# Coexistence and mixing in $^{182,184}$Hg

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I have used a simple two-state mixing model to analyze $0 \leftrightarrow 2$ transition matrix elements in $^{182,184}$Hg. Fits provide $0^+$ and $2^+$ mixing in both nuclei, and values of basis-state $E2$ matrix elements. I agree with previous work that the ground state contains most of the less-collective basis state, but the converse is true for the $2^+$. It turns out that the potential mixing matrix elements are not independent of $J$, as is commonly assumed. I also include an analysis with newly revised matrix elements.

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## I. INTRODUCTION

Even Hg nuclei, with $A \approx 180$–$188$, have been long described as having a nearly spherical ground state (g.s.), accompanied by a deformed rotational band built on a low-lying $0^+$ state [1–7]. Various workers have estimated the mixing between these coexisting structures [8–12]. Dracoulis [8] used excitation energies and a variable moment of inertia model to extract the unperturbed $0^+$ energies, and hence the $0^+$ mixing. He then assumed the mixing matrix element was independent of $J$ and estimated mixing for other $J$ values. His results for $0^+$ and $2^+$ mixing in $^{180,182,184}$Hg are listed in Table I. In $^{182}$Hg, Bindra *et al.* [9] used $2^+$ energies and a two-parameter band mixing calculation to obtain an interaction strength of 83 keV as responsible for mixing the $2^+$ states. Richards *et al.* [10] used a method similar to Dracoulis, again assuming $V$ independent of $J$, and obtained mixing parameters. Grahn *et al.* [11] estimated the mixing amplitude for the $2^+$ state in $^{182}$Hg by using an average of the $|Qt|$ values of the $6^+$, $8^+$, and $10^+$ states. Their result was about 20% in intensity. Gaffney *et al.* [12] used a similar model to Dracoulis to update these mixings. These results are listed in Table I. Presumably, differences in the two sets of estimates for $A = 180$ and 182 are caused by the fact that the later work included additional states. In most analyses, the $4^+$ mixing was small, 4–7%, with the majority being deformed.

One oddity is that even though the mixing changes considerably across the chain of isotopes [8,10,12]—especially for $2^+$ states—the $2_1^+$ energy and its $E2$ strength to the g.s. remain nearly constant [13]. Chakraborty *et al.* [14], in their work on shape coexistence in $^{94}$Zr, stressed the importance of having $E2$ strengths, rather than just energies, in treatments of coexistence and mixing. The relevant $E2$ matrix elements are now available in $^{182,184}$Hg [13], and the present purpose is to use them in a simple mixing model.

## II. ANALYSIS AND RESULTS

As elsewhere, I write

$$0_1 = a\,0_g + b\,0_e, \quad 0_2 = -b\,0_g + a0_e,$$
$$2_1 = A\,2_g + B\,2_e, \quad 2_2 = -B\,2_g + A\,2_e,$$

and I define $M_g = \langle 0_g|E2|2_g\rangle, \quad M_e = \langle 0_e|E2|2_e\rangle$.

Then, with the labeling in Table II, we have $M_0 = aAM_g + bBM_e$, and similarly for the other transitions. I assume the $E2$ operator does not connect $g$ to $e$.

With four known quantities (the transition matrix elements) and four unknowns—$0^+$ mixing, $2^+$ mixing, $M_g$ and $M_e$—the fit is generally unique. A set of parameters can be found to fit the central values of the $M$'s exactly, and the uncertainties in the fit parameters can be computed from the experimental uncertainties in the $M$'s. This fit to the $E2$ transition matrix elements does not make use of energies, or of any other observable.

Energies of the low-$J$ members of the first two bands in various Hg isotopes are plotted in Fig. 1. Available $0 \leftrightarrow 2$ $E2$ transition matrix elements are plotted vs $A$ in Fig. 2. Experimental $0 \leftrightarrow 2$ $E2$ matrix elements for $^{182}$Hg [13] are listed in Table II. A brief discussion of phases is needed at this point. There is no physical content in the absolute phase for any state. In the present circumstance, this means that no observable is changed if all the matrix elements involving a given state have their signs changed. In my phase convention, $M_0$ and $M_3$ must be positive, but $M_1$ and $M_2$ can have either sign because they involve destructive interference. However, if $M_e$ is significantly larger than $M_g$, all $M$'s will be positive. In the experimental results of Bree *et al.*, $M_3$ is negative. To be consistent with my phases, I must therefore change all the signs involving either $0_2$ or $2_2$. The results are called alternatives 1 and 2 in Table II.

