α-decay systematics for superheavy nuclei

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In this study, the α-decay half-life of 160 superheavy nuclei (SHN) with $105 \leqslant Z \leqslant 120$ has been calculated using six empirical formulas: the scaling law of Brown (SLB), the modified scaling law Brown formula (MSLB), the Yibin *et al.* formula (YQZR), the modified Yibin *et al.* formula (MYQZR), the Viola-Seaborg formula (VS), and the modified Viola-Seaborg formula (MVS). The formulas were readjusted with new coefficients using the experimental α-decay half-life data of 356 nuclei. The six models were tested for their accuracy by comparing with the experimental results, and the MYQZR reveals best agreement. Later on the calculations have been extended to cover the specified range of SHN isotopes. The obtained results from MYQZR can be adopted to predict acceptable α-decay half-lives for the studied SHN nuclei.

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I. INTRODUCTION

The alpha radioactivity was observed by Rutherford and Geiger in the second decade of the 20th century. Alpha decay is considered as one of the most important decay modes to study and provide information on the nuclear structure like ground state lifetime, nuclear spin and parity, and nuclear interaction [1–15].

First, the α decay described by Gamow and Gurney-Condon [16,17] under the effect of the quantum tunneling effect can be considered as the first successful quantum description of nuclear phenomenon.

The first empirical formula for α-decay half-life was done by Geiger and Nuttal [18]. Since then, many empirical and semiempirical formulas have been presented and developed by different authors [19–35]. These empirical formulas aid the experimentalists to find accurate values of α-decay half-lives during the experiment design.

In the present article, we modified the scaling law Brown (SLB) [36] and Yibin *et al.* (YQZR) [37] formulas by adding nuclear asymmetry parameter *(N-Z)/A* inserted as a new influential physical quantity.

Also, we described the α-decay half-life from ground-state to ground-state transitions of 356 selected nuclei (separated to four sets of nuclei into *e-e*, *e-o*, *o-e*, and *o-o*) by using six empirical formulas like the scaling law Brown formula (SLB), the modified scaling law Brown formula (MSLB), the Yibin *et al.* formula (YQZR), the modified Yibin *et al.* formula (MYQZR), the Viola-Seaborg formula (VS), and the modified Viola-Seaborg formula (MVS). For each formula and each sets of nuclei, a new coefficient for the empirical formulas was obtained.

The following section explains the formalism of six empirical formulas and tabulation of new coefficients for each set of nuclei. In Sec. III, we apply each model to evaluate the α-decay half-lives for each isotope and compare the results with the experimental values; the accuracy of each model was tested through statistical estimation of standard deviation, root mean square, and the difference between experimental and calculated half-lives. Also the application of the models has been extended to the SHN nuclei within a specified isotope range; the results are tabulated and shown in figures for each set of isotopes. Finally in Sec. IV the list of conclusions has been presented.

II. FORMALISM OF α-DECAY HALF-LIVES

Since the beginning of the last century, the attempts to formalize alpha decay have been started. Geiger and Nuttall in 1911 set a linear relationship between the logarithms of α-decay half-life with the inverse square root of alpha decay energy ${Q_\alpha}^{-1/2}$ [38]:

$$\ln T = a + b\, Q^{-1/2},$$

where a and b are Z (number of protons of the parent nucleus) dependent fitting parameters. After this achievement, many studies were directed toward the modification and improvement of this principal formula using analytical [23,24,25], semiempirical [21,28], and empirical methods [31,32]. The trials lead to the suggestion of new formulas which subsequently yield better agreement between the experimental and calculated α-decay half-lives. Here in this work four of these formulas [the scaling law of Brown (SLB), the modified scaling law of Brown (MSLB), the Viola-Seaborg formula (VS), and the modified Viola-Seaborg formula (MVS)] were involved (in their previous proposed forms) beside a modified

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TABLE I. The coefficient of the scaling law of Brown (SLB).

