

## Dynamically assisted nuclear fusion

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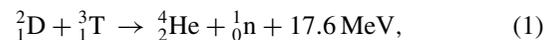
We consider deuterium-tritium fusion as a generic example for general fusion reactions. For initial kinetic energies in the keV regime, the reaction rate is exponentially suppressed due to the Coulomb barrier between the nuclei, which is overcome by tunneling. Here, we study whether the tunneling probability could be enhanced by an additional electromagnetic field, such as an x-ray free electron laser (XFEL). We find that the XFEL frequencies and field strengths required for this dynamical assistance mechanism should come within reach of present-day or near-future technology.

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**Introduction.** Tunneling is ubiquitous in physics. Examples include field ionization in atomic physics and  $\alpha$  decay in nuclear physics. The Gamov picture [1] explains the Geiger-Nuttall law [2] via tunneling of the  $\alpha$  particle through the Coulomb barrier of the remaining nucleus. In the opposite process, nuclear fusion, the two nuclei must also overcome their Coulomb barrier, typically by tunneling, before they can fuse. As an extreme example, the Sauter-Schwinger effect predicts the creation of electron-positron pairs out of the vacuum by a strong electric field  $E$ , which can be understood as tunneling from the Dirac sea [3–7]. The exponential dependence characteristic of tunneling leads to a strong suppression of the pair-creation probability  $P_{e^+e^-} \sim \exp\{-\pi E_S/E\}$  for electric fields  $E$  too far below the Schwinger critical field  $E_S$  determined by the mass  $m_e$  of the electron and the elementary charge  $q$  via  $E_S = m_e^2 c^3 / (q\hbar) \approx 1.3 \times 10^{18}$  V/m. Verifying this prediction has been one motivation for reaching these ultra-high field strengths  $E$ . As we shall see below, these theoretical and experimental efforts may also prove useful for assisting nuclear fusion.

Even though tunneling is usually taught in the first course on quantum mechanics, our understanding is still far from complete, especially in time-dependent scenarios, see [8–13]. Interesting phenomena in this context include the Franz-Keldysh effect [14,15] or the Büttiker-Landauer traversal time [16]. For the Sauter-Schwinger effect, it has been found that the pair-creation probability can be drastically enhanced by an additional weaker but time-dependent field [17–25], even if its frequency scale  $\omega$  is well below the mass gap of  $2m_e c^2 \approx 1$  MeV. As another surprise, this enhancement mechanism, i.e., the dynamically assisted Sauter-Schwinger effect, strongly depends on the concrete temporal (or spatiotemporal) dependence of the assisting field [26], such as a Sauter  $1/\cosh^2(\omega t)$  or Gaussian  $\exp\{-(\omega t)^2\}$  pulse or a sinusoidal profile  $\cos(\omega t)$ . In the following, we study whether and how tunneling in nuclear fusion could be dynamically assisted, for example, by the additional electromagnetic field of an x-ray free electron laser (XFEL) [27].

**Model.** We consider deuterium-tritium fusion



where the initial kinetic energies  $\mathcal{E}$  of the nuclei are in the keV regime and thus typical length scales (such as the tunneling distance) of order picometer. (As indicated above, D and T denote  ${}^2_1\text{H}$  and  ${}^3_1\text{H}$ , respectively.) Hence we may describe the two nuclei as nonrelativistic point particles with masses  $m_D$  and  $m_T$  and positions  $\mathbf{r}_D(t)$  and  $\mathbf{r}_T(t)$ . Their dynamics is governed by the Lagrangian

$$L = \frac{m_D}{2} \dot{\mathbf{r}}_D^2 + \frac{m_T}{2} \dot{\mathbf{r}}_T^2 - V(|\mathbf{r}_D - \mathbf{r}_T|) + q\dot{\mathbf{r}}_D \cdot \mathbf{A}(t, \mathbf{r}_D) + q\dot{\mathbf{r}}_T \cdot \mathbf{A}(t, \mathbf{r}_T), \quad (2)$$

where the potential  $V(|\mathbf{r}_D - \mathbf{r}_T|)$  contains the Coulomb repulsion at large distances and the nuclear attraction at short distances (of order Fermi). The vector potential  $\mathbf{A}$  represents the field of the XFEL.