The alt. 1 matrix elements lead to a solution that is listed in Table III. The $2^+$ mixing is seen to be nearly identical to that of Dracoulis. In the alt. 1 solution for $^{182}$Hg, the values of $M_g$ and $M_e$ of 1.17 and 3.28 eb, respectively are virtually identical to the results of 1.2 and 3.3 in Ref. [13]. These authors used the mixing of [12] and varied $M_g$ and $M_e$.

Experimental $E2$ matrix elements for $^{184}$Hg are listed in Table IV [13]. Results of the fit are also listed in Table III. The value of $M_e$ that results from this fit is close to that for $^{182}$Hg, but $M_g$ is drastically different. It does not seem reasonable that the $g$ basis state could have changed so much from 182 to 184. Therefore, I have also investigated a fit with all four matrix elements positive for $^{182}$Hg (alt. 3) and a fit for $^{184}$Hg with $M_2$ negative (as in $^{182}$Hg). These results are also listed in

 

TABLE I. Previous estimates of mixing in various Hg nuclei.

| | | $a^2$ | | | |
|---|---|---|---|---|---|
| $A$ | $J$ | Dracoulis | Gaffney *et al.* | Grahn *et al.* | Richards *et al.* |
| 180 | 0 | 0.927 | 0.960 | | |
| | 2 | 0.342 | 0.596 | | |
| 182 | 0 | 0.838 | 0.923 | | 0.929 |
| | 2 | 0.203 | 0.290 | 0.20 | 0.244 |
| 184 | 0 | 0.946 | 0.946 | | 0.947 |
| | 2 | 0.527 | 0.514 | | 0.491 |

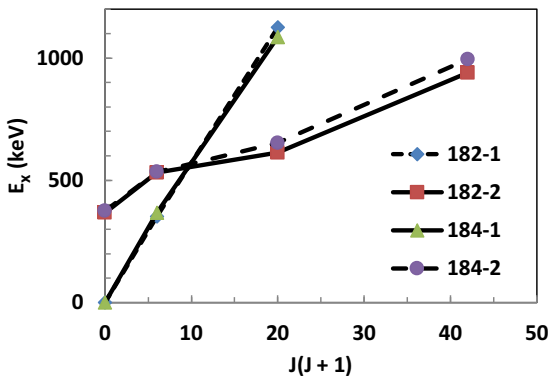


FIG. 1. Energies of the low-$J$ members of the two lowest bands in $^{182,184}$Hg.

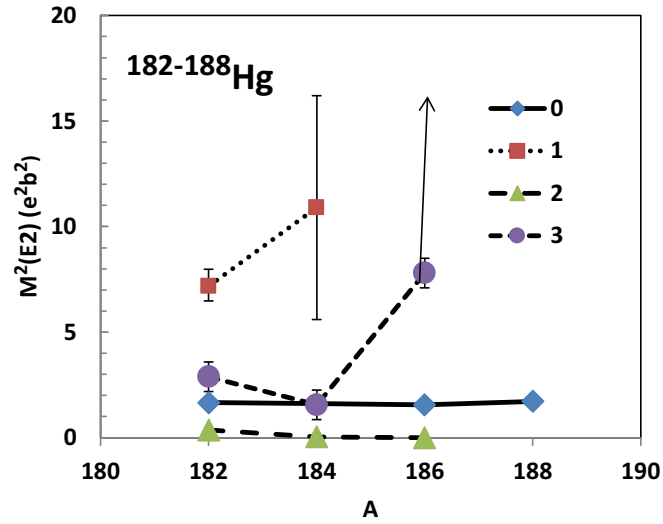


FIG. 2. Squares of $0 \leftrightarrow 2$ transition matrix elements [13] are plotted vs $A$. Labeling is as in Tables II and IV.

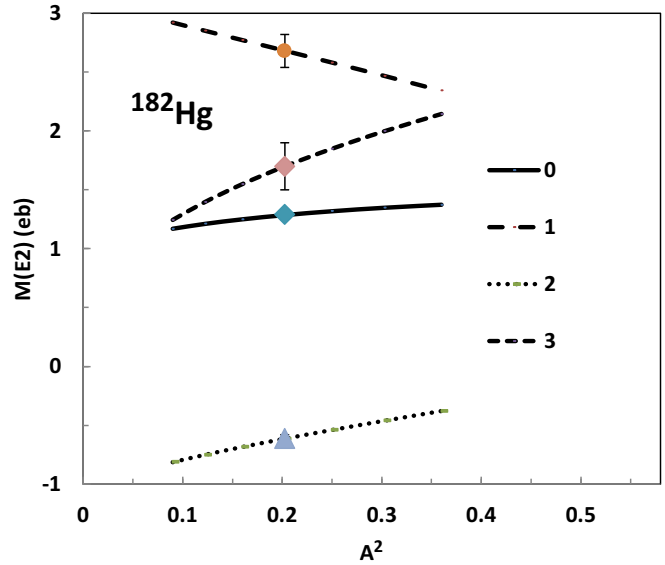


FIG. 3. Computed matrix elements in $^{182}$Hg are plotted vs $2^+$ mixing.

