Pure spin-3/2 representation with consistent interactions

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We have investigated the use of a pure spin-3/2 propagator with consistent interaction Lagrangians to describe the property of spin-3/2 resonance. For this purpose, we use the antisymmetric tensor spinor representation. By using the primary and secondary constraints, we obtain the interaction fields that have the correct degrees of freedom. To visualize the result, we calculate the contribution of spin-3/2 Δ resonance to the total cross section of pion scattering and pion photoproduction off the nucleon. The result confirms that the scattering and photoproduction amplitudes obtained from the pure spin-3/2 representation with consistent interaction Lagrangians exhibit the required property of resonance. Therefore, the formalism can be used for phenomenological investigations in the realm of nuclear and particle physics.

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I. INTRODUCTION

In nuclear and particle physics, the formulation of spin-3/2 particle constitutes an arduous and long-standing problem. So far, such a particle is commonly represented by the Rarita-Schwinger (RS) field [1], which is described by the tensor product of vector $(\frac{1}{2},\frac{1}{2})$ and Dirac $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ fields. Mathematically, the result of this product is well known, i.e., $(1,\frac{1}{2})\oplus(\frac{1}{2},1)\oplus(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ [2], which shows that the RS field contains two fields: the $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ and the Dirac fields. The latter can be eliminated by using an orthogonality relation and as a result we obtain a spin-3/2 field that simultaneously contains a spin-1/2 background: the $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ field.

The RS field has also another fundamental problem called the Velo-Zwanziger problem [3]. This problem originates from the noncausal propagation of the wave front when the derivative terms of the RS field are gauged with the electromagnetic field. It was shown that the Velo-Zwanziger problem is related to the violation of constraints [4]. The interaction of spin-3/2 field with other fields should be constructed to have the same symmetry as the free field Lagrangian in order to preserve the correct degrees of freedom. For example, the earliest version of the $\pi N\Delta$ coupling that has an off-shell parameter [5] does not possess the local symmetry of the RS field [6]. Such a problem could be solved by introducing the gauge-invariant (GI) interaction to decouple the unphysical spin-1/2 background from the calculated transition amplitude [6].

Actually, the formalism of spin-3/2 particle can be presented by the pure spin-3/2 field $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$, which is clearly free from the spin-1/2 background. However, the problem with this field is that it uses an eight-dimensional spinor since the spin-3/2 operator is represented by 4×4 matrices. The free eight-dimensional field has been formulated by Weinberg [7], and it was still intricate enough to construct the corresponding interaction Lagrangian due to the noncovariant form of the eight-dimensional field, until Acosta *et al.* [8] could embed the pure spin-3/2 field into a totally antisymmetric tensor of second rank. Since the components of the tensor are spinor, such representation is called the antisymmetric tensor spinor (ATS). The ATS representation is formed by a tensor product of antisymmetric field and Dirac spinor

$$[(1,0)\oplus(0,1)]\otimes\left[\left(\tfrac{1}{2},0\right)\oplus\left(0,\tfrac{1}{2}\right)\right]$$
$$=\left[\left(\tfrac{3}{2},0\right)\oplus\left(0,\tfrac{3}{2}\right)\right]\oplus\left[\left(1,\tfrac{1}{2}\right)\oplus\left(\tfrac{1}{2},1\right)\right]$$
$$\oplus\left[\left(\tfrac{1}{2},0\right)\oplus\left(0,\tfrac{1}{2}\right)\right]. \quad (1)$$

In the ATS representation, the pure spin-3/2 field is projected out by the Lorentz projection operator.

In the previous paper, we have briefly reported the use of pure spin-3/2 propagator to describe the Δ resonance in the πN scattering [9]. It was shown that the conventional GI interaction Lagrangian cannot describe the resonance behavior of the spin-3/2 Δ baryon, unless the interaction was modified by adding an extra momentum dependence. Obviously, there was a lack of theoretical basis to support this solution. Furthermore, the theoretical consistency of such an *ad hoc* interaction Lagrangian could be questioned. In this paper, we present the complete results of our investigation on the pure spin-3/2 formalism. We first discuss the ATS formalism and its problem in describing the properties of a resonance. In Ref. [8], this problem was not observed since the proposed phenomenological application is Compton scattering, in which the spin-3/2 particle is on shell and does not resonate. For the sake of simplicity, we choose the π-N scattering to visualize the present problem. Then, we search for the consistency requirement in the interaction Lagrangians and present an example of consistent interaction Lagrangians for hadronic and electromagnetic interactions.

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By using these Lagrangians and the pure spin-3/2 propagator, we show that this formalism can be used for the purpose of phenomenological applications.

We organize this paper as follows. In Sec. II, we present the formalism of ATS and the corresponding problem to describe the resonance properties. In Sec. III, we explain the construction of consistent interaction Lagrangians. Section IV exhibits the numerical result and visualization of the resonance behavior of the pure spin-3/2 representation in the pion scattering and pion photoproduction processes. Finally, in Sec. V, we summarize our investigation and conclude our findings.

II. ATS AND ITS PROBLEM

In what follows, we briefly summarize the ATS formalism and show that this formalism has trouble describing the properties of a resonance. We have discussed this topic in our previous Rapid Communication [9]. Let us start with the Casimir operator $F = \frac{1}{4}J_{\mu\nu}J^{\mu\nu}$ with $J^{\mu\nu}$ being the angular momentum operator. For the field $|(a,b)\rangle$, this Casimir operator has the eigenvalue equation

$$F|(a,b)\rangle = C(a,b)|(a,b)\rangle, \tag{2}$$

with the eigenvalue $C(a,b) = a(a+1) + b(b+1)$. By using this Casimir operator, we can construct the projection operator that can remove the $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ and $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ fields from the ATS. The projection operator reads

$$\mathcal{P} = \frac{\left[F - C\left(1,\frac{1}{2}\right)\right]\left[F - C\left(\frac{1}{2},0\right)\right]}{\left[C\left(\frac{3}{2},0\right) - C\left(1,\frac{1}{2}\right)\right]\left[C\left(\frac{3}{2},0\right) - C\left(\frac{1}{2},0\right)\right]}. \tag{3}$$