Sets	a	b
Even-even	9.210 67	−49.588 40
Even-odd	9.717 86	−51.608 75
Odd-even	10.041 41	−53.457 69
Odd-odd	9.018 62	−47.882 99

version of each formula which were all dependent on the calculation of the α-decay half-lives.

A. Scaling law of Brown (SLB)

In 1992, Brown [36] proposed a universal scaling law by the linear dependence of the $\log_{10} T_{1/2}$ on ${Z_d}^{0.6}{Q_\alpha}^{-1/2}$ quantity for α-decay half-lives of even-even, even-odd, odd-even, and odd-odd parents and is given as

$$\log_{10} T_{1/2}(s) = aZ_d^{0.6}Q_\alpha^{-1/2} + b, \tag{1}$$

where Z_d is the charge number of the daughter nuclei, Q_α is the decay energy of the α-particle, and a and b are coefficients adjusted by the least square fitting of the experimental data for each set, Table I.

B. Modified scaling law of Brown (MSLB)

In the present work, we modify the former SLB relation by adding asymmetry dependent (I and I^2) terms which are linearly related to the logarithm of α-decay half-lives, and the MSLB takes the form

$$\log_{10} T_{1/2}(s) = aZ_d^{0.6}Q_\alpha^{-1/2} + b + cI + dI^2, \tag{2}$$

where Z_d is the charge number of daughter nuclei, Q_α is the decay energy of the α particle, and I is the asymmetry term, $I = \frac{N-Z}{A}$, while a, b, c, and d are the obtained fitting parameters for all the studied sets of nuclei, Table II.

C. Viola-Seaborg formula (VS)

Viola and Seaborg [38,39] proposed a generalized Geiger-Nuttal law as a new empirical formula for α-decay half-lives as

$$\log_{10} T_{1/2}(s) = \frac{aZ + b}{\sqrt{Q}} + cZ + d, \tag{3}$$

where the half-life is given in seconds, and decay energy (Q) in MeV, Z is the proton number of parent nuclei, and a, b, c, and d are the evaluated coefficients given in Table III.

TABLE II. The coefficient of the modified scaling law of Brown (MSLB).

Sets	a	b	c	d
Even-even	9.041 75	−49.629 00	6.878 29	−4.214 97
Even-odd	9.553 19	−49.516 66	−40.593 39	194.661 81
Odd-even	9.945 88	−53.031 98	−8.029 64	48.388 98
Odd-odd	9.131 98	−49.967 73	5.231 86	21.657 46

TABLE III. The coefficient of the Viola-Seaborg (VS) formula.

Sets	a	b	c	d
Even-even	1.466 06	6.945 76	−0.179 00	−34.725 60
Even-odd	1.939 81	−19.843 20	−0.282 51	−30.243 60
Odd-even	1.800 75	−6.340 49	−0.224 45	−36.166 97
Odd-odd	1.665 98	1.435 53	−0.195 27	−36.997 09

D. Modified Viola-Seaborg formula (MVS)

The modification of the Viola-Seaborg formula was also done by adding the two asymmetry dependent (I and I^2) terms that are linearly related to the logarithm of α-decay half-lives; the MVS is given as [40]

$$\log_{10} T_{1/2}(s) = \frac{aZ + b}{\sqrt{Q}} + cZ + d + eI + fI^2. \tag{4}$$

For this MSV formula, the fitting with experimental data yielded the adjusted coefficients a, b, c, d, e, and f as listed in Table IV.

E. YQZR formula (YQZR)

Based on the Ni *et al.* (NRDX) formula [37] for alpha and cluster decay, Yibin *et al.* presented a new empirical formula by introducing the angular momentum (L) parameter to the NRDX formula; the (YQZR) empirical formula was given as [30]

$$\log_{10} T_{1/2}(s) = a\sqrt{\mu}Z_1Z_2Q^{-1/2} + b\sqrt{\mu}(Z_1Z_2)^{1/2} + c\frac{L(L+1)}{\mu\sqrt{Z_1Z_2}A_1^{1/6}} + d, \tag{5}$$

where Z_1 and Z_2 are the proton number of daughter and alpha nuclei, A_1 is the mass number of daughter nuclei, μ is the reduced mass, L is the angular momentum, and a, b, c, d, and e are free parameter coefficient sets for all nuclei and are given in Table V.