For initial kinetic energies  $\mathcal{E}$  between 1 and 10 keV, the outer classical turning point  $r_{\mathcal{E}}$  where  $V(r_{\mathcal{E}}) = \mathcal{E}$  lies between 1.4 pm and 140 fm, which then determines the remaining tunneling distance. Since the XFEL wavelength ( $\geq 50$  pm) is much larger than that, we may approximate the vector potential  $\mathbf{A}(t, \mathbf{r})$  by a purely time-dependent field  $\mathbf{A}(t)$ . As a result, the center-of-mass motion decouples from the relative coordinate  $\mathbf{r}_- = \mathbf{r}_D - \mathbf{r}_T$ , whose dynamics is governed by

$$L_- = \frac{\mu}{2} \dot{\mathbf{r}}_-^2 - V(|\mathbf{r}_-|) + q_{\text{eff}} \dot{\mathbf{r}}_- \cdot \mathbf{A}(t), \quad (3)$$

with the reduced mass  $\mu = (m_D^{-1} + m_T^{-1})^{-1}$  and the effective charge  $q_{\text{eff}} = q(m_T - m_D)/(m_T + m_D) \approx q/5$ .

**Deformation of potential.** Let us first estimate the tunneling probability without the  $\mathbf{A}$  field via the WKB approximation. For low initial kinetic energies  $\mathcal{E}$ , the short-range details of the nuclear attraction are not important and the tunneling exponent is dominated by the long-range behavior of  $V$ , which

gives (for  $s$  waves)

$$P \sim \exp \left\{ -\pi \sqrt{\frac{2\mu c^2}{\mathcal{E}}} \alpha_{\text{QED}} \right\}, \quad (4)$$

where  $\alpha_{\text{QED}} \approx 1/137$  is the fine structure constant. Of course, this expression is analogous to the Geiger-Nuttall law for  $\alpha$  decay [2]. Inserting an energy  $\mathcal{E} = 1$  keV and the reduced mass  $\mu \approx 1.12$  GeV, the above tunneling exponent is  $P \sim 10^{-15}$  (for  $\mathcal{E} = 10$  keV, it is  $10^{-5}$ ).

At the classical turning point  $r_{\mathcal{E}}$  (minimum distance) of around 1.4 pm (for an energy  $\mathcal{E}$  of 1 keV), the Coulomb field strength is around  $7 \times 10^{14}$  V/m. As a result, near-future ultra-strong optical lasers or XFEL approaching this field strength regime could deform the potential barrier and thereby enhance (or suppress) tunneling significantly. For example, for a constant electric field of  $35 \times 10^{14}$  V/m, the factor of  $\pi$  in the exponent (4) is replaced by  $8/3 \approx 2.7$ . Note that due to the exponential scaling of the tunneling probability  $P$ , even moderate deformations can have a strong effect, e.g.,  $\pi \rightarrow 8/3$  in the exponent (4) implies  $P \sim 10^{-15} \rightarrow 10^{-13}$ .

*Floquet approach.* However, while the frequency of an optical laser is so low that this deformation can be treated within the quasi-static approximation, the temporal variations of an XFEL are too fast and hence should be taken into account. In fact, as we shall see below, this time dependence can strongly enhance the tunneling probability.