Acosta *et al.* [8] have shown that this projection operator can be written as

$$\mathcal{P}_{\alpha\beta\gamma\delta} = \tfrac{1}{8}(\sigma_{\alpha\beta}\sigma_{\gamma\delta} + \sigma_{\gamma\delta}\sigma_{\alpha\beta}) - \tfrac{1}{12}\sigma_{\alpha\beta}\sigma_{\gamma\delta}\,, \tag{4}$$

with

$$\sigma_{\alpha\beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta]\,. \tag{5}$$

This projection operator assures that the ATS formalism has only the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ representation. In the ATS representation, the pure spin-3/2 spinor is obtained by operating a pure spin-3/2 projection operator to the GI RS spinor [8], i.e.,

$$w^{\mu\nu}(\mathbf{p},\lambda) = 2\mathcal{P}^{\mu\nu}{}_{\alpha\beta}U^{\alpha\beta}(\mathbf{p},\lambda)\,, \tag{6}$$

where $\lambda = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$ are the z components of the spin-3/2 operator eigenvalues and $U^{\alpha\beta}(\mathbf{p},\lambda)$ is the GI RS spinor, given by

$$U^{\alpha\beta}(\mathbf{p},\lambda) = \frac{1}{2m}[p^\alpha\mathcal{U}^\beta(\mathbf{p},\lambda) - p^\beta\mathcal{U}^\alpha(\mathbf{p},\lambda)], \tag{7}$$

with $\mathcal{U}^\alpha(\mathbf{p},\lambda)$ being the RS vector spinor. Clearly, except for the normalization constant $(2m)^{-1}$, the GI RS spinor $U^{\alpha\beta}(\mathbf{p},\lambda)$ given in Eq. (7) is identical to the GI RS field tensor $\Delta^{\mu\nu} = \partial^\mu\Delta^\nu - \partial^\nu\Delta^\mu$ given in Ref. [6]. Therefore, the difference between the ATS and the GI RS representations is in their projection operators. The ATS projection operator is completely different from the common projection operator in RS field. The former projects out the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ field, whereas the latter projects out the $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ field.

In the pure spin-3/2 representation, the corresponding propagator can be written as [8]

$$S_{\alpha\beta\gamma\delta}(p) = \frac{\Delta_{\alpha\beta\gamma\delta}(p)}{p^2 - m^2 + i\epsilon}\,, \tag{8}$$

where

$$\Delta_{\alpha\beta\gamma\delta}(p) = \left(\frac{p^2}{m^2}\right)\mathcal{P}_{\alpha\beta\gamma\delta} - \left(\frac{p^2 - m^2}{m^2}\right)1_{\alpha\beta\gamma\delta}\,, \tag{9}$$

and $1_{\alpha\beta\gamma\delta}$ is the identity in the ATS space, i.e.,

$$1_{\alpha\beta\gamma\delta} = \tfrac{1}{2}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})1_{4\times4}\,. \tag{10}$$

By using the orthogonality relation for the projection operator $\gamma^\mu\mathcal{P}_{\mu\nu\rho\sigma} = 0$, one may easily prove that the pure spin-3/2 spinor satisfies $\gamma_\mu w^{\mu\nu}(\mathbf{p},\lambda) = 0$. This relation can be used to reduce the number of degrees of freedom (DOF) in the ATS representation, i.e., $6\times4 = 24$, by $4\times4 = 16$. As expected, the pure spin-3/2 field in the ATS representation has 24–16 = 8 DOF.

Finally, for the purpose of the phenomenological application, such as meson-nucleon scattering, it is important to note that the free Lagrangian for the pure spin-3/2 field in the ATS representation can be written as [8]

$$\mathcal{L} = (\partial^\mu\Psi^{\alpha\beta})\Gamma_{\mu\nu\alpha\beta\gamma\delta}(\partial^\nu\Psi^{\gamma\delta}) - m^2\Psi^{\mu\nu}\Psi_{\mu\nu}\,, \tag{11}$$

where

$$\Gamma_{\mu\nu\alpha\beta\gamma\delta} = 4g^{\sigma\rho}\mathcal{P}_{\alpha\beta\rho\mu}\mathcal{P}_{\sigma\nu\gamma\delta} \tag{12}$$

and $\Psi^{\mu\nu}$ is the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ field. The kinetic term of the Lagrangian is invariant under the following gauge transformation,

$$\Psi^{\mu\nu} \to \Psi^{\mu\nu} + \xi^{\mu\nu}\,, \tag{13}$$

where the antisymmetric tensor $\xi^{\mu\nu}$ is given by

$$\xi^{\mu\nu} = \gamma^\mu\partial^\nu\xi - \gamma^\nu\partial^\mu\xi\,. \tag{14}$$

As stated in the introduction, we have found that the ATS formalism has a problem in describing the properties of a resonance. To explain this problem, let us consider the elastic πN scattering with a Δ resonance in the intermediate state. The corresponding Feynman diagram is displayed in Fig. 1, in which the momenta of all involved particles are shown for our convenience. In the literature, we note that the popular

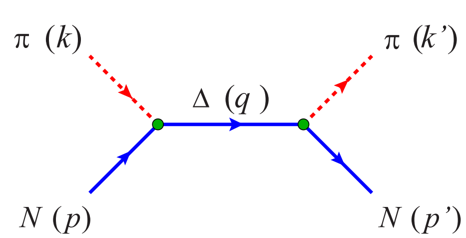

FIG. 1. Feynman diagram for the elastic πN scattering with a Δ resonance in the intermediate state.

choice of Lagrangian for the $\pi N\Delta$ interaction reads [6]

