Mass relations of corresponding mirror nuclei

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In this paper we report two simple relations of masses between corresponding mirror nuclei, the first of which is based on the regularity of empirical neutron-proton interactions, and the second of which is based on the regularity of the one-nucleon separation energy. We demonstrate that, for $N \geqslant 10$, the empirical neutron-proton interaction of given nucleus with neutron number $N-1$ and proton number Z (we use the convention that $N=Z$ in this paper), or abbreviated by the $(N-1, Z)$ nucleus, equals the neutron-proton interaction of its corresponding mirror nucleus, i.e., the $(N, Z-1)$ nucleus; we also demonstrate that one-proton separation energy S_p and one-neutron separation energy S_n of the $(N-k, Z)$ nucleus $(k = 1, 2, 3, 4)$ equals one-neutron separation energy S_n and one-proton separation energy S_p, respectively, of the $(N, Z-k)$ nucleus, after a simple correction of Coulomb energy and proton-neutron mass difference are considered. Numerical experiments show that these correlations provide us with a remarkably accurate approach to predict masses and separation energies of some proton-rich nuclei with neutron numbers from 10–46. Our predicted masses of proton-rich nuclei are tabulated as a Supplemental Material of this paper.

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I. INTRODUCTION

Nuclear mass, or equivalently binding energy, is one of the most important and fundamental quantities for a given nucleus. Nuclear mass could be conveniently used to extract pairing gaps, neutron-proton interactions, energies of α and β decays, and information on the shell evolution; furthermore, nuclear mass database is one of the key inputs in studying astrophysical processes of the universe. Great efforts have been devoted to describe the state-of-the-art atomic-mass-evaluation database and to predict unknown masses, and along this line, we mention here the Duflo-Zuker mode [1], the finite range droplet model (FRDM) [2,3], the Skyrme Hartree-Fock-Bogoliubov theory [4,5], the Weizsäcker-Skyrme (WS) mass model [6–9], and the spherical relativistic continuum Hartree-Bogoliubov (RCHB) theory [10]. These models are usually called global models.

These global models are both interesting and important. From another perspective, there are also many efforts, called local mass formulas, which are applicable to given nucleus and its neighboring nuclei in the nuclear chart. Along this line we mention the Audi-Wapstra extrapolation method [11,12], mass relations based on neutron-proton (n-p) interactions [13–15], and the Garvey-Kelson mass relations (GKs) [16–32].

Aside from the global models and local formulas, there are some formulas that do not cover the whole nuclear chart but cover much larger regions than the local mass relations, e.g., for pairs of mirror nuclei. Predicting nuclear masses by using the isospin symmetry between mirror nuclei is actually an old idea. In Ref. [33] the mass difference between mirror nuclei, or more generally, between isospin multiplets was discussed in terms of Coulomb energy and neutron-proton mass difference. A recent examination of mass relation for mirror nuclei was performed in Ref. [34], where it was shown that with proper corrections of Coulomb energy and shell effect one is able to describe masses of mirror nuclei with the root-mean-squared deviation around 120–290 keV.

To proceed our discussion below, we take the convention of $Z = N$ for simplicity. Although the Wigner energy prohibits the applicability of the Garvey-Kelson relations for nuclei with $N \approx Z$, in general, it was shown in Refs. [17,18,35] that the transverse Garvey-Kelson relation

$$\begin{aligned} &M(Z+1, N-1) + M(Z-1, N) + M(Z, N+1) \\ &\quad - M(Z, N-1) - M(Z+1, N) - M(Z-1, N+1) \\ &\quad = 0, \end{aligned} \tag{1}$$

works remarkably well. This applicability is a reflection of the isospin symmetry between mirror nuclei. Based on this idea, the above relations was generalized to a number of sophisticated Garvey-Kelson mass relations for mirror nuclei by Tian and collaborators [36], with the resultant root-mean-squared deviation of 398 keV.

The purpose of this paper is to report two interesting relations related to masses of mirror nuclei. We demonstrate

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that some empirical neutron-proton interactions (to be defined later) of the $(N-1, Z)$ nucleus are very close to those of the $(N, Z-1)$ nucleus, and that one-nucleon separation energy of the $(N, Z-k)$ $[k = 1, 2, 3, 4]$ nucleus is close to that of the $(N-k, Z)$ nucleus after correction of Coulomb energy and the neutron-proton mass difference. These simple relations provide us with a new approach to predict, at a remarkable accuracy, some masses of proton-rich nuclei, which are hitherto unaccessible experimentally.