In order to study this enhancement, let us first assume an oscillating time dependence  $\mathbf{A}(t) = A_z \mathbf{e}_z \cos(\omega t)$  and use a Floquet ansatz (see, e.g., [28,29])

$$\psi(t, \mathbf{r}) = \sum_{n=-\infty}^{+\infty} \psi_n(\mathbf{r}) \exp \{-i\mathcal{E}t/\hbar - in\omega t\}, \quad (5)$$

where  $\mathbf{r} = \mathbf{r}_-$  denotes the relative coordinate from now on. Assuming that the external vector potential  $\mathbf{A}(t)$  is a small perturbation, we employ perturbation theory and split the total Hamiltonian  $\hat{H}(t)$  into the stationary unperturbed part  $\hat{H}_0$  plus the time-dependent perturbation  $\hat{H}_1(t) = \hat{H}_A \cos(\omega t)$ . The zeroth order  $\hat{H}_0 \psi_0(\mathbf{r}) = \mathcal{E} \psi_0(\mathbf{r})$  represents the solution in the absence of the XFEL and we choose it to be a  $p$  wave  $\psi_0(\mathbf{r}) = \psi_0^p(r) \cos \vartheta$ . Of course, for  $p$  waves we have to take the angular momentum barrier into account. However, comparing the angular momentum barrier for  $\ell = 1$  with the Coulomb potential, we see that the latter dominates for radii larger than the reduced Compton wavelength  $\lambda_C$  divided by  $\alpha_{\text{QED}}$ , in our case, 24 fermi. Consequently, the angular momentum barrier becomes only relevant at very short distances  $r \ll r_{\mathcal{E}}$ .

Following this strategy, the first Floquet side bands  $\psi_{\pm 1}(\mathbf{r})$  are (to first order in  $\mathbf{A}$ ) determined by

$$(\mathcal{E} - \hat{H}_0 \pm \hbar\omega) \psi_{\pm 1}(\mathbf{r}) = \hat{H}_A \psi_0(\mathbf{r}), \quad (6)$$

together with the appropriate boundary conditions. As expected from the selection rules, the first-order wave functions  $\psi_{\pm 1}(\mathbf{r})$  contain  $s$ - and  $d$ -wave contributions, where we focus on the most important part  $\psi_{\pm 1}^s(\mathbf{r}) = \psi_{\pm 1}^s(r)$  in the following.

Then Eq. (6) turns into an ordinary second-order differential equation for  $\psi_{\pm 1}^s(r)$  which can be solved numerically.

However, we may also obtain an analytical estimate: The zeroth-order  $\psi_0(\mathbf{r})$  represents a wave which is incident with energy  $\mathcal{E}$  from the outside, i.e., it is oscillating for radii  $r$  larger than the turning point  $r_{\mathcal{E}}$  and has an exponential (tunneling) tail for smaller radii  $r < r_{\mathcal{E}}$ . As a result, the source term  $\hat{H}_A \psi_0(\mathbf{r})$  in Eq. (6) is negligibly small near the origin  $r \ll r_{\mathcal{E}}$  and assumes its maximum near the turning point  $r_{\mathcal{E}}$ .

Now, let us first construct a particular solution of the inhomogeneous differential equation (6) which is also zero near the origin. Then, integrating equation (6) toward larger radii, we see that this particular solution remains negligible until we approach the turning point  $r_{\mathcal{E}}$  where the source term  $\hat{H}_A \psi_0(\mathbf{r})$  starts to play a role. For large radii, this particular solution then contains the forced oscillation with  $\exp\{\pm ik_{\mathcal{E}} r\}$  corresponding to the initial kinetic energy  $\mathcal{E} = \hbar^2 k_{\mathcal{E}}^2 / (2\mu)$  plus the two locally homogeneous solutions with  $\exp\{\pm ik_{\mathcal{E} + \hbar\omega} r\}$  corresponding to the higher energy  $\mathcal{E} + \hbar\omega = \hbar^2 k_{\mathcal{E} + \hbar\omega}^2 / (2\mu)$ . However, this particular solution does not satisfy the correct boundary conditions for large radii, because we do not have an incident wave with this higher energy  $\mathcal{E} + \hbar\omega$ . Thus, in order to correct this, we have to add a homogeneous solution of Eq. (6) which precisely cancels this incident wave. This homogeneous solution corresponds to a wave which is incident with energy  $\mathcal{E} + \hbar\omega$ , mostly reflected back to  $r \rightarrow \infty$ , but also contains a small tunneling amplitude at the origin, for which we can use the same WKB estimate as in (4), but now with  $\mathcal{E}$  being replaced by  $\mathcal{E} + \hbar\omega$ .