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\pi}\right)\bar{\Delta}^\mu \Theta_{\mu\nu}(z) N \partial^\nu \pi + \text{H.c.}, \quad (15)$$

where Δ^μ, N, and π denote the Δ-baryon vector spinor, nucleon spinor, and pion field, respectively. The tensor $\Theta_{\mu\nu}(z)$ is given by

$$\Theta_{\mu\nu}(z) = g_{\mu\nu} - \left(z + \tfrac{1}{2}\right)\gamma_\mu\gamma_\nu . \quad (16)$$

Note that the constant z in Eq. (16) is arbitrary and conventionally called the off-shell parameter. As stated before, this Lagrangian does not posses any local symmetries of the RS field, and as a consequence it induces the unphysical lower-spin DOF, which is called spin-1/2 background [6]. To decouple this unphysical background from the Δ-exchange amplitude, Pascalutsa and Timmermans introduce a GI interaction, which is given by [6]

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\pi m_\Delta}\right)\bar{N}\gamma_5\gamma_\mu \tilde{\Delta}^{\mu\nu}\partial_\nu \pi + \text{H.c.}, \quad (17)$$

where $\tilde{\Delta}^{\mu\nu}$ is the dual tensor of GI RS field tensor $\Delta^{\mu\nu}$. The latter is given by

$$\Delta_{\mu\nu} = \partial_\mu \Delta_\nu - \partial_\nu \Delta_\mu . \quad (18)$$

This GI interaction yields the Δ-exchange amplitude

$$\Gamma^\mu(k')S_{\mu\nu}(q)\Gamma^\nu(k) = \frac{(g_{\pi N\Delta}/m_\pi)^2}{\not{q} - m_\Delta}\frac{q^2}{m_\Delta^2}P^{(3/2)}_{\mu\nu}(q)k'^\mu k^\nu , \quad (19)$$

with $P^{(3/2)}_{\mu\nu}$ the spin-3/2 projection operator in the RS field, i.e.,

$$P^{(3/2)}_{\mu\nu}(q) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3q^2}(\not{q}\gamma_\mu q_\nu + q_\mu\gamma_\nu\not{q}) . \quad (20)$$

In analogy to the GI interaction described above, we can also construct the $\pi N\Delta$ interaction in the ATS formalism by changing the GI RS field tensor to the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ representation,

$$\mathcal{L}_{\pi N\Delta} = g_{\pi N\Delta}\bar{N}\gamma_5\gamma_\mu\tilde{\Psi}^{\mu\nu}\partial_\nu\pi + \text{H.c.}, \quad (21)$$

where $\Psi^{\mu\nu}$ is the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ field tensor and $\tilde{\Psi}^{\mu\nu}$ is its dual tensor. In terms of vertex factor used in many phenomenological applications [10,11], the interaction Lagrangian given in Eq. (21) can be translated as

$$\Gamma_{\mu\nu}(k) = g_{\pi N\Delta}\gamma_5\gamma_\mu k_\nu . \quad (22)$$

Thus, the Δ-exchange amplitude in the ATS formalism can be written as $\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k)$, where $\tilde{S}^{\mu\nu\rho\sigma}$ is defined by

$$\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \tfrac{1}{4}g^2_{\pi N\Delta}\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\kappa\lambda}\gamma_5\gamma_\mu S_{\alpha\beta\kappa\lambda}(q)\gamma_\rho\gamma_5 k'_\nu k_\sigma . \quad (23)$$

By using Eqs. (8) and (22), we can directly calculate Eq. (23), and it is easy to show that the nonvanishing amplitude is only the term obtained from the contraction with the identity $1_{\alpha\beta\kappa\lambda}$, since on the right-hand side of Eq. (23) the contraction with $\mathcal{P}_{\alpha\beta\kappa\lambda}$ vanishes due to the orthogonality relation $\gamma^\alpha\mathcal{P}_{\alpha\beta\kappa\lambda} = 0$ and the fact that $\tilde{\sigma}^{\mu\nu} = -\gamma_5\sigma^{\mu\nu}$. By calculating this nonvanishing Δ-exchange amplitude, we obtain that

$$\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \frac{g^2_{\pi N\Delta}\left(q^2 - m_\Delta^2\right)}{m_\Delta^2(q^2 - m_\Delta^2 + i\epsilon)}\left(g^{\nu\sigma} + \frac{1}{2}\gamma^\nu\gamma^\sigma\right)k'_\nu k_\sigma . \quad (24)$$

Obviously, Eq. (24) does not show the behavior of a resonance, since at the resonance pole ($q^2 = m_\Delta^2$) the amplitude is equal to zero, instead of being maximum. Thus, we may conclude that the interaction Lagrangian given by Eq. (21) cannot be used for calculating the resonance contribution.

The source of problem is coming from the GI interaction Lagrangian given by Eq. (21), i.e., the contraction between γ matrix and the projection operator of pure spin-3/2 field $\mathcal{P}_{\alpha\beta\kappa\lambda}$ vanishes. It is also obvious that this problem can be easily solved by modifying the interaction Lagrangian, e.g., by replacing the γ matrix with a partial derivative,

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\bar{N}\gamma_5\partial^\mu\Psi_{\mu\nu}\partial^\nu\pi + \text{H.c.}, \quad (25)$$

with the corresponding vertex factor

$$\Gamma^{\mu\nu}(k) = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\gamma_5 q^\mu k^\nu . \quad (26)$$

By using this vertex factor, we can calculate the Δ-exchange amplitude to obtain

$$\begin{aligned}\Gamma^{\mu\nu}(k')S_{\mu\nu\rho\sigma}(q)\Gamma^{\rho\sigma}(k) &= \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)^2\gamma_5 q^\mu S_{\mu\nu\rho\sigma}(q)\gamma_5 q^\rho k'^\nu k^\sigma \\ &= \frac{g^2_{\pi N\Delta}k'^\nu k^\sigma}{q^2 - m_\Delta^2 + i\epsilon}\left[\frac{q^4}{4m_\Delta^4}P^{(3/2)}_{\nu\sigma}(q) - \left(\frac{q^2 - m_\Delta^2}{2m_\Delta^4}\right)(q^2 g_{\nu\sigma} - q_\nu q_\sigma)\right],\end{aligned} \quad (27)$$

which is different from the result of GI interaction given by Eq. (19) by the second term. This term is significant only at energies far beyond the resonance pole and, as in Eq. (24), does not show the property of a resonance. Nevertheless, the result given by Eq. (27) is very interesting, because at the resonance pole, i.e., $q^2 = m_\Delta^2$, the second term vanishes and the Δ-exchange amplitude is proportional to that obtained from the GI RS interaction, i.e., Eq. (19).