F. Modified YQZR formula (MYQZR)

Similarly, as we did with the SLB relation, the YQZR formula has been modified by adding the two asymmetry dependent (I and I^2) terms that are linearly related to the logarithm of α-decay half-lives; the MYQZR will be

$$\log_{10} T_{1/2}(s) = a\sqrt{\mu}Z_1Z_2Q^{-1/2} + b\sqrt{\mu}(Z_1Z_2)^{1/2} + c\frac{L(L+1)}{\mu\sqrt{Z_1Z_2}A_1^{1/6}} + d + eI + fI^2. \tag{6}$$

When the MYQZR formula is tested for the experimental data of the α-decay half-lives using the least square fitting, new free coefficients a, b, c, d, e, and f were obtained, and are presented in Table VI.

III. RESULTS AND DISCUSSION

In this work, a total of 356 nuclei classified to the four sets of even-even, even-odd, odd-even, and odd-odd have been selected to study their α-decay half-lives. Six formulas

TABLE IV. The coefficient of the modified Viola-Seaborg (MVS) formula.

Sets	*a*	*b*	*c*	*d*	*e*	*f*
Even-even	1.526 13	5.689 75	−0.174 39	−36.542 43	6.080 68	−39.581 99
Even-odd	1.928 93	−22.699 30	−0.316 05	−23.701 47	−44.578 35	181.894 29
Odd-even	1.867 12	−9.256 24	−0.224 03	−36.142 37	−6.000 62	−5.204 09
Odd-odd	1.731 98	−4.094 32	−0.223 99	−33.266 85	−24.087 92	92.687 67

TABLE V. The coefficient of the Yibin *et al.* formula (YQZR).

Sets	*a*	*b*	*c*	*d*
Even-even	0.400 02	−1.498 80	0	−11.709 70
Even-odd	0.441 01	−1.433 89	5.353 46	−18.112 24
Odd-even	0.439 01	−1.394 39	4.934 99	−19.017 46
Odd-odd	0.430 57	−1.423 02	4.320 26	−16.937 51

TABLE VI. The coefficient of the modified Yibin *et al.* formula (MYQZR).

Sets	*a*	*b*	*c*	*d*	*e*	*f*
Even-even	0.411 07	−1.449 14	0	−14.870 85	13.386 18	−61.471 07
Even-odd	0.442 47	−1.417 06	5.258 60	−16.755 11	−28.422 24	93.534 85
Odd-even	0.446 95	−1.317 32	4.947 11	−21.249 56	−1.837 58	−16.494 10
Odd-odd	0.433 11	−1.405 14	4.388 54	−17.145 06	−7.397 68	21.414 28

TABLE VII. The RMS deviation of the models SLB, MSLB, VS, and MVS.

Formula	Even-even $n = 137$	Even-odd $n = 90$	Odd-even $n = 66$	Odd-odd $n = 63$
SLB	0.5714	1.2333	1.0020	0.9352
MSLB	0.5141	0.9233	0.9652	0.7551
VS	0.3065	0.7844	0.7180	0.6362
MVS	0.2646	0.6567	0.6899	0.5932
YQZR	0.3165	0.4663	0.4252	0.4649
MYQZR	0.2636	0.4191	0.3832	0.4606

TABLE VIII. ΔT different between experimental and theoretical formulas.