As a result, we find that the solution  $\psi_{\pm 1}^s(r)$  of Eq. (6) satisfying the correct boundary conditions must also contain a small amplitude at the origin, which gives us the dynamically assisted tunneling probability

$$P \sim \exp \left\{ -\pi \sqrt{\frac{2\mu c^2}{\mathcal{E} + \hbar\omega}} \alpha_{\text{QED}} \right\}. \quad (7)$$

With an initial energy  $\mathcal{E}$  of 1 keV and an XFEL frequency  $\hbar\omega$  of 10 keV, for example, the above tunneling exponent is enhanced by ten orders of magnitude. Of course, while we are mostly interested in the exponent (as the leading-order contribution), one must also take the prefactor in front of the exponent into account. Apart from geometrical factors (e.g., stemming from the angular dependences in three dimensions), this prefactor scales with  $q_{\text{eff}}^2 A_z^2$ , i.e., with the XFEL intensity. Thus the probability is proportional to the number of incident XFEL photons, which indicates that this enhancement mechanism should also work with incoherent photons.

By numerically integrating Eq. (6), we may arrive at quantitative results for the prefactor, where we find that it actually grows for decreasing  $\omega$ , see also [30]. However, if  $\omega$  becomes too small, the above Floquet approach breaks down and it becomes necessary to consider higher bands  $|n| \geq 2$ . From the lowest-order ( $n = 1$ ) result in Eq. (7) with  $\mathcal{E} = \omega = 1$  keV, for example, we conclude that the dynamical assistance requires electric field strengths of  $10^{15}$  V/m, which is similar to those required for the deformation of the potential discussed above. However, at those field strengths, the perturbative treatment above becomes questionable (see the next paragraph).

The replacement  $\mathcal{E} \rightarrow \mathcal{E} + \hbar\omega$  in (7) is typical for the Franz-Keldysh effect to lowest order, which describes dynamically assisted tunneling in the perturbative regime<sup>1</sup>. As explained above, it is a consequence of the dressed Floquet state [Eq. (5)] containing the side bands [Eq. (6)]. For higher side bands  $n = 2, 3, \dots$ , one would expect terms with  $\mathcal{E} \rightarrow \mathcal{E} + 2\hbar\omega$  and so on, where the exponential enhancement is even stronger while the prefactor is also more suppressed (e.g., with  $q_{\text{eff}}^4 A^4$ ) for low intensities. As in the dynamically assisted Sauter-Schwinger effect, one would expect that higher orders can dominate in this case, see [31].

*Büttiker-Landauer approach.* To go beyond the lowest order Floquet approach above, we study the WKB exponent  $S(t, r)$  in a space-time dependent setting. Considering a central collision of the two nuclei along the  $z$  axis, we assume vanishing angular momentum, i.e.,  $\partial_\vartheta S = \partial_\varphi S = 0$ . However, we have checked that including an angular dependence such as  $S = S(t, r, \vartheta)$  does not affect the following results significantly, which is consistent with our previous observation that the angular momentum barrier is not crucial for the parameters considered here.

Employing the WKB ansatz  $\psi = \mathcal{A} \exp\{iS/\hbar\}$ , we obtain the usual eikonal (Hamilton-Jacobi) equation

$$\partial_t S(t, r) + \frac{[\partial_r S(t, r) - q_{\text{eff}} A_z(t)]^2}{2\mu} + V(r) = 0, \quad (8)$$

with the static potential barrier  $V(r)$  while the time-dependent XFEL field is represented by  $A_z(t)$ , see [32]. As the next step (see also [16,33–35]), we split the eikonal function  $S(t, r) = S_0(t, r) + S_1(t, r)$  into the zeroth-order solution  $S_0(t, r)$  of the static tunneling problem  $\partial_t S_0 + (\partial_r S_0)^2/(2\mu) + V(r) = 0$  with  $\partial_t S_0 = -\mathcal{E}$ , plus the corrections  $S_1(t, r)$  induced by the XFEL field  $A(t)$ . Linearizing (8) in those quantities  $S_1$  and  $A$  yields the first-order equation

$$\left( \frac{\partial}{\partial t} + \frac{\partial_r S_0}{\mu} \frac{\partial}{\partial r} \right) S_1(t, r) = q_{\text{eff}} A_z(t) \frac{\partial_r S_0}{\mu}. \quad (9)$$