To conclude this section, we may safely say that although for certain types of interaction Lagrangians the ATS representation cannot exhibit the property of a resonance required for use in phenomenological studies of hadronic physics, we are still able to choose different interactions to overcome this issue. However, it is obvious that such a solution does not have a strong theoretical basis and it is also possible that the suitable interaction found in this way is not unique. Therefore, we need a systematic mechanism to determine the genuine interaction through a number of relevant constraints. This is the topic of our discussion in the following section.

III. CONSTRUCTION OF THE CONSISTENT INTERACTION LAGRANGIANS

The problem of constraint in the interacting RS field can be solved by constructing the interaction that has the same symmetry as the free field one [12]. To this end, it is essential to check the impact of the interaction Lagrangian on the constraint and to carry out the Dirac-Faddeev quantization [13]. This procedure can be generally applied to the higher spin baryons such as nucleon resonances. In the case of the interaction terms that are separable from the free field, i.e., $\mathcal{L} = \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int}$, such procedure can be easily carried out. However, in the case of the interaction terms that originate from the gauged free field, we again face the Velo-Zwanziger problem. Such a gauged spin-3/2 field represents the spin-3/2 lepton undergoing electroweak interaction with photon, $W^\pm$ and Z^0 bosons. We note that the search for excited state of leptons was performed by the ATLAS Collaboration [14]. It has been proposed that the excited state of leptons has a spin-3/2 state [15]. To this end, we can use the pure spin-3/2 field to find the gauged electroweak Lagrangian of spin-3/2 lepton, as pure spin-3/2 field is free from the Velo-Zwanziger problem [3].

We start again with the free Lagrangian for pure spin-3/2 in the ATS representation given by Eq. (11). The conjugate momenta of the fields are given by

$$\bar{\pi}_{\gamma\delta} = \frac{\partial\mathcal{L}}{\partial(\partial^0\psi^{\gamma\delta})} = (\partial^\mu\bar{\psi}^{\alpha\beta})\Gamma_{\mu0\alpha\beta\gamma\delta}, \tag{28}$$

$$\pi_{\gamma\delta} = \frac{\partial\mathcal{L}}{\partial(\partial^0\bar{\psi}^{\gamma\delta})} = \Gamma_{0\nu\gamma\delta\alpha\beta}(\partial^\nu\psi^{\alpha\beta}), \tag{29}$$

and they can be expressed as functions of the field "velocity" $\bar{v}^{\alpha\beta} = \partial^0\bar{\psi}^{\alpha\beta}$ and $v^{\alpha\beta} = \partial^0\psi^{\alpha\beta}$, i.e.,

$$\bar{\pi}_{\mu\nu} = \bar{v}^{\alpha\beta}\mathcal{P}_{\alpha\beta}{}^{\rho}{}_{0}(g_{\mu\rho}\gamma_\nu - g_{\nu\rho}\gamma_\mu)\gamma_0 + (\partial^i\bar{\psi}^{\alpha\beta})\Gamma_{i0\alpha\beta\mu\nu}, \tag{30}$$

$$\pi_{\mu\nu} = \gamma_0(\gamma_\mu g_{\rho\nu} - \gamma_\nu g_{\rho\mu})\mathcal{P}^{\rho}{}_{0\alpha\beta}v^{\alpha\beta} + \Gamma_{0j\mu\nu\alpha\beta}(\partial^j\psi^{\alpha\beta}). \tag{31}$$

Because of the noninvertible property of idempotent operator $P_{\alpha\beta\gamma\delta}$, the "velocity" cannot be expressed as a linear combination of conjugate momenta. The primary constraints arise from the condition that not all momenta are linearly independent, with the relations

$$\bar{\theta}^{(1)}_\rho = \bar{\pi}_{\rho\sigma}\gamma^\sigma, \quad \theta^{(1)}_\rho = \gamma^\sigma\pi_{\rho\sigma}\,. \tag{32}$$

Next, the Hamiltonian density of pure spin-3/2 field is given by

$$\mathcal{H}_{3/2} = \bar{\pi}_{\gamma\delta}v^{\gamma\delta} + \bar{v}^{\gamma\delta}\pi_{\gamma\delta} - \mathcal{L}\,, \tag{33}$$

and the total Hamiltonian reads

$$\begin{aligned}\mathcal{H}_T &= \bar{\lambda}^\rho\theta^{(1)}_\rho + \bar{\theta}^{(1)}_\rho\lambda^\rho + \mathcal{H}_{3/2}\\ &= \bar{\lambda}^\rho\theta^{(1)}_\rho + \bar{\theta}^{(1)}_\rho\lambda^\rho - (\partial^i\bar{\psi}^{\alpha\beta})\Gamma_{ij\alpha\beta\gamma\delta}(\partial^j\psi^{\gamma\delta})\\ &\quad - [\bar{\pi}_{\gamma\delta} - (\partial^i\bar{\psi}^{\alpha\beta})\Gamma_{i0\alpha\beta\gamma\delta}]\\ &\quad \times\left[\pi^{\gamma\delta} - \Gamma_{0j}{}^{\gamma\delta}{}_{\alpha\beta}(\partial^j\psi^{\alpha\beta})\right] + m^2\psi^{\mu\nu}\psi_{\mu\nu}\,.\end{aligned} \tag{34}$$

The conditions of $\{\bar{\theta}^{(1)}_\rho, \mathcal{H}_T\} = 0$ and $\{\theta^{(1)}_\rho, \mathcal{H}_T\} = 0$ create the secondary constraints

$$\bar{\theta}^{(2)}_\rho = \bar{\psi}_{\rho\sigma}\gamma^\sigma, \quad \theta^{(2)}_\rho = \gamma^\sigma\psi_{\rho\sigma}\,. \tag{35}$$

Thus, the pure spin-3/2 fields end up with the secondary constraints, as the conditions of $\{\bar{\theta}^{(2)}_\rho, \mathcal{H}_T\} = 0$ and $\{\theta^{(2)}_\rho, \mathcal{H}_T\} = 0$ will be satisfied if the coefficients λ^ρ and $\bar{\lambda}^\rho$ fulfill the relation $4\lambda^\rho = \gamma^\rho\gamma_\mu\lambda^\mu$ and $4\bar{\lambda}^\rho = \bar{\lambda}^\mu\gamma^\rho\gamma_\mu$.