	Even-even		Even-odd		Odd-even		Odd-odd	
Formula	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
SLB	−0.9348	2.4758	−3.1751	4.2444	−2.4497	3.9708	−2.0412	2.4607
MSLB	−0.7904	2.1366	−1.9374	3.4669	−2.7317	3.5861	−1.3860	2.1426
VS	−0.8917	0.6976	−2.1071	1.0894	−2.6422	1.3887	−1.8018	2.3339
MVS	−1.0583	0.5866	−1.7918	1.5694	−2.4912	1.2452	−1.8836	2.1380
YQZR	−0.9225	0.6691	−1.2097	1.5045	−0.9481	1.0474	−0.9257	2.1188
MYQZR	−1.0617	0.5540	−1.2059	1.2923	−0.9913	0.9510	−0.9506	2.1117

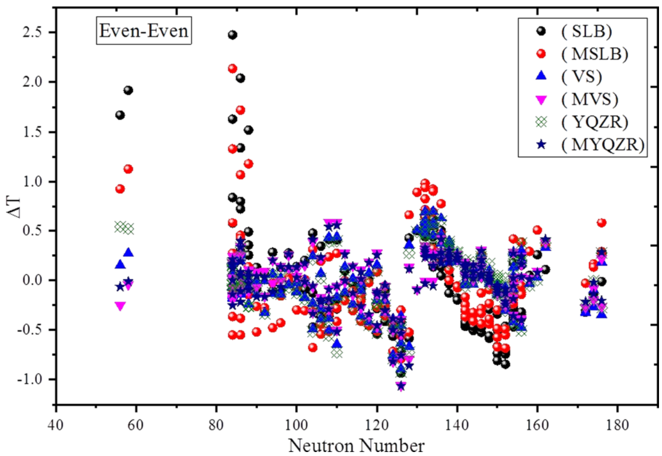

FIG. 1. The difference between theoretical and experimental α-decay half-lives for all formulas of set even-even.

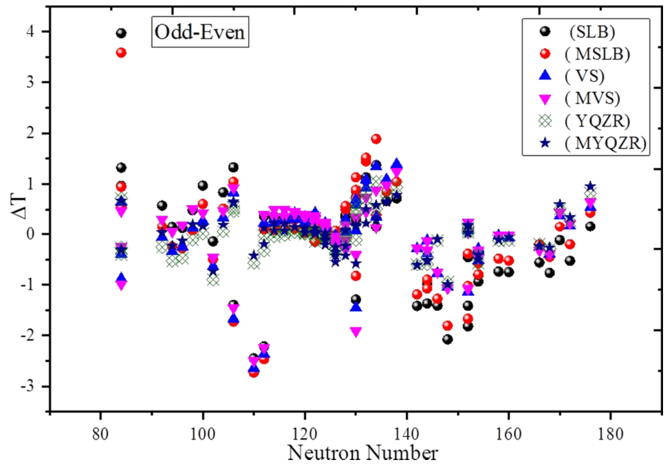

FIG. 3. The difference between theoretical and experimental α-decay half-lives for all formulas of set odd-even.

describing the logarithm of α-decay half-lives, $\log_{10}T_{1/2}$, have been tested using the measured experimental data. Each of the SLB, VS, YQZR formulas and their modified versions MSLB, MVS, and MYQZR were applied using the corresponding recalculated coefficients mentioned in the previous section. The reliability of the formulas is obtained using the root mean square expression,

$$\sigma = \left\{ \frac{1}{n} \sum_{i=1}^{N} \left[\ln\left(T_{\frac{1}{2},i}^{\text{calc.}}\right) - \ln\left(T_{\frac{1}{2},i}^{\text{expt.}}\right) \right]^2 \right\}^{1/2},$$

where $T_{1/2,i}^{\text{calc.}}$ and $T_{1/2,i}^{\text{expt.}}$ are the calculated and experimental α-decay half-lives of the nuclei, and n is the number of nuclei involved for each set.

The calculated RMS for each model is presented in Table VII. The results show the advantage of the MYQZR model over the other five models for all the studied sets of nuclei.