Employing the boundary condition  $S_1(t, r_\varepsilon) = 0$ , this equation has the solution

$$S_1(t, r) = q_{\text{eff}} \int_{r_\varepsilon}^r dr' A_z[t - \tau(r) + \tau(r')], \quad (10)$$

with the well-known WKB expression [16,36,37]

$$\tau(r) = \int_{r_\varepsilon}^r \frac{dr'}{\sqrt{2[\mathcal{E} - V(r')]/\mu}} \rightsquigarrow \frac{d\tau}{dr} = \frac{\mu}{\partial_r S_0}. \quad (11)$$

<sup>1</sup>Since Eq. (7) has the same form as Eq. (4), but just with an increased energy, one could be tempted to arrive at the simple picture that the nuclei just increase their initial kinetic energy by absorbing XFEL photons. However, this simple picture can be rather misleading: Due to momentum conservation, the gain in kinetic energy of a nucleus by absorbing a keV photon is negligible. Even if we consider the (classical) acceleration of a nucleus by an XFEL field consisting of many coherent photons with a frequency of  $\hbar\omega = 1$  keV and an ultra-high field strength of order  $10^{15}$  V/m, the ponderomotive energy of the quivering motion is negligible in comparison to the kinetic energy of 1 keV. (For higher frequencies, the ponderomotive energy would be even smaller.)

For classically allowed propagation  $\mathcal{E} > V$ , all the quantities  $S_0(r)$  and  $\tau(r)$  and thus also  $S_1(t, r)$  are real. For tunneling  $\mathcal{E} < V$ , however,  $S_0(r)$  and  $\tau(r)$  become imaginary and thus  $S_1(t, r)$  will be complex in general. Very analogous to the Sauter-Schwinger effect, the imaginary part of  $S_1(t, r)$  then determines the enhancement (or suppression) of the tunneling probability. Note that  $\tau$  is precisely the Büttiker-Landauer traversal time for tunneling, i.e., the imaginary turning time in the instanton picture.

According to Eq. (10), the tunneling exponent is determined by the analytic continuation of the vector potential  $A(t)$  to complex times (see also [38]), again in close analogy to the Sauter-Schwinger effect. As a result, we also find a qualitative difference [26] between a Sauter  $E(t) = \dot{A}(t) = E_0/\cosh^2(\omega t)$  and a Gaussian pulse  $E(t) = E_0 \exp\{-(\omega t)^2\}$  as well as a sinusoidal profile  $E(t) = E_0 \cos(\omega t)$  here. Let us first consider a sinusoidal profile which grows exponentially as  $\exp\{\omega|\tau|\}$  for large imaginary times  $\tau$ . In analogy to Eq. (4), we may estimate the maximum imaginary turning time (again neglecting the finite size of the nuclei) via

$$\frac{\mathcal{E}|\tau|}{\hbar} = \frac{\pi}{4} \sqrt{\frac{2\mu c^2}{\mathcal{E}}} \alpha_{\text{QED}}. \quad (12)$$

Apart from the factor  $1/4$ , we find the same expression as in the WKB tunneling exponent (4). For  $\mathcal{E} = 1$  keV, we get  $\mathcal{E}|\tau|/\hbar \approx 8.6$ . Thus, for frequencies  $\omega$  in the keV regime,  $\omega|\tau|$  is a large number, which allows us to approximate our result (10) further. Calculating  $S_1$  near the origin, the integral (10) receives its maximum contribution near the turning point  $r_\varepsilon$  (similar to the Floquet approach above). For an oscillating time-dependence  $A_z(t)$ , we may thus estimate this integral by

$$\frac{S_1}{\hbar} \approx \frac{i q_{\text{eff}} A_z e^{i\omega t} \mathcal{E}^2}{2\mu c \alpha_{\text{QED}} (\hbar\omega)^2} \exp \left\{ \hbar\omega \frac{\pi}{4} \sqrt{\frac{2\mu c^2}{\mathcal{E}^3}} \alpha_{\text{QED}} \right\}. \quad (13)$$