TABLE II. Relevant transition matrix elements in $^{182}$Hg.

| | | | $M(E2)$ (eb)[a] | | | |
|---|---|---|---|---|---|---|
| Label | Initial | Final | Bree *et al.* | alternate 1[b] | alternate 2[c] | alternate 3[d] |
| $M_0$ | $2_1$ | $0_1$ | $1.29^{+0.04}_{-0.03}$ | $1.29^{+0.04}_{-0.03}$ | $1.29^{+0.04}_{-0.03}$ | $1.29^{+0.04}_{-0.03}$ |
| $M_1$ | $2_1$ | $0_2$ | $-2.68^{+.15}_{-0.13}$ | $2.68^{+.13}_{-0.15}$ | $-2.68^{+.15}_{-0.13}$ | $2.68^{+.13}_{-0.15}$ |
| $M_2$ | $2_2$ | $0_1$ | $-0.61(3)$ | $-0.61(3)$ | 0.61(3) | 0.61(3) |
| $M_3$ | $2_2$ | $0_2$ | $-1.7(2)$ | 1.7(2) | 1.7(2) | 1.7(2) |

[a] $M^2\ (E2;\ i \to f) = (2J_i + 1)\ B(E2;\ i \to f)$ .
[b] Obtained by changing signs of all matrix elements involving $0_2$.
[c] Obtained by changing signs of all matrix elements involving $2_2$.
[d] Assuming all $M$'s are positive, as in $^{184}$Hg [13].

TABLE III. Results of fit for $0 \leftrightarrow 2$ transitions in $^{182,184}$Hg.

| | $^{182}$Hg | | $^{184}$Hg | |
|---|---|---|---|---|
| Quantity | alt. 1[a] | alt. 3[b] | original[c] | modified[d] |
| $a^2$ | 0.929 | 0.834 | 0.885(24) | 0.904 |
| $A^2$ | 0.202 | 0.268 | 0.111(9) | 0.095 |
| $M_g$ | 1.17 eb | 0.160 eb | 0.239(25) eb | 0.615 eb |
| $M_e$ | 3.28 eb | 3.48 eb | 3.75(10) eb | 3.71 eb |

[a]Only $M_2$ negative. Consistent with [13].
[b]All $M$'s positive.
[c]All $M$'s positive, as in Ref. [13].
[d]Only $M_2$ negative.

Table III. With the alt. 3 solution in $^{182}$Hg, $M_e$ is even closer to that of $^{184}$Hg, and $M_g$ is only slightly smaller. The authors of Ref. [13] were also unable to reproduce all the signs of the matrix elements. Of course, if the sign of the experimental $M_2$ matrix element is incorrect, its magnitude could also be suspect. It would be useful to look at this matrix element again. The sign is part of the matrix element. If the sign is wrong, then the matrix element is wrong.

With $M_g$ and $M_e$ fixed, computed $M$'s for $^{182}$Hg vary with $2^+$ mixing as depicted in Fig. 3. Note that the predicted values of $M_0$ and $M_2$ vary more slowly than do $M_1$ and $M_3$. Also, the latter two have the largest uncertainties.

Note from $M_g$ and $M_e$ that the $e$ basis-state band is significantly more collective than $g$. Also, the $0^+$ mixing is reasonably close to previous results, but the $2^+$ mixing differs appreciably from both (which were quite different from each other). These mixing parameters can now be combined with the physical energies to obtain the matrix element responsible for the mixing. These are listed in Table V. For $^{184}$Hg, the $0^+$ result is larger than that of Dracoulis, but $V_2$ is smaller. In $^{182}$Hg, $V_0$ is smaller and again $V_2$ is significantly smaller. My value for $V_2$ in $^{182}$Hg is close to that of Bindra *et al.*

With the $V$'s and the experimental energies, the energies of the basis states can be computed. These are plotted in Fig. 4. Note the closeness of the $2^+$ states in both nuclei.