In the case of massive pure spin-3/2 Lagrangian, the number of DOF can be counted as follows. Each of the fields $\psi^{\mu\nu}$ and its conjugate momentum $\pi^{\mu\nu}$ have $6\times 4 = 24$ components, so in total they have 48 components. Each of the primary and secondary constraints reduces the DOF by $4\times 4 = 16$ components. Hence, the number of independent components is $48 - 2\times 16 = 16$. This number describes the number of independent components of the $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ representation in phase space. Thus, each of the fields and its conjugate momentum have eight independent components. For the massless case, the primary constraints become the first-class constraints as the conditions of $\{\bar{\theta}^{(1)}_\rho, \mathcal{H}_T\} = 0$ and $\{\theta^{(1)}_\rho, \mathcal{H}_T\} = 0$ are identities. Therefore, the coefficients λ^ρ and $\bar{\lambda}^\rho$ cannot be determined.

The general form of phase-space integral is

$$\begin{aligned}Z = \int \mathcal{D}\psi^{\mu\nu}\mathcal{D}\bar{\psi}^{\mu\nu}\sqrt{\det\{\theta_\alpha,\theta_\beta\}}\prod_{n=1}^{2}\delta(\theta_n)\delta(\bar{\theta}_n)\\ \times \exp i\int d^4x[\bar{\pi}_{\mu\nu}v^{\mu\nu} + \bar{v}^{\mu\nu}\pi_{\mu\nu} - \mathcal{H}_{3/2}]\,.\end{aligned} \tag{36}$$

The Poisson bracket of all constraint combinations are independent of the field. All constraints are proportional to the field or conjugate momentum. There are only two possibilities of the Poisson bracket of constraints, either zero or proportional to the γ matrices. Therefore, the determinant factor is just a normalizing constant that can be ruled out from the integral.

Generally, the interaction Lagrangian with pure spin-3/2 field can be written as

$$\mathcal{L} = \mathcal{L}_{3/2} + \bar{J}_{\mu\nu}\Psi^{\mu\nu} + \bar{\Psi}^{\mu\nu}J_{\mu\nu}\,. \tag{37}$$

The interaction terms affect the secondary constraints, so the latter becomes

$$\begin{aligned}\bar{\theta}^{(2)}_\rho &= (m^2\bar{\psi}_{\rho\sigma} - \bar{J}_{\rho\sigma})\gamma^\sigma,\\ \theta^{(2)}_\rho &= \gamma^\sigma(m^2\psi_{\rho\sigma} - J_{\rho\sigma})\,.\end{aligned} \tag{38}$$

Because the interaction terms consist of other fields, we should constrain $J_{\mu\nu}$ in such a way that it will not affect the functional determinant of the constraints. To this end, we pick the constraint as $\gamma^\mu J_{\mu\nu} = 0$ and $\bar{J}_{\mu\nu}\gamma^\nu = 0$. In the pure spin-3/2 field formalism, incidentally, the projection operator $\mathcal{P}_{\mu\nu\rho\sigma}$ has the orthogonality relation $\gamma^\mu\mathcal{P}_{\mu\nu\rho\sigma} = 0$ and $\mathcal{P}_{\mu\nu\rho\sigma}\gamma^\rho = 0$. As a consequence, the interaction Lagrangian could contain such projection operator and for the simplest consistent $\pi N\Delta$ interaction Lagrangian we have

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\bar{N}\gamma_5\mathcal{P}_{\mu\nu\rho\sigma}\partial^\rho\psi^{\mu\nu}\partial^\sigma\pi + \text{H.c.}, \tag{39}$$

whereas for the electromagnetic transition the corresponding $\gamma N\Delta$ Lagrangian reads

$$\mathcal{L}_{\gamma N\Delta} = \bar{N}(g_1\mathcal{P}_{\rho\sigma\mu\nu} + g_2\gamma_\rho\mathcal{P}_{\sigma\alpha\mu\nu}\partial^\alpha)\psi^{\mu\nu}F^{\rho\sigma} + \text{H.c.} \tag{40}$$

IV. NUMERICAL RESULT AND VISUALIZATION

As in the previous report [9], we can explore the behavior of the pure spin-3/2 propagator by comparing the $\Delta(1232)$ resonance contribution to the total cross section of elastic πN scattering, obtained from the pure spin-3/2 propagator and the conventional ones. For this purpose, it is important to include the resonance width Γ in the resonance propagator, i.e., by replacing $i\epsilon \to i\Gamma m_\Delta$.