This paper is organized as follows. In Sec. II, we study the empirical neutron-proton interactions, and in Sec. III we study separation energies. Finally in Sec. IV we summarize the results of this paper.

II. NEUTRON-PROTON INTERACTIONS

The residual neutron-proton interaction plays an important role in the evolution of the nuclear properties, such as single-particle structure, nuclear collectivity, and deformation [37–40], and has attracted much attention [13–15,41–44]. The neutron-proton interaction between the last i neutrons and j protons for the nucleus with neutron number N and proton number Z is defined by

$$V_{in-jp}(N, Z) = -M(N, Z) + M(N-i, Z) + M(N, Z-j) - M(N-i, Z-j). \quad (2)$$

We denote the difference between the neutron-proton interaction V_{in-jp} of two-mirror nuclei, the $(N-k, Z)$ and $(N, Z-k)$ pair by

$$\Delta V_{in-jp}(N-k, Z) = V_{in-jp}(N-k, Z) - V_{jn-ip}(N, Z-k), \quad (3)$$

where the $N = Z$ convention is used to ensure the $(N-k, Z)$ and $(N, Z-k)$ pair are two-mirror nuclei. Suppose that the neutron-proton interactions of two-mirror nuclei are equal. One has

$$\Delta V_{1n-1p}(N-1, Z) = 0. \quad (4)$$

This relation was pointed out by Jänecke in Ref. [35] many years ago, and was recently exemplified for a few cases by Zhang *et al.* in Ref. [45].

An examination of the relation in Eq. (4) and relation $\Delta V_{2n-2p}(N-1, Z) = 0$ by using the AME2016 database [46] is presented in Fig. 1, one sees that these two simple relations works remarkably well for $N \geqslant 10$. The root-mean-squared deviation (RMSD) of this relation is very small: For the relation in Eq. (4), namely, $\Delta V_{1n-1p}(N-1, Z) = 0$, all deviations are well below 200 keV except one case with a large uncertainty; the RMSD value is only 77 keV and 73 keV, respectively, for Eq. (4) and the relation $\Delta V_{2n-2p}(N-1, Z) = 0$, with $N-1 \geqslant 10$. We note without details that the exception of the relation with the largest uncertainty is originated from the proton-rich nucleus ^{44}V whose experimental uncertainty of mass is large. If one excluded this exceptional nucleus, the above RMSD value would be reduced to 43 keV and 73 keV, respectively, for relations $\Delta V_{1n-1p}(N-1, Z) = 0$ and $\Delta V_{2n-2p}(N-1, Z) = 0$.

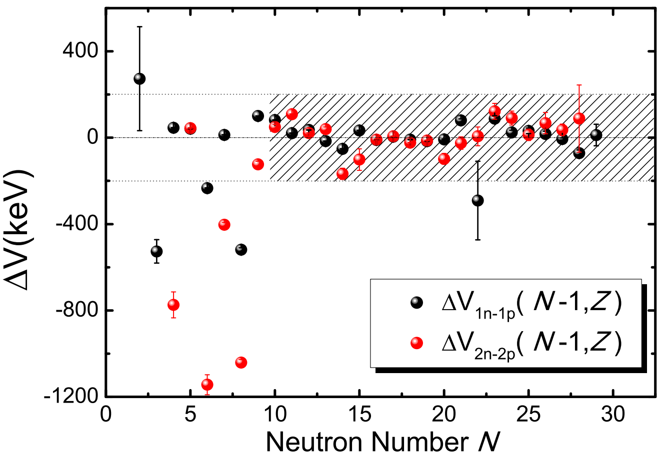

FIG. 1. $\Delta V_{1n-1p}(N-1, Z)$ and $\Delta V_{2n-2p}(N-1, Z)$ $(N = Z)$ extracted from the AME2016 database [46] versus neutron number N. One sees that $\Delta V_{1n-1p}(N-1, Z) = 0$ and $\Delta V_{2n-2p}(N-1, Z) = 0$ works very well for $N \geqslant 10$. Note that the result of $\Delta V_{1n-2p}(N-1, Z)$ is precisely the same as that of $\Delta V_{1n-1p}(N-1, Z)$, and the result of $\Delta V_{2n-1p}(N-1, Z)$ is a simple combination of $\Delta V_{2n-1p}(N-1, Z)$ and $\Delta V_{2n-2p}(N-1, Z)$. Therefore plots of $\Delta V_{2n-1p}(N-1, Z)$ and $\Delta V_{1n-2p}(N-1, Z)$ are not necessary here.