## III. ANALYSIS WITH REVISED MATRIX ELEMENTS

After the present paper was submitted, a new paper [15] appeared, with many of the same co-authors as Bree *et al.*, with some of the $E2$ transition matrix elements significantly different. Reference [15] used new information, primarily involving gamma-ray branching ratios [16], to re-evaluate the results of Bree *et al.* The original and revised $0 \leftrightarrow 2$ matrix

TABLE IV. Relevant transition matrix elements in $^{184}$Hg.

| Label | Initial | Final | $M(E2)$ (eb)[a] |
|---|---|---|---|
| $M_0$ | $2_1$ | $0_1$ | 1.27(3) |
| $M_1$ | $2_1$ | $0_2$ | 3.3(8) |
| $M_2$ | $2_2$ | $0_1$ | 0.21(2) |
| $M_3$ | $2_2$ | $0_2$ | 1.25(28) |

[a]Reference [13].

TABLE V. Potential matrix elements.

| | | | $V$ (keV) | | |
|---|---|---|---|---|---|
| $A$ | $J$ | $\Delta E$ (keV) | present | Dracoulis | Bindra *et al.* |
| 182 | 0 | 335 | 86 or 125 | 144 | |
| | 2 | 179 | 72 or 79 | 144 | 83 |
| 184 | 0 | 375 | 119 or 110 | 84 | |
| | 2 | 167 | 52 or 49 | 84 | |

elements are compared in Table VI. Note that the revised matrix elements labeled $M_1$ are drastically different from the original ones. Changes in the others are less severe, and $M_0$'s are unchanged. The most significant aspects of the revised results are as follows:

(1) The value of $M_1$ in $^{182}$Hg is now very poorly determined, having changed from $-2.68$ eb, with a small uncertainty, to $[-2.2,\ 0.9]$ eb.
(2) The absolute value of $M_1$ in $^{184}$Hg has been drastically reduced, from 3.3(8) eb to 1.2 eb. The sign has also changed.
(3) The drastic reduction in $M_1$ in both nuclei has reduced the values of the invariant $\Sigma M^2$ considerably—from 12–14 $e^2b^2$ to something near 4 $e^2b^2$.

I have redone the fits described above for the revised matrix elements. Reference [15] explained clearly and thoroughly the situation regarding signs. Briefly, the absolute phase for any state has no meaning. Of course, the signs of the extracted matrix elements depend on the assumed sign convention. In order to be consistent with my sign convention, $M_2$ must be positive. This can be accomplished by changing the signs of matrix elements involving $0_2$ or $2_2$ in $^{182}$Hg. For the fit, I have chosen to change those for $2_2$, as noted in the footnotes to the table. The large range of the revised $M_1$ for $^{182}$Hg presents a problem. It turns out there is no solution for values near the upper limit, but a continuous range of solutions exists for most of the range. I have listed the solution for the midpoint of the range and for the lower limit (Table VII). As would be expected, the solutions using revised data are considerably different from the original ones.

For $^{184}$Hg, the major change is for the magnitude and sign of $M_1$, from 3.3 eb originally to $-1.2$ eb in the revision. The dominant effect of this modification is a drastic reduction

TABLE VI. Original [13] and revised [15] $0 \leftrightarrow 2$ matrix elements (eb) in $^{182,184}$Hg.

| | 182 | | 184 | |
|---|---|---|---|---|
| Label | original | Revised | original | revised |
| $M_0$ | $1.29^{+0.04}_{-0.03}$ | 1.29(4) | 1.27(3) | 1.27(3) |
| $M_1$ | $-2.68^{+0.15}_{-0.13}$ | $[-2.2, 0.9]$ | 3.3(8) | $-1.2^{+0.3}_{-0.2}$ |
| $M_2$ | $-0.61(3)$ | $-0.6(1)$[a] | 0.21(2) | 0.348(14) |
| $M_3$ | $-1.7(2)$ | $-1.25(30)$[a] | 1.25(28) | $0.93^{+0.20}_{-0.25}$ |

[a]For the fits, I changed the signs of all matrix elements involving $2_2$ (see text).

TABLE VII. Fit results using revised matrix elements [15].

| | 182 | | |
|---|---|---|---|
| Quantity | sol. 1[a] | sol. 2[b] | 184 |
| $a^2$ | 0.61 | 0.14 | 0.39(12) |
| $A^2$ | 0.95 | 0.88 | 0.92(28) |
| $M_g$(eb) | 1.45 | 2.69 | 1.80(6) |
| $M_e$(eb) | 1.38 | 1.09 | 0.89(13) |
| $\Sigma\, M^2(e^2b^2)$ | 4.01 | 8.42 | $4.03^{+0.81}_{-0.67}$ |

[a]Using $M_1 = -0.65$ eb.
[b]Using $M_1 = -2.2$ eb.

in the total $0 \leftrightarrow 2$ strength, as evidenced by the absolute values of $M_g$ and $M_e$. The sum of $M^2$ has been reduced from 14.1 $e^2b^2$ to 4.04 $e^2b^2$. This total strength in $^{184}$Hg is close to that for $^{182}$Hg at the midpoint of the range for $M_1$. At this point, the solutions for 182 and 184 are similar.