The elastic πN scattering amplitude is traditionally written as

$$\mathcal{M} = \bar{u}(p', s')(A + B\not{Q})u(p, s)\,, \tag{41}$$

with $Q = (k + k')/2$. By using the RS propagator with GI interaction, the amplitude obtained from the Feynman diagram depicted in Fig. 1, i.e., Eq. (19), can be decomposed into

$$A = G_1\big\{m_N\big(3k'\cdot k - 2p\cdot k - m_\pi^2 - 2q\cdot k' q\cdot k/q^2\big) + m_\Delta\big(3k'\cdot k - 2p\cdot k - m_\pi^2 - 2m_\pi^2 q\cdot Q/q^2\big)\big\}, \tag{42}$$

$$B = G_1\big\{3k'\cdot k - m_\pi^2 + 2m_N^2 - 2q\cdot k' q\cdot k/q^2 + q(k'-k) + 2m_\Delta m_N(1 - q\cdot Q/q^2)\big\}\,, \tag{43}$$

with

$$G_1 = q^2\, g_{\pi N\Delta}^2\big/\big[3m_\pi^2 m_\Delta^2\big(q^2 - m_\Delta^2 + i\Gamma m_\Delta\big)\big]. \tag{44}$$

In the case of pure spin-3/2 propagator with the hadronic vertex given by Eq. (26), i.e., Eq. (27), we obtain

$$A = G_2\big[\big(q^4\big/12m_\Delta^4\big)\big(3k'\cdot k - m_\pi^2 - 2p\cdot k - 2m_\pi^2\, q\cdot Q/q^2\big) - \big\{\big(q^2 - m_\Delta^2\big)\big/2m_\Delta^4\big\} \times (q^2 k'\cdot k - q\cdot k q\cdot k')\big]\,, \tag{45}$$

$$B = \big(q^4 m_N G_2\big/6m_\Delta^4\big)(1 - q\cdot Q/q^2)\,, \tag{46}$$

with

$$G_2 = g_{\pi N\Delta}^2\big/\big[q^2 - m_\Delta^2 + i\Gamma m_\Delta\big]. \tag{47}$$

Finally, if we used the consistent interaction Lagrangian given by Eq. (39), we noted that the scattering amplitude becomes more simple, i.e.,

$$\mathcal{M} = G_3\, \bar{u}(p', s')\, \gamma_5\, q_\mu k'_\nu\, \mathcal{P}^{\mu\nu\alpha\beta}\, \gamma_5\, q_\alpha k'_\beta\, u(p, s)\,, \tag{48}$$

where

$$G_3 = \frac{g_{\pi N\Delta}^2}{m_\Delta^2\big(s - m_\Delta^2 + i m_\Delta \Gamma_\Delta\big)}\,. \tag{49}$$

By decomposing Eq. (48) into Eq. (41), we obtain

$$A = G_3\big(q^2 m_\pi^2 - q\cdot k'^2\big)\big/6, \tag{50}$$

$$B = 0\,. \tag{51}$$

By using the standard method [16], we can calculate the cross section from the scattering amplitude $\mathcal{M}$ given by Eq. (41). Note that the amplitudes obtained from the three different models are completely different. For the sake of comparison, we use different coupling constants in order to produce comparable total cross sections. Obviously, this would not raise a problem since almost all of the coupling constants in the phenomenological applications are fitted to reproduce the experimental data.

By taking point-particle approximation, the total cross sections obtained from the three models are depicted in Fig. 2(a). The resonance behavior centered around $W \approx 1.25$ GeV is produced by all models, including the background phenomenon shown by the increase of total cross section as the energy increases for $W \gtrsim 1.40$ GeV. The phenomenon originates from the momentum dependence in the numerator of Eq. (27). The resonance background has another effect, i.e., shifting the resonance peak slightly from its original position at 1.232 GeV to higher energy. We also observe from Fig. 2(a) that the backgrounds obtained from the pure spin-3/2 models are significantly smaller than that of the RS model at $W \approx 1.40$ GeV. The reason can be traced back to the second term in the square bracket of Eq. (27). For $W \gg 1.40$ GeV, the first term of Eq. (27) becomes dominant, since $q^4 = W^4$, and the total contribution starts to diverge. From Fig. 2(a) it is also interesting to note that the pure spin-3/2 model with consistent interaction yields moderate background compared to the other two models and does not show a divergence behavior at high energies.

In the covariant Feynman diagrammatic approach, the phenomenon of large background contribution is found to be natural. Alternatively, one can also interpret this background as the contribution of a Z diagram [17], i.e., a production of a

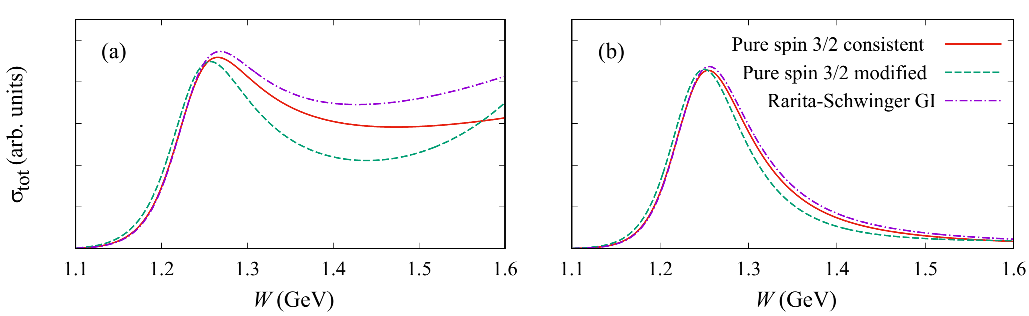

FIG. 2. Contribution of the $\Delta(1232)$ resonance to the $\pi N \to \pi N$ scattering total cross section in arbitrary units (arb. units) calculated by using the pure spin-3/2 models with consistent [Eq. (39)] and modified [Eq. (25)] interaction Lagrangians as well as the Rarita-Schwinger model with GI interaction as a function of total c.m. energy W. Panels (a) and (b) are obtained from the calculations without and with hadronic form factors in the hadronic vertices, respectively. Note that to simplify the comparison we do not use the same value of coupling constant in all calculations.

particle and an antiparticle in the intermediate state not considered in Fig. 1. Note that the phenomenon does not exist in the multipoles approach, where a perfect resonance structure can be produced by using the Breit-Wigner parameterization [18].

The large background contribution can disturb the nature of the resonance itself and might induce other difficulties such as the problem to fit experimental data, especially in a covariant isobar model [10], in which a large number of resonances are included while the individual resonance peaks are no longer distinguishable due to the proximity of their masses. To alleviate this problem, it is customarily to use hadronic form factor (HFF) in each hadronic vertex shown in Fig. 1. For a brief discussion of the HFF along with its problem with the gauge invariance, we refer the reader to Refs. [19,20].