The relation $\Delta V_{in-jp}(N-2, Z) = 0$ works also very well. In the third column of Table I we present the RMSD values of $\Delta V_{in-jp}(N-2, Z) = 0$ for $(i, j) = (1, 1), (1, 2), (2, 1), (2, 2)$ with $N-k \geqslant 10$. The RMSD values of these cases are from 64–152 keV. For relation $\Delta V_{in-jp}(N-3, Z) = 0$, the RMSD values are 133 and 136 keV for $(i, j) = (1, 1), (1, 2)$, respectively; however, they are very large for $(i, j) = (2, 1), (2, 2)$. On the other hand, we should note that the number of relation $\Delta V_{in-jp}(N-3, Z) = 0$ is only one for $(i, j) = (2, 1), (2, 2)$; therefore it is not very clear whether the relation $\Delta V_{in-jp}(N-3, Z) = 0$ does not work for $(i, j) = (2, 1), (2, 2)$, or these two cases with very large deviations are given by exotic structure of certain nucleus in this region, or some of relevant data should be reexamined.

It is instructive to perform numerical experiments of extrapolation by using our simple mass relations. To exemplify this approach, we choose the relation $\Delta V_{in-ip} = 0$ with $i = j = 1$, $i = j = 2$, and $(i = 2, j = 1)$, for $k = 1$. We rewrite these three relations as below.

$$M(N-2, Z) = M(N, Z-2) + M(N-1, Z) - M(N, Z-1) + M(N-2, Z-1) - M(N-1, Z-2), \quad (5)$$

TABLE I. The RMSD (in unit of keV, denoted by σ_k) of relations $\Delta V_{in-jp}(N-k, Z)$ and the number (denoted by $\mathcal{N}_k$) of $\Delta V_{in-jp}(N-k, Z)$ extracted by the AME2016 database for each case, for $k = 1$–3 and different i, j values, with the isotope ^{44}V excluded.

	σ_1	$\mathcal{N}_1$	σ_2	$\mathcal{N}_2$	σ_3	$\mathcal{N}_3$
ΔV_{1n-1p}	43	19	64	16	133	5
ΔV_{2n-1p}	55	18	110	6	868	1
ΔV_{1n-2p}	43	19	76	17	136	5
ΔV_{2n-2p}	73	19	152	6	1071	1

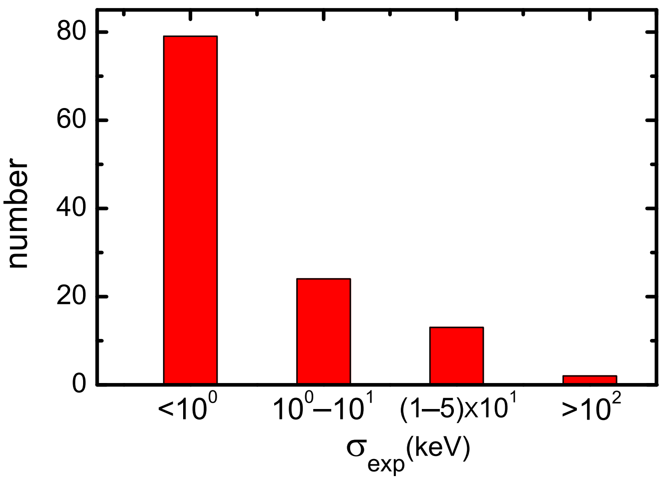

FIG. 2. Histogram plot for the number of nuclei involved in our predictions, versus experiment uncertainty $\sigma_{\rm exp}$ compiled in the AME2016 database. One sees that very few data have experimental uncertainties larger than 100 keV. Therefore the uncertainties of our predicted results come essentially from the uncertainties of Eqs. (5)–(7).

