-8	0-8	6-8
	$1p_{3/2}$	0-4	0-4	0-4	0-4
	$0f_{5/2}$	0-2	0-2	0-2	0-6
	$1p_{1/2}$	0-2	0-2	0-2	0-2
	$1s_{1/2}$	0-2	0-2	0-2	0-2
Neutron	$1s_{1/2}$	0-2	2-2	0-2	0-2
	$0d_{3/2}$	4-4	4-4	4-4	4-4
	$0f_{7/2}$	0-0	0-0	0-4	8-8
	$1p_{3/2}$	0-0	0-0	0-4	0-4
	$0f_{5/2}$	0-0	0-0	0-0	0-6
	$1p_{1/2}$	0-0	0-0	0-0	0-2
	$1s_{1/2}$	0-0	0-0	0-2	0-0

TABLE IV. Two-nucleon removal amplitudes for the different $2p$ emitters including the s^2 contributions.

k	^{45}Fe $3/2^+$	^{48}Ni 0^+	^{54}Zn 0^+	^{67}Kr $3/2^-$	^{67}Kr $1/2^-$
$1s_{1/2}$	0.158	0.142	0.066	0.039	0.035
$0f_{7/2}$	1.046	0.991	0.210	0.115	0.103
$0f_{5/2}$	0.134	0.109	0.219	0.918	0.765
$1p_{3/2}$	0.218	0.156	0.710	0.397	0.434
$1p_{1/2}$	0.085	0.062	0.252	0.382	0.293
$2s_{1/2}$	0.012	0.011	0.005	0.007	0.006

E. The case of ^{67}Kr

Grigorenko and coworkers [40] acknowledged that their model cannot reproduce the experimental data for ^{67}Kr. Instead, they proposed that for this particular nucleus the masses and therefore the one- and two-proton separation energies are such that the one-proton emission is energetically allowed. Therefore, a mix of one- and two-proton decay channels could decrease the decay half-life.

A different explanation is favored by Wang and Nazarewicz [41]. These authors studied the decay of ^{67}Kr in a Gamow coupled-channels framework and find that the deformation couplings increase the $2p$ decay width and thus decrease the decay half-life. This finding is in agreement with the fact that nuclei in the krypton region are strongly deformed, whereas the nuclei in the iron-to-zinc region are rather expected to be spherical.

In Ref. [26] it was shown that the shell model is able to describe the collective features observed in the region of ^{68}Se, and this results in a mixture of TNA involving the $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ shells (see Table III). To understand the difference between our hybrid model and the Gamow-model results [41], one needs to understand how the Gamow-model radial wave functions compare to those used by Grigorenko and coworkers [40] and how the Gamow-model transfer amplitude is expressed in terms of TNA.

An experimental study by means of a time-projection chamber can most likely distinguish between the two explanations. If the one-proton emission channel is open, then the energy-difference distribution of the two protons is expected to be distinctively different for the standard case of "true $2p$ emission" where the energy difference has a Gaussian-type shape with the maximum of the probability distribution corresponding to an equal energy sharing between the two protons. If a different distribution is observed, as predicted by Grigorenko and coworkers [40], then the one-proton emission channel might be indeed open and a mix of $1p$ and $2p$ emission is at work.

IV. CONCLUSIONS

We have analyzed the experimental partial two-proton emission half-lives available in the literature and compared them to theoretical values determined by means of a hybrid model. This approach uses the half-life values (i.e., the dynamical part of the decay process) from the three-body model of Grigorenko and coworkers and the spectroscopic factors (i.e., the nuclear-structure part of the decay) from the nuclear shell model. We believe that this hybrid approach is presently the best way to describe the $2p$ emission process and the global parameter of the decay which is the half-life of the process. It allows for a comparison of experimental values within the bounds of the two theoretical extremes (the incoherent and coherent sums). It is important to include the small s^2 components in the decay.

Similarly to the present hybrid model, the theoretical predictions from the three-body approach could be combined with the CI TNA to yield "shell-model corrected three-body angular distributions." However, this approach cannot replace a model which treats the emission dynamics and the nuclear structure part in a coherent manner. The Gamow coupled-channels framework might be a way to include both aspects consistently. However, presently only calculations for ^{48}Ni and ^{67}Kr are available [41].

On the experimental side, it is necessary to reduce the errors in Q_{2p}, the $2p$ branching ratios, and $T_{1/2}$ as well as to add new emitters to broaden the experimental body of data for the comparison with theoretical predictions. In particular, a reduction of the error in Q_{2p} for ^{45}Fe would be most useful, as all other experimental parameters are reasonably well known for this nucleus.

ACKNOWLEDGMENTS

B.A.B. acknowledges support from NSF Grant No. PHY-1811855. B.B. and J.G. have received support from the Région Nouvelle Aquitaine, Grant No. 2014-1R60402-00003319.

TABLE V. The theoretical half-lives (in ms, except for ^{19}Mg where we use picoseconds) determined with the incoherent and the coherent sum of the different amplitudes contributing to the emission process are compared to the experimental $2p$ emission half-lives (in ms, ^{19}Mg in picoseconds). For the four heavier nuclei, we compare also our calculations with and without the s^2 contributions.

		$T_{1/2}$ without s^2 contribution		$T_{1/2}$ with s^2 contribution	
	Expt. $T_{1/2}$	Incoherent sum	Coherent sum	Incoherent sum	Coherent sum
^{19}Mg $1/2^-$	4.0(15)			$0.73^{+1.5}_{-0.17}$	$0.20^{+0.40}_{-0.05}$
^{45}Fe $3/2^+$	3.6(4)	20(8)	6.6(26)	5.9(24)	1.8(7)
^{48}Ni 0^+	4.1(20)	5.1(29)	1.8(11)	1.3(6)	0.43(22)
^{54}Zn 0^+	1.9(6)	1.8(8)	0.9(4)	1.7(8)	0.6(3)
^{67}Kr $3/2^-$	20(11)	850(390)	320(140)	820(380)	250(110)
^{67}Kr $1/2^-$	20(11)	904(420)	290(130)	940(430)	360(160)

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