In spite of the objection that the HFF introduces new free parameters, it should be noted that the existence of HFF is inevitable because the baryon is not a point particle. Furthermore, the use of HFF is also important to eliminate the divergence of the scattering amplitude. Thus, in the present work we include the HFF and adopt the dipole HFF as in the previous work in the form of [19]

$$F = \Lambda^4 / \left[\Lambda^4 + \left(q^2 - m_\Delta^2\right)^2\right], \tag{52}$$

where the hadronic cutoff is chosen to be $\Lambda = 0.5$ GeV in order to produce a reasonable resonance structure in the total cross section. By including this HFF, we obtain the result shown in Fig. 2(b), in which a perfect resonance structure for all models is displayed. Compared to the pure spin-3/2 models, the RS structure is slightly shifted to the right. This is understandable if we compare the original contributions of all models (without HFF) as shown in Fig. 2(a). Therefore, apart from its different formulation, the pure spin-3/2 propagator still shows the usual resonance structure as in the conventional RS propagator. Furthermore, Fig. 2 clearly indicates that to obtain the natural property of a resonance, the use of HFF is mandatory in the covariant Feynman diagrammatic approach.

The next application of our present work is the contribution of spin-3/2 Δ resonance to the pion photoproduction off a nucleon. As shown in the previous work, the choice of inappropriate electromagnetic interaction could fail to generate the correct property of a resonance in the cross section [9]. Thus, in what follows we will calculate the Δ resonance contribution to the total cross section with a consistent interaction and compare the result with those of previous works.

The corresponding Feynman diagram is depicted in Fig. 3. The hadronic vertex factor can be obtained from Eq. (39), i.e.,

$$\Gamma^{\mu\nu}_{\pi N\Delta} = \frac{g_{\pi N\Delta}}{m_\Delta^2}\gamma_5 (p+k)_\rho q_\sigma \mathcal{P}^{\mu\nu\rho\sigma}, \tag{53}$$

whereas the electromagnetic one obtained from Eq. (40) can be written as

$$\Gamma^{\alpha\beta}_{\gamma N\Delta} = \mathcal{P}^{\rho\sigma\alpha\beta}\left\{ g_1 (k_\rho \epsilon_\sigma - \epsilon_\rho k_\sigma) + \frac{g_2}{m_\Delta}(p+k)_\sigma (\not{k}\epsilon_\rho - k_\rho \not{\epsilon})\right\}. \tag{54}$$

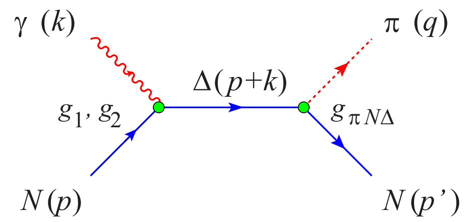

FIG. 3. Feynman diagram for the πN photoproduction with a Δ resonance in the intermediate state. The electromagnetic and hadronic couplings are indicated in the diagram.

Note that in Eqs. (53) and (54) we have inserted additional m_Δ in the denominator to make the coupling constants dimensionless. Furthermore, only two couplings are independent in this case, since the dual tensor of ATS is proportional to the tensor itself, i.e., $\tilde{\Psi}_{\mu\nu} = -\gamma_5 \Psi_{\mu\nu}$. This is different from the case of GI interaction [6,10].

By using the propagator given in Eq. (8) and the interaction vertex factors of Eqs. (53) and (54), we may write the production amplitude as

$$\begin{aligned}\mathcal{M} &= \bar{u}_{N'}\,\Gamma^{\rho\sigma}_{\pi N\Delta}\,\frac{\Delta_{\rho\sigma\gamma\delta}}{s - m_\Delta^2 + i m_\Delta \Gamma_\Delta}\,\Gamma^{\gamma\delta}_{\gamma N\Delta}\,u_N \\ &= \bar{u}_{N'}\,\gamma_5 (p+k)_\mu q_\nu\,\mathcal{P}^{\mu\nu\alpha\beta}\,[G_1 (k_\alpha \epsilon_\beta - \epsilon_\alpha k_\beta) \\ &\quad + G_2\,(\not{k}\epsilon_\alpha - k_\alpha \not{\epsilon})(p+k)_\beta]\,u_N\,,\end{aligned} \tag{55}$$

where m_Δ and Γ_Δ are the mass and width of Δ, respectively, $s = (p+k)^2$, and we have used the relation

$$\mathcal{P}^{\rho\sigma\mu\nu}\Delta_{\rho\sigma\gamma\delta}\mathcal{P}^{\alpha\beta\gamma\delta} = \mathcal{P}^{\mu\nu\alpha\beta}. \tag{56}$$

Furthermore, in Eq. (55) we have defined

$$G_1 = -\frac{g_1\, g_{\pi N\Delta}}{m_\Delta^2\left(s - m_\Delta^2 + i m_\Delta\Gamma_\Delta\right)}, \tag{57}$$

$$G_2 = -\frac{g_2\, g_{\pi N\Delta}}{m_\Delta^3\left(s - m_\Delta^2 + i m_\Delta\Gamma_\Delta\right)}. \tag{58}$$