$$\begin{aligned} M(N-3,Z) = {} & M(N,Z-3) + M(N-1,Z) \\ & - M(N,Z-1) + M(N-2,Z-1) \\ & - M(N-1,Z-2) + M(N-3,Z-2) \\ & - M(N-2,Z-3), \end{aligned} \tag{6}$$

$$\begin{aligned} M(N-3,Z) = {} & M(N,Z-3) + M(N-1,Z) \\ & - M(N,Z-1) + M(N-3,Z-1) \\ & - M(N-1,Z-3), \end{aligned} \tag{7}$$

with $N = Z$.

We make use of these three relations and extrapolate some mass excesses from the AME1995 database to the AME2016 database [46]. The resultant RMSD value of our extrapolated eleven proton-rich nuclei is very small: only 47 keV. Therefore, it is very interesting to apply them and to predict unknown masses for the proton-rich nuclei based on the AME2016 database [46]. Here the uncertainty of our predictions by using each of Eqs. (5)–(7) is defined by the square root of corresponding $(\sigma_1)^2$ of Table I plus a sum of squared uncertainties $\sigma^2_{\rm exp}$ of masses involved in corresponding formula. Very luckily, experimental uncertainties of masses in this region are very small, as shown in Fig. 2. Therefore theoretical uncertainties of our predicted values by using each of the formulas are always small.

From Eqs. (6)–(7), we have here up to two possible predicted results, each with a theoretical value (m_i) and an uncertainty (σ_i), our final predicted result is defined by

$$m = F\frac{m_1}{(\sigma_1)^2} + F\frac{m_2}{(\sigma_2)^2}, \tag{8}$$

$$F = \frac{1}{\frac{1}{(\sigma_1)^2} + \frac{1}{(\sigma_2)^2}},$$

TABLE II. Predicted mass excesses and corresponding uncertainties of our predictions (in units of keV) by using Eqs. (5)–(7).

N	Z	A	$\text{mass}^{\text{excess}}_{\text{pred}}$	σ_{pred}
27	30	57	−32778	45
28	31	59	−34019	60
29	31	60	−39988	60
29	32	61	−34001	92
30	32	62	−42328	70
30	33	63	−33832	125
31	33	64	−39666	102
31	34	65	−33484	136
32	34	66	−42059	116
32	35	67	−32935	137
33	35	68	−38712	90
33	36	69	−32755	171
34	36	70	−41579	142

and our predicted uncertainty σ is defined by

$$\sigma = \sqrt{F}. \tag{9}$$

By using Eqs. (5)–(7), we predict 13 nuclear masses above the $N = Z$ line, and present our predicted results in Table II.

III. SEPARATION ENERGIES

In Refs. [33,34], mass relations between mirror nuclei have been performed. In those cases, mass differences between mirror nuclei with neutron number and proton numbers being $(N-k, Z)$ and $(N, Z-k)$, are highly k dependent. The parameter sets in Ref. [34] are determined by empirical neutron-proton mass difference and Coulomb energy difference, as well as a phenomenological shell effect. The RMSD values in Ref. [34] are from 126–289 keV. In this section we report the relations of separation energies for mirror nuclei, which yield substantially smaller RMSD values with much less parameters.

We begin with masses differences of Refs. [33,34],

$$\begin{aligned} & M(N-k,Z) - M(N,Z-k) \\ & \quad = a_c k(A-k)^{2/3} + k(M_p - M_n), \end{aligned} \tag{10}$$

where the $Z = N$ convention is used as above, $A = N + Z$ is the mass number, a_c is the Coulomb term coefficient, and M_p and M_n are the masses of a free proton and a neutron.