Another way to compare the accuracy of each model will be estimating the difference between their predicted half-lives and the experimental one. This is done using

$$\Delta T = \ln\left(T_{\frac{1}{2},i}^{\text{calc.}}\right) - \ln\left(T_{\frac{1}{2},i}^{\text{expt.}}\right).$$

These differences are all tabulated in Table VIII in which the accuracy of MYQZR prevails over those of the other models, except the minimum deviations of the even-even set listed in column 2. This deviation has no significant effect as can almost be compensated for by the maximum values of the same set listed in column 3. In Figs. 1–4 we also plot the resulted ΔT versus neutron number for the studied sets of even-even, even-odd, odd-even, and odd-odd nuclei, respectively, in which the more adequate MYQZR formula is obvious.

The predictions of the α-decay half-life by the six adopted models have been extended to 160 superheavy nuclei (SHN) with $105 \leqslant Z \leqslant 120$. Consequently, the available experimental SHN α-decay half-lives were compared with the calculated results of each model as shown in Figs. 5(a)–8(d) which show

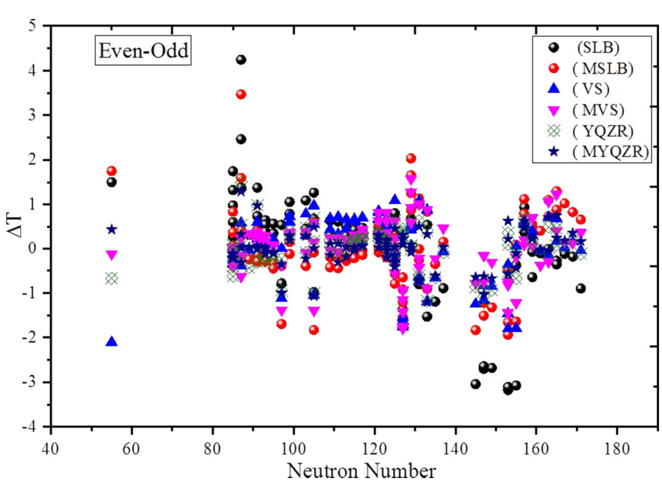

FIG. 2. The difference between theoretical and experimental α-decay half-lives for all formulas of set even-odd.

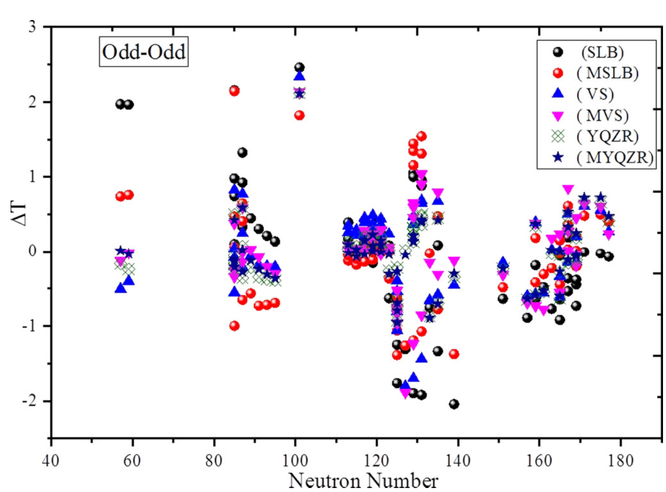

FIG. 4. The difference between theoretical and experimental α-decay half-lives for all formulas of set odd-odd.