Apart from the WKB prefactor  $\mathcal{A}$ , the time average of the probability  $|\exp\{iS_0/\hbar + iS_1/\hbar\}|^2$  is given by the zeroth-order term  $\exp\{-2|S_0|\}$  multiplied by  $\mathfrak{J}_0(2|S_1|/\hbar)$ , where  $\mathfrak{J}_0$  is the modified Bessel function of the first kind. For small arguments, it behaves as  $1 + |S_1|^2/\hbar^2$  and for large arguments, it scales with  $\exp\{2|S_1|/\hbar\}/\sqrt{4\pi|S_1|/\hbar}$ . Note, however, that our linearized approach (10) breaks down when  $|S_1|$  becomes too large. The double exponential dependence of the probability on  $\omega$  is typical for the Büttiker-Landauer approach (in oscillating fields) and shows that the required field strength is actually weaker than expected from the lowest order Floquet approach above.

The dynamical assistance sets in when  $S_1/\hbar$  approaches order unity. For  $\mathcal{E} = \omega = 1$  keV, this requires field strengths of order  $10^{13}$  V/m. For  $\mathcal{E} = 9$  keV and  $\omega = 27$  keV, the required field strength from Eq. (13) is even lower, but in this regime, the accuracy of our approximations is a bit less reliable. Nevertheless, the main mechanism should still persist.

Turning the argument around, we find that the threshold frequency  $\omega_*$  where the enhancement mechanism sets in is determined by the inverse Büttiker-Landauer traversal time  $1/|\tau|$  multiplied by the logarithm  $\ln E_0$  of the field strength  $E_0$ . This is very reminiscent of the dynamically assisted

Sauter-Schwinger effect for an oscillatory time dependence [26]. Indeed, we find the same qualitative dependence on the pulse shape in both cases: For a Gaussian profile  $E(t) = E_0 \exp\{-(\omega t)^2\}$ , the threshold frequency  $\omega_*$  scales with  $\omega_* \sim \sqrt{\ln E_0/|\tau|}$ , while  $\omega_* \sim 1/|\tau|$  is nearly independent of the field strength  $E_0$  for a Sauter pulse  $E(t) = E_0/\cosh^2(\omega t)$ .

*Outlook: Assistance by electrons.* For an XFEL, time dependences such as a Gaussian or Sauter pulse may be hard to realize experimentally. However, the Coulomb field of a particle such as an electron passing through (or close by) the smallest gap of the two nuclei would more correspond to a pulse-like time dependence (cf. the idea in [39]). Note that the quasistatic deformation of the potential due to the Coulomb field of an electron would correspond to the well-known screening; but we are interested in the dynamical assistance of the tunneling process. Of course, the assumption of an external (i.e., classical) and spatially homogeneous field describes an XFEL field quite well, but it is not such a good approximation for the Coulomb field of an electron.

Nevertheless, one would expect that the dynamical assistance mechanism does also apply (qualitatively) to this case. To obtain a first rough estimate, let us employ time-dependent perturbation theory with respect to the Coulomb interaction between the electrons and the nuclei. The  $\hat{H}_0$  problem of the two nuclei could in principle again be diagonalized in terms of the center of mass and relative coordinates. However, let us simplify this problem even more by fixing the position of the tritium nucleus (formally corresponding to the limit  $m_T \rightarrow \infty$ ) and considering the motion of the deuterium nucleus in the external potential  $V(\mathbf{r}_D)$ . In second quantization, the Coulomb interaction Hamiltonian reads