## IV. COMPARISON WITH OTHER MODELS

The authors of Ref. [15] have compared their revised results with those of a simplified mixing model and with three theoretical models, namely the interacting boson model with configuration mixing (IBM-CM), generalized Bohr Hamiltonian (GBH), and adiabatic time-dependent Hartree-Fock-Bogoliubov (ATDHFB). The IBM-CM approach contains seven parameters per isotope, plus two other parameters that refer to the whole isotopic chain. These were obtained through a least-squares fit to the available experimental information. The BMF and GBH calculations contain no parameters that are fitted to the properties of the Hg nuclei. It is not surprising then that the IBM-CM model calculations reproduce the experimental data better as compared to BMF and GBH. However, for the low-lying $2^+$ and $0^+$ states the comparison with theory was less successful. Reference [15] states that "the excitation energies of the different configurations are not correctly reproduced and their relative positions are reversed."

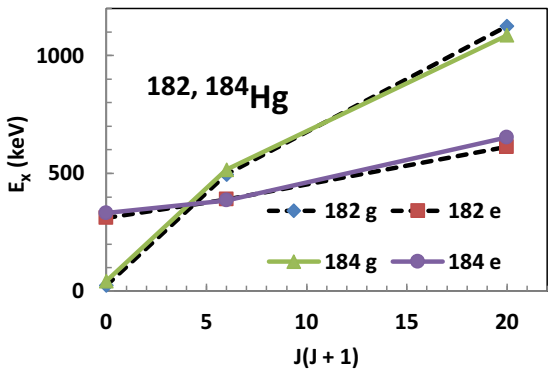


FIG. 4. Energies of basis states in $^{182,184}$Hg.

In their two-state mixing calculations, the observed physical states are written as linear combinations of two unmixed structures with specific mixing amplitudes. These, taken from Ref. [12], were derived from a fit of the known higher-lying level energies in the rotational bands, built upon the first two $0^+$ states, using a variable moment of inertia model. States with spin $J > 4$ were assumed to be weakly mixed and to manifest a rotational-like character. Stronger mixing was deduced for states with spin $J = 2$, reaching the maximum of mixing for $^{184}$Hg. As evidenced by their Fig. 16, the revised matrix elements are better described by their two-state mixing model than in the original analysis (Fig. 4 of [13]). However, those results differ from mine in one important respect—my analysis (Table VII) results in more mixing for $0^+$ states than for $2^+$ in $^{184}$Hg. Also, as noted earlier, except for extreme circumstances, my two-state mixing analysis fits the central values of the matrix elements exactly, and uncertainties in the fit parameters can be computed from the uncertainties of the matrix elements. Furthermore, in my approach, no assumptions are made about the structure of the basis states, except for the assumption that $g$ and $e$ are not connected by the $E2$ operator. Properties of the basis states arise from the analysis. In the present case, with the revised matrix elements, basis states $g$ are found to be somewhat more collective than $e$. With the new mixing amplitudes, the mixing potentials in $^{184}$Hg become $V_0 = 182(4)$ and $V_2 = 46(6)$, both in keV. Of course, the poor determination of $M_1$ in $^{182}$Hg does not allow a meaningful fit for that nucleus.

## V. SUMMARY

I have applied a simple two-state mixing model to $0 \leftrightarrow 2$ transitions in $^{182,184}$Hg. Results provide $0^+$ and $2^+$ mixing amplitudes in both nuclei, and values of basis-state $E2$ matrix elements. I agree with previous work that the g.s. contains most of the less-collective basis state, but the converse is true for the $2^+$. I find that the potential mixing matrix elements are not independent of $J$, as is commonly assumed. An ambiguity exists in the sign of the $2_2 \rightarrow 0_1$ matrix element, which is small in absolute magnitude. In the experimental paper [13], this $M$ has opposite signs in the two nuclei. Consequently, I performed fits for both signs in both nuclei. Perhaps surprisingly, $M_e$ and both $0^+$ and $2^+$ mixing differ very little with the two choices of sign. However, $M_g$ is quite different with the two signs. The fact that Ref. [13] was unable to reproduce the signs of all the matrix elements in $^{184}$Hg may be related to this sign problem. An analysis using newly revised matrix elements [15] finds much less collectivity in both nuclei, as evidenced by the sum of $M^2$. A fit to $^{182}$Hg is hampered by the large allowed range of $M_1$ in that nucleus.

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