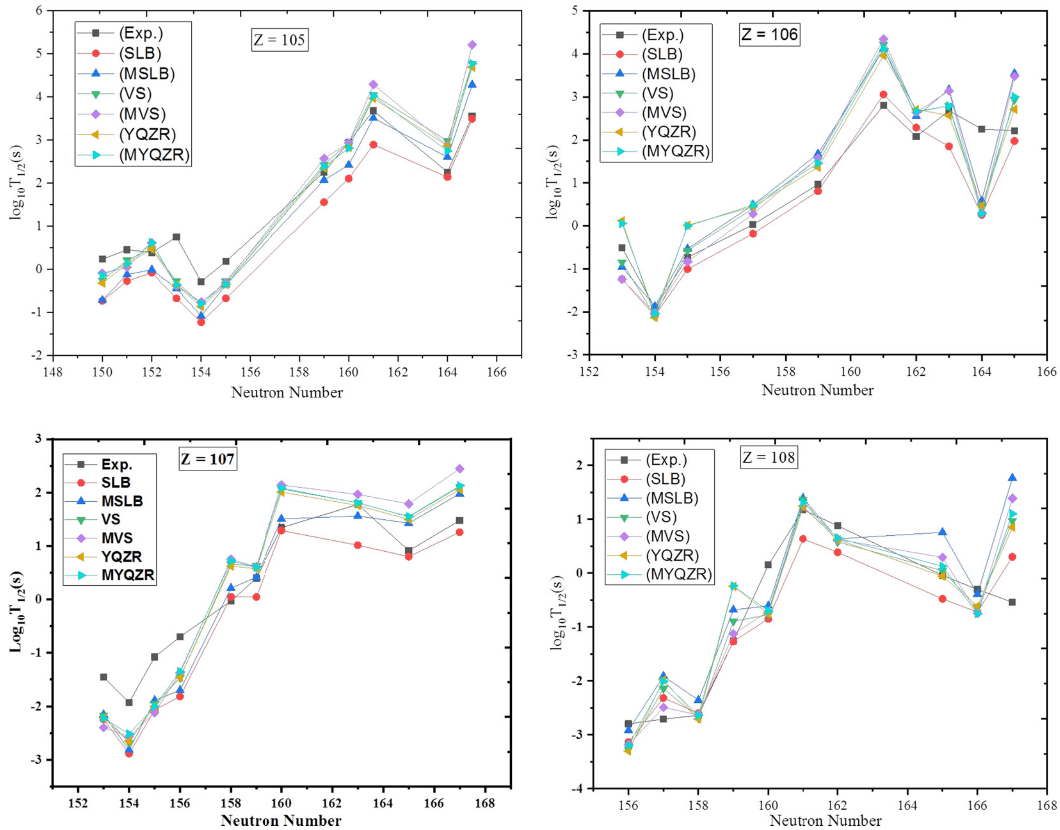

FIG. 5. The predicted logarithm of half-lives against neutron number by the six models compared with experimental results for $Z = 105$, 106, 107, and 108.