To calculate the total cross section, we decompose the reaction amplitude $\mathcal{M}$ into the form functions A_i [10]

$$\mathcal{M} = \bar{u}_N(p') \sum_{i=1}^{4} A_i(s,t,u)\, M_i\, u_N(p), \tag{59}$$

with the gauge and Lorentz invariant matrices M_i

$$M_1 = \gamma_5\,\not{\epsilon}\not{k}, \tag{60}$$

$$M_2 = 2\gamma_5 (q\cdot\epsilon P\cdot k - q\cdot k P\cdot\epsilon), \tag{61}$$

$$M_3 = \gamma_5 (q\cdot k\not{\epsilon} - q\cdot\epsilon\not{k}), \tag{62}$$

$$M_4 = i\epsilon_{\mu\nu\rho\sigma}\gamma^\mu q^\nu \epsilon^\rho k^\sigma, \tag{63}$$

where $P = \frac{1}{2}(p + p')$ and ϵ is the photon polarization. By performing the decomposition, we obtain

$$A_1 = 2\{k \cdot (q - p) - \tfrac{2}{3}m_\pi^2 + \tfrac{2}{3}q \cdot (p + k)\}G_1 + \tfrac{1}{3}m_N\big(9p \cdot k + 5m_\pi^2 - 8q \cdot (p + k)\big)G_2 , \quad (64)$$

$$A_2 = 0 , \quad (65)$$

$$A_3 = \tfrac{2}{3}m_N G_1 + \{s + \tfrac{2}{3}m_\pi^2 - \tfrac{4}{3}m_N^2 - 3q \cdot (p + k)\}G_2 , \quad (66)$$

$$A_4 = -\tfrac{4}{3}m_N G_1 + \{2s + \tfrac{2}{3}m_\pi^2 - \tfrac{4}{3}m_N^2 - 3q \cdot (p + k)\}G_2 , \quad (67)$$

from which we can calculate the total cross section [21]. Note that the form functions A_i for non-ATS models can be found in the previous works [10,22].

Different from pion scattering, in pion photoproduction there is only one hadronic vertex. Nevertheless, in the photoproduction the hadronic form factor still plays an important role to suppress the background contribution at high energies. Note that a very soft form factor leads to very strong suppression of the cross section. Although it produces an ideal resonance bump, the resonance contribution could become very small and might distort the physics behind it. On the other hand, a very hard form factor could fail to suppress the cross section at high energies and, as a consequence, could fail to create the resonance bump. Thus, as in the previous example, we include the hadronic form factor given by Eq. (52), albeit with a different hadronic cutoff, i.e., $\Lambda = 0.8$ GeV, to obtain reasonable values of photoproduction total cross section.

In the present work, we scale all peaks of the total cross sections to the same value. This is required merely for the sake of comparison, but we believe that this is still acceptable since we set all coupling constants to unity in the numerical calculation. Moreover, in this visualization, we merely want to see the structure of a resonance produced by different representations of spin 3/2. The result is shown in Fig. 4, where we compare the contribution of the $\Delta(1232)$ resonance to the total cross section of pion photoproduction off a nucleon according to the model of pure spin 3/2 with consistent interaction, the RS model with the GI interaction [12], the RS model with non-GI interaction [22], and the modified RS with GI interaction [23], where in the latter we only use two electromagnetic couplings, instead of four as in the original version [6]. Obviously, all models exhibit a peak as the basic behavior required for a resonance. Surprisingly, the pure spin-3/2 with consistent interaction (ATS) and the Rarita-Schwinger with non-GI interaction (RS2) prescriptions yield a similar structure. Presumably this is because the similar structure of the two models. The Rarita-Schwinger with GI interaction (RS1) indicates a larger background. Previously, we suspect that this property originates from the larger number of coupling constants used in this model. However, the use of only two of these coupling constants (RS1r), i.e., only g_1 and g_2, does not reduce the background. This is caused by the destructive effect of the other two couplings, g_3 and g_4, that is missing in the RS1r model. Nevertheless, the most important point to note here is that the pure spin-3/2 model with consistent interaction produces the correct property of resonance as in the conventional models.

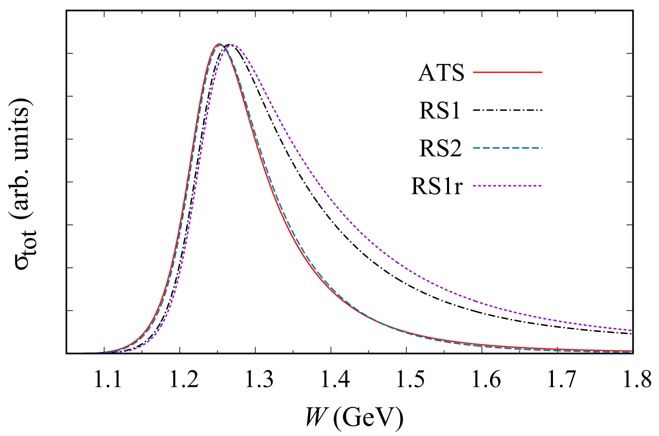

FIG. 4. Total cross sections of the πN photoproduction calculated from the contribution of the $\Delta(1232)$ resonance in arbitrary units according to the ATS prescription with consistent interaction Lagrangians (ATS), Rarita-Schwinger with GI interaction (RS1 [12]), Rarita-Schwinger with non-GI interaction (RS2 [22]), and Rarita-Schwinger with GI interaction with only two electromagnetic couplings (RS1r), as a function of the total c.m. energy W. Note that for the sake of comparison the total cross section peaks are scaled to the same value.

V. SUMMARY AND CONCLUSION

We have proposed the use of pure spin-3/2 propagator along with the consistent interaction Lagrangians in the phenomenological studies of nuclear and particle physics. To this end, we employ the ATS representation to describe the corresponding projection operator. We have shown that the ATS formalism has a problem to exhibit the resonance behavior, unless the interaction Lagrangian is slightly modified, i.e., by replacing the γ matrix with a partial derivative. However, the choice of a partial derivative seems to be arbitrary. To obtain a more systematic procedure and the correct degrees of freedom, we determine a number of constraints required by the interaction. In this work, we give the simplest example of consistent interactions that satisfy these constraints. To visualize the result, we apply the pure spin-3/2 propagator and consistent interactions to calculate the contribution of $\Delta(1232)$ resonance in the pion scattering and pion photoproduction off a nucleon. The obtained total cross sections in the two cases indicate that the pure spin-3/2 propagator with consistent interaction Lagrangians exhibits the required property of a resonance. Thus, we have proven that the proposed spin-3/2 propagator along with the consistent interaction Lagrangian can be directly used for phenomenological investigations in the realm of nuclear and particle physics.

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