Now let us define one-neutron separation energy $S_n(N, Z)$ and one-proton separation energy $S_p(N, Z)$,

$$S_n(N,Z) = M(N-1,Z) - M(N,Z) + M_n, \tag{11}$$

$$S_p(N,Z) = M(N,Z-1) - M(N,Z) + M_p. \tag{12}$$

We next define two quantities, Δ_n and Δ_p,

$$\begin{aligned} \Delta_n(N-k,Z) = {} & S_n(N-k,Z) - S_p(N,Z-k) \\ & + (M_p - M_n), \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta_p(N-k,Z) = {} & S_p(N-k,Z) - S_n(N,Z-k) \\ & + (M_n - M_p). \end{aligned} \tag{14}$$

TABLE III. The RMSD (in keV) of Eqs. (15)–(16), the pair number of mirror nuclei with $k \leqslant 4$ (denoted by $\mathcal{N}$), and resultant parameters a_c and C in Eqs. (15)–(16) optimized by using the AME2016 [46].

Δ	RMSD	$\mathcal{N}$	a_c	C
Δ_n	113	43	718	-1833
Δ_p	132	68	702	$+1637$

For short we denote these Δ_n and Δ_p by Δ. By using the above definitions and Eqs. (10)–(14), we obtain

$$\begin{aligned}\Delta_n(N-k,Z) &= M(N-1-k,Z) - M(N-k,Z) \\ &\quad - M(N,Z-k-1) + M(N,Z-k) \\ &= a_c\delta_c^n + (M_p - M_n),\end{aligned} \tag{15}$$

$$\begin{aligned}\Delta_p(N-k,Z) &= M(N-k,Z-1) - M(N-k,Z) \\ &\quad - M(N-1,Z-k) + M(N,Z-k) \\ &= a_c\delta_c^p + (M_n - M_p),\end{aligned} \tag{16}$$

where

$$\delta_c^n = (k+1)(A-k-1)^{2/3} - k(A-k)^{2/3}, \tag{17}$$

$$\delta_c^p = (k-1)(A-k-1)^{2/3} - k(A-k)^{2/3}, \tag{18}$$

correspond to the Coulomb-energy correction, which are based on the simple formula of Coulomb energy

$$V_\mathrm{c} = \frac{a_c Z(Z-1)}{A^{1/3}}.$$

We abbreviate the above δ_c^n and δ_c^p by using δ_c. For $k=1$, Eq. (16) is reduced to Eq. (10). Therefore, Eqs. (15)–(16) are generalizations of the key formula in Ref. [34].

In Fig. 3, we present the values of Δ defined in Eqs. (13)–(14) and extracted by using the AME2016 database [46], versus δ_c defined in Eqs. (17)–(18). One sees there exist strong linear correlations between those Δ and δ_c. These correlations are independent of k, and this independence is very useful and convenient, as one is able to predict masses of $(N-k,Z)$ nuclei (with different k) based on the unified trajectory.

We rewrite the simple relations in short as below

$$\Delta = a_c\delta + C. \tag{19}$$

For each of Δ_n and Δ_p, we adjust the above coefficients a_c and C to optimize the correlation. The resultant coefficients a_c and C are listed in Table III. We note that values of a_c here are close to that in the Weizsäcker formula, and the values of C are in the reasonable consistence with the neutron-proton mass difference. It is also noted that Eq. (19) for Δ_n with $k=0$ is equivalent to the case for Δ_p with $k=1$. The RMSD values of the simple Δ-δ correlation (which are 113 and 132 keV for one-nucleon separation energy here) are substantially smaller than those obtained in Ref. [34] (which are about 126–300 keV, depending on the value of k) for binding energies.

Clearly, the simple and remarkable Δ-δ correlation provides us with an approach to predict unknown nuclear masses