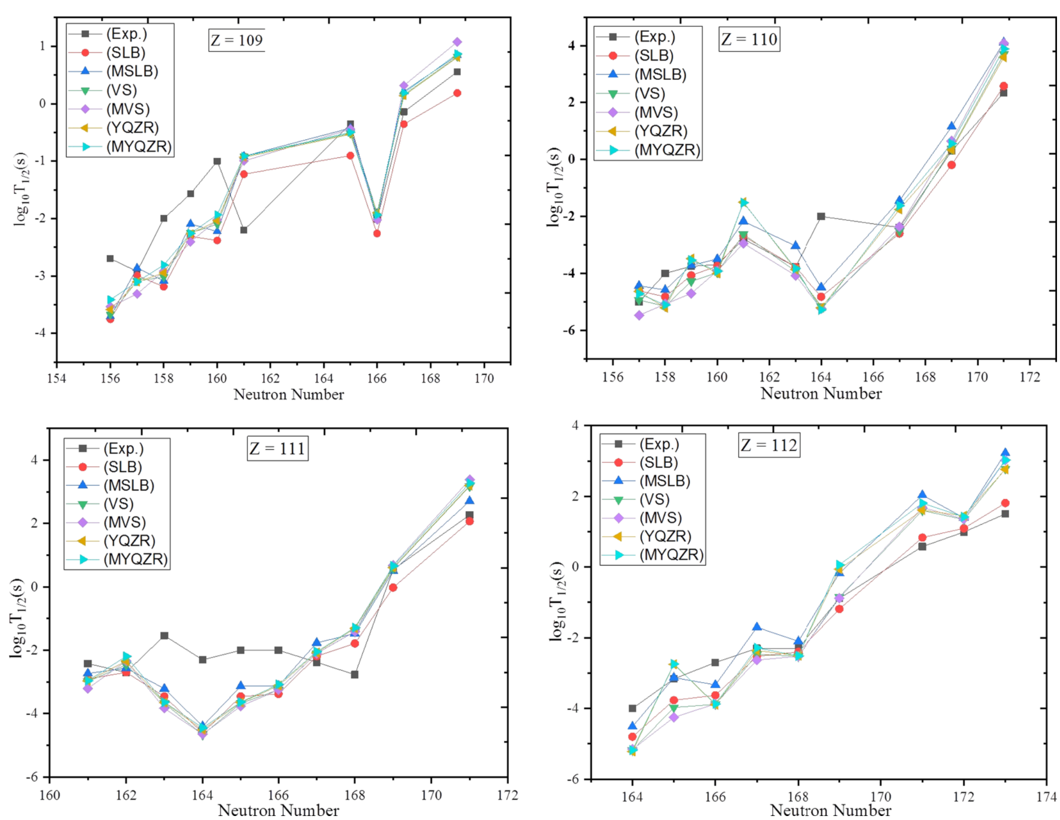

FIG. 6. The predicted logarithm of half-lives against neutron number by the six models compared with experimental results for $Z = 109$, 110, 111, and 112.

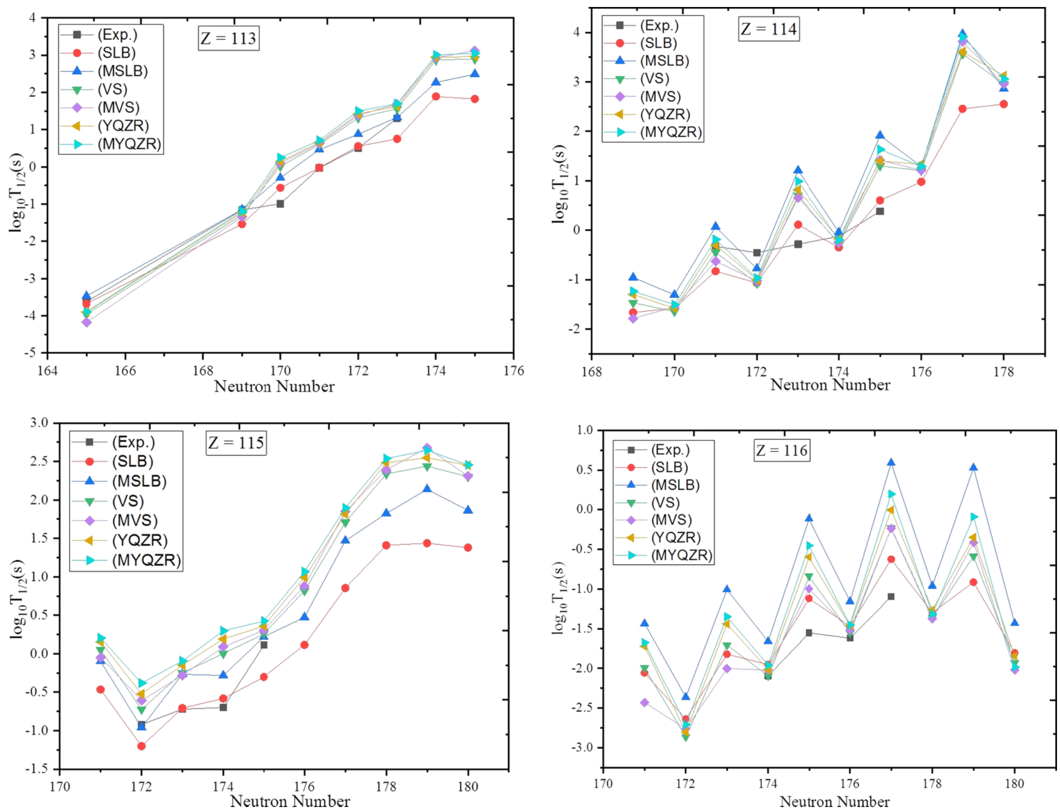

FIG. 7. The predicted logarithm of half-lives against neutron number by the six models compared with experimental results for $Z = 113$, 114, 115, and 116.