$$\hat{H}_{\text{int}} = -q^2 \int d^3 r_D \int d^3 r_e \frac{\hat{\rho}_D(\mathbf{r}_D) \hat{\rho}_e(\mathbf{r}_e)}{4\pi \epsilon_0 |\mathbf{r}_D - \mathbf{r}_e|}, \quad (14)$$

where  $\hat{\rho}_D(\mathbf{r}_D) = \hat{\Psi}_D^\dagger(\mathbf{r}_D) \hat{\Psi}_D(\mathbf{r}_D)$  is the deuterium and  $\hat{\rho}_e(\mathbf{r}_e) = \hat{\Psi}_e^\dagger(\mathbf{r}_e) \hat{\Psi}_e(\mathbf{r}_e)$  the electron density operator. Let us consider the transition from an initial electron state with the energy  $\mathcal{E}_e^{\text{in}}$  to a final state with the energy  $\mathcal{E}_e^{\text{out}} = \mathcal{E}_e^{\text{in}} - \Delta\mathcal{E}$ . Then, the excess energy  $\Delta\mathcal{E}$  is transferred to the deuterium. Its initial state is incident with an initial energy  $\mathcal{E}$ . As before, the associated wave function decays exponentially for  $|\mathbf{r}_D| < r_\mathcal{E}$ . As the final state, we consider a wave function which is peaked near the origin (due to the nuclear attraction by the tritium) and decays exponentially for larger radii (inside the Coulomb barrier). However, due to the excess energy  $\Delta\mathcal{E}$ , this final state has an energy  $\mathcal{E} + \Delta\mathcal{E}$  and thus its exponential decay is slower and given by Eq. (7) with  $\hbar\omega$  being replaced by  $\Delta\mathcal{E}$ . Hence, the spatial overlap integral over  $r_D$  is again peaked near the turning point  $|\mathbf{r}_D| \approx r_\mathcal{E}$  and yields an exponential suppression

as in (7). The remaining  $r_e$  integral is not exponentially suppressed and is mainly determined by the probability that the electron is indeed close enough to assist dynamically. In this case, the field strength of the electron is also large enough.

*Conclusions.* Even though nuclear physics is customarily associated with very high field strengths and energies (in the MeV to GeV range), we find that nuclear fusion could be assisted at much lower scales, which should come within reach of present-day or near-future XFEL facilities (or with electrons), see [40]. Apart from the deformation of the potential barrier, the time dependence plays a crucial role for assisting tunneling through the Coulomb barrier, in close analogy to the dynamically assisted Sauter-Schwinger effect.

Within the lowest order Floquet approximation, we found that the tunneling exponent is enhanced according to (7). In order to go beyond the lowest-order Floquet approximation, we generalized the Büttiker-Landauer approach to this case and derived the first corrections  $S_1$  to the tunneling exponent in Eq. (10). Note that dynamically assisted tunneling has already been observed experimentally in several other scenarios, see, e.g., [41–43].

The proposed dynamical assistance mechanism should also work for other fusion reactions. An important example is deuterium-deuterium fusion. In this case, the above approximation  $\mathbf{A}(t, \mathbf{r}) \approx \mathbf{A}(t)$  is not adequate because  $q_{\text{eff}} = 0$  and we have to include the spatial dependence of the XFEL field. For an XFEL wavelength of 50 pm and distances of order 1 pm, this results in a suppression by a factor of around 1/50, which is partly compensated for by the fact that  $q_{\text{eff}} \approx q/5$  is now replaced by  $q$ . On the other hand, this suppression does not apply to the dynamical assistance by electrons sketched in Eq. (14).

In summary, our understanding of tunneling is still far from complete and offers surprises which motivate further studies. For example, the limitation of perturbative and linearized approaches necessitates the development of fully nonperturbative methods, perhaps in analogy to the world-line instanton technique in the Sauter-Schwinger effect, see, e.g., [44–49].

After understanding the main mechanism better, the next step would be to study whether it could be observed experimentally and which scenario (e.g., beam-beam or beam-target fusion, thermal or inertial fusion) might be most suitable. These findings could then determine the potential for possible future technological applications.

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