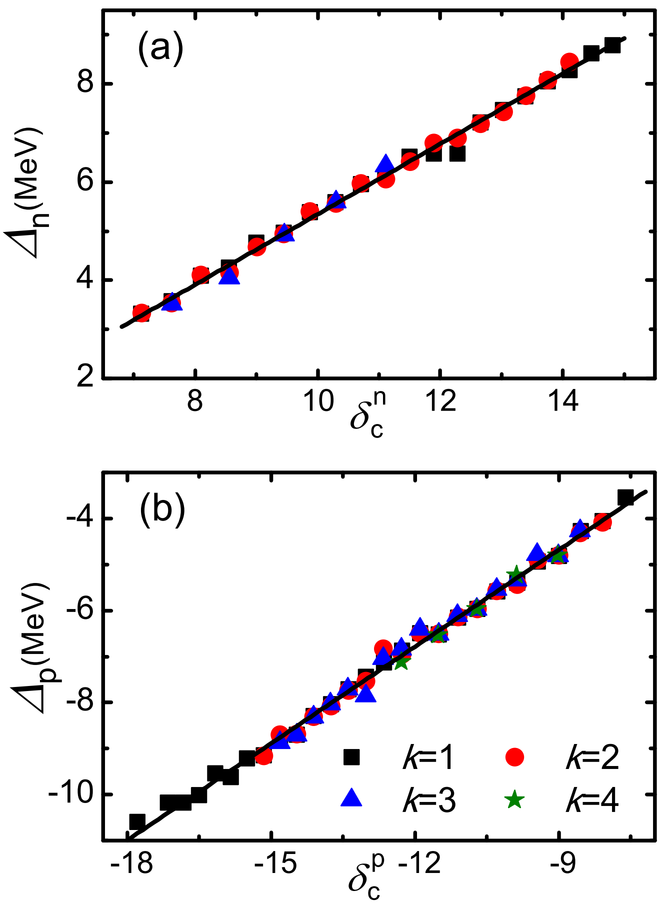

FIG. 3. Linear correlation of Δ extracted from the AME2016 database and δ_c defined in Eqs. (17)–(18). One sees that these correlations are k independent, which enables us to explore masses far from the $N=Z$ line. Here we exclude cases with $(N-k) < 10$. (a): Δ_n; (b): Δ_p.

by extrapolations in high accuracy for proton-rich nuclei with $N \geqslant 10$. We rewrite Eqs. (15)–(16) as below

$$\begin{aligned}M(N-1-k,Z) &= M(N-k,Z) + M(N,Z-1-k) \\ &\quad - M(N,Z-k) + a_c\delta_c^n + C,\end{aligned} \tag{20}$$

$$\begin{aligned}M(N-k,Z) &= M(N-k,Z-1) - M(N-1,Z-k) \\ &\quad + M(N,Z-k) - a_c\delta_c^p - C.\end{aligned} \tag{21}$$

They are used in extrapolations to unknown masses. From these two formulas, we have up to two possible predictions for given nucleus. In this case, our predicted masses are their averaged value with the weight of uncertainties, as in Eqs. (8)–(9).

Before going to our predictions, it is interesting to demonstrate the predictive power of Eqs. (20)–(21) by numerical experiments. For the extrapolation from the AME2003 database to the AME2016 database, our RMSD value is only 102 keV for 11 proton-rich nuclei in the region that we consider above. This RMSD value is substantially smaller than the RMSD (303 keV) given in Ref. [34]. This means that Eqs. (20)–(21)

are very powerful in extrapolations to the proton-rich nuclei for which experimental data is not yet available.

By using Eqs. (20)–(21), we predict 58 nuclear masses for proton-rich nuclei with N from 10–50 and $k \leqslant 4$, based on the AME2016 database [46]; our predicted results are tabulated as a Supplemental Material of this paper [47].

IV. SUMMARY

To summarize, in this paper we report two regularities, one of which is related to neutron-proton interactions, and the other of which is related to the separation energies, for corresponding mirror nuclei. These regularities provides us with a simple but powerful approach to predict unknown neutron-rich nuclear masses with very competitive accuracy, as demonstrated by numerical experiments of extrapolations from the AME1995 database to the AME2016 database, and from AME2003 database to the AME2016 database.

The first regularity is that, neutron-proton interactions of two mirror nuclei, particularly those with neutron number N is smaller than proton number by one or two and their mirror nuclei, are very close to each other. This regularity enables us to predict six proton-rich nuclear masses with theoretical uncertainty ($\sigma_{\rm th}$) below 100 keV, and other seven proton-rich nuclear masses with $\sigma_{\rm th}$ around 100–200 keV.

The second regularity is that, difference between one-nucleon separation energies of corresponding mirror nuclei exhibits a linear correlation with the Coulomb correction term, typically with the RMSD values below 150 keV. This regularity reduces to two formulas of extrapolation to unknown masses, and by which we are able to predict about 60 unknown proton-rich nuclear masses, typically with theoretical uncertainties below 250 keV.

We believe that our predicted nuclear masses and separation energies, via sagacious application of the isospin symmetry, are useful to all studies in which these masses are involved.

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