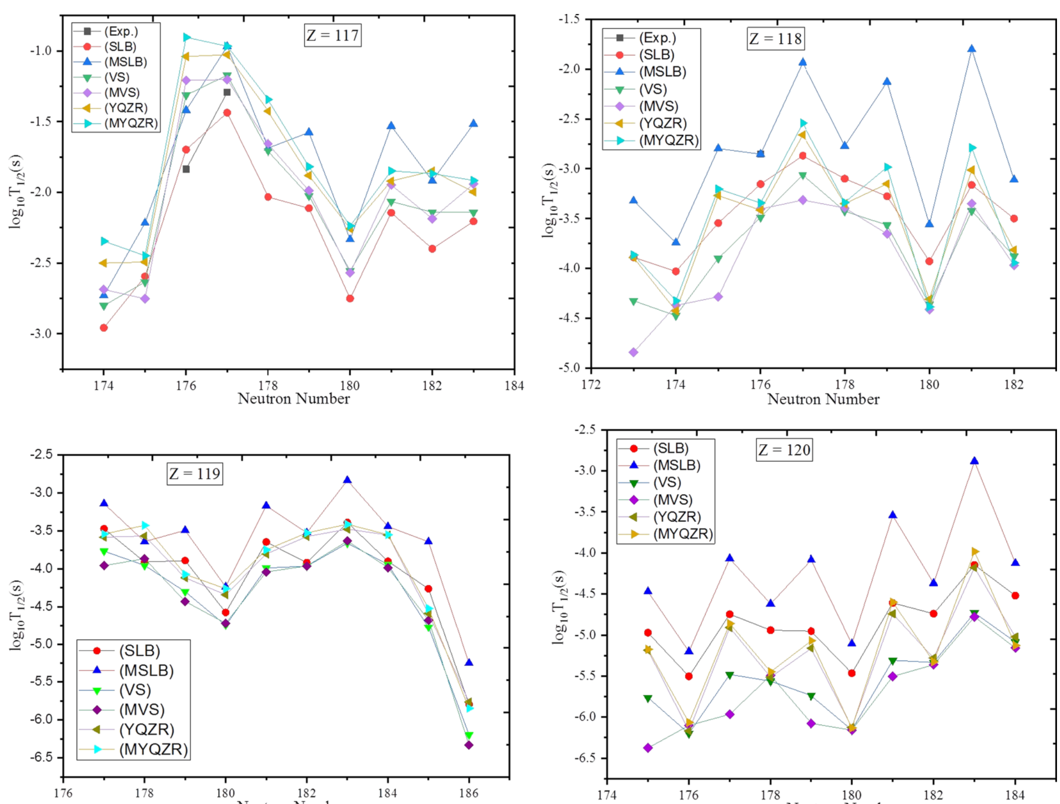

FIG. 8. The predicted logarithm of half-lives against neutron number by the six models compared with experimental results for $Z = 117$, 118, 119, and 120.

the plot of $\log T_{1/2}$ against neutron number N for the isotope sets of $Z = 105, 106, \ldots, 119$, and 120, respectively.

For several isotopes and the $Z = 119$ isotope set, there were no available experimental data to be compared with. Despite this shortage, generally, the MYQZR model reveals an acceptable prediction of the SHN α-decay half-lives in comparison to the other five tested models. So it can be depended on for the theoretical calculation of the alpha decay half-lives for those nonmeasured SHN isotopes.

IV. CONCLUSION

The modification of the three empirical formulas of SLB, YQZR, and VS were performed through adding the nuclear isospin terms and then new adjusting parameters have been obtained. The α-decay half-lives calculated from the modified YQZR formula (MYQZR) shows a better agreement with the experimental data. The extended application of the formula also predicts acceptable results for the studied superheavy nuclei which aids the fast calculations of their α-decay half-lives within the specified studied range.

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