# **Nucleon resonances with hidden charm in** *γp* reactions

Jia-Jun Wu,<sup>1</sup> T.-S. H. Lee,<sup>2</sup> and Bing-Song Zou<sup>1,3,4</sup>

<sup>1</sup>*School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

<sup>2</sup>*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>3</sup>*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China* <sup>4</sup>*School of Physics and Electronics, Central South University, Hunan 410083, China*

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The excitations of nucleon resonances with hidden charm,  $N_{c\bar{c}}^*$ , in  $\gamma p$  reactions are investigated by using the predictions from the available meson-baryon (*MB*) coupled-channel models of  $N_{c\bar{c}}^*$ , with  $MB =$  $\rho N$ ,  $\omega N$ ,  $J/\psi N$ ,  $\bar{D}\Lambda_c$ ,  $\bar{D}^*\Lambda_c$ ,  $\bar{D}\Sigma_c$ ,  $\bar{D}^*\Sigma_c$ ,  $\bar{D}\Sigma_c^*$ . For the  $\gamma p \to J/\psi p$  process, we first apply the model of Donnachie and Landshoff to calculate the Pomeron-exchange amplitudes with the parameters determined from fitting the available total cross section data up to invariant mass  $W = 300$  GeV. We then add the resonant  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  amplitudes to examine the effects of  $N_{c\bar{c}}^*$  excitations on the cross sections of  $\gamma p \to J/\psi p$ in the near threshold energy region covered by the recent experiments at Jefferson Lab. The  $N_{c\bar{c}}^* \to MB$  transition matrix elements are determined from the partial decay widths predicted by the considered meson-baryon coupled-channel models of  $N_{c\bar{c}}^*$ . The  $\gamma p \to N_{c\bar{c}}^*$  transition amplitudes are calculated from the vector meson dominance (VMD) model as  $\gamma p \to V p \to N_{c\bar{c}}^*$  with  $V = \rho$ ,  $\omega$ ,  $J/\psi$ . The total  $\gamma p \to J/\psi p$  amplitudes then depend on an off-shell form factor, parametrized as  $F_V(q^2) = \Lambda_V^4/[\Lambda_V^4 + (q^2 - m_V^2)^2]$ , which is needed to account for the  $q^2$  dependence of the photon-vector meson coupling constant  $\frac{e m_V^2}{f_V}$  of the VMD model. It was found that, with  $\Lambda_V = 0.55$  GeV, the predicted total cross sections are within the range of the data in the energy region near the *J*/ $\psi$  production threshold. We then demonstrate that the  $N_{c\bar{c}}^*$  can be most easily identified in the differential cross sections at large angles where the contribution from Pomeron exchange becomes negligible. With the same VMD model and the same coupled-channel models of  $N_{c\bar{c}}^*$ , we also calculate the resonant amplitudes for the  $\gamma p \to V p \to N_{c\bar{c}}^* \to \bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$  processes. By adding the nonresonant amplitudes due to the exchange of  $\bar{D}^{*0} (\bar{D}^0)$ , we then predict the cross sections of  $\gamma p \to \bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$  for additional experimental tests of the available meson-baryon coupled-channel models of  $N_{c\bar{c}}^*$ .

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### **I. INTRODUCTION**

It is well recognized that the interaction between a nucleon  $(N)$  and a  $c\bar{c}$  system of a charm quark  $(c)$  and an anticharm quark  $(\bar{c})$  is mainly due to gluon-exchange mechanisms. All of the earlier investigations  $[1-8]$  indicated that the  $c\bar{c}$ -*N* interaction is attractive. This implies the possible existence of nuclear systems with hidden charm, as investigated in Refs.  $[8-10]$ . For the baryon number  $B = 1$  system, it was proposed [\[11\]](#page-11-0) in 2010 that there exist excited nucleons with  $c\bar{c}$  components in the mass range of 4.0–5.0 GeV within a meson-baryon coupled-channel model. Such baryons with hidden charm were subsequently also predicted  $[12-18]$  as molecular states made of anticharmed mesons and charmed baryons (such as  $\bar{D}^{(*)}\Sigma_c^{(*)}$ ). Alternatively, they are described as compact pentaquark states made of colored quark clusters  $[19]$  or a mixture of the two configurations  $[20]$ . The masses from these earlier predictions are qualitatively consistent with the mass  $(m)$  and width  $(\Gamma)$  of two pentaquark states  $(P_c)$  identified from analyzing the  $J/\psi$ -p invariant mass distributions of the  $\Lambda_c^* \to K J/\psi \, p$  decays measured by the LHCb Collaboration [\[21,22\]](#page-11-0) in 2015. Their results are listed in the left part of Table [I.](#page-1-0)

The resonance peaks in the  $J/\psi$ -*p* invariant mass distributions from the LHCb measurement [\[21\]](#page-11-0) motivated a lot of theoretical efforts [\[24–](#page-11-0)[68\]](#page-12-0). Roughly speaking, there are three different interpretations of these peaks:

- (1) They are due to the excitations of meson-baryon molecular systems which could be made of (1) anticharm mesons and charm baryons [\[24–46\]](#page-11-0), (2) baryons and charmonium [\[47,48\]](#page-11-0), (3) the mixture  $[49,50]$  of (1) and (2).
- (2) They could be multiquark states within the conventional constituent quark model [\[51,52,54\]](#page-11-0) or cluster states pictured as a diquark-diquark-antiquark system [\[55–](#page-11-0)[58\]](#page-12-0) or a diquark-triquark system [\[59\]](#page-12-0).
- (3)  $P_c(4450)$  may not be a resonance state because it is close to the Triangle Singularity (TS) [\[60–62\]](#page-12-0) and the observed narrow peak is purely due to the kinematic effect, although for some quantum numbers of the *Pc* state preferred in Ref. [\[21\]](#page-11-0), such as  $3/2^-$  or  $5/2^+$ , the TS cannot explain the peak as shown in Ref. [\[63\]](#page-12-0).

With the the new results from the LHCb Collaboration [\[23\]](#page-11-0), these theoretical interpretations can be better tested. By analyzing the data, which are about a factor of 9 larger than what they analyzed in 2015, the LHCb Collaboration obtained three clean peaks which are interpreted as the excitations of

<span id="page-1-0"></span>

	2015		2019			
$P_c$	m			$\boldsymbol{m}$		
$P_c(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$P_c(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	
$P_c(4450)$	$4449.8 \pm 1.7 \pm 2.5$	$39 \pm 5 \pm 19$	$P_c(4440)$ $P_c(4457)$	$4440.3 \pm 1.3_{-4.7}^{+4.1}$ $4457.3 \pm 0.6_{-1.7}^{+4.1}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$ $6.4 \pm 2.0^{+5.7}_{-1.9}$	

TABLE I. The masses  $[m \text{ (MeV)}]$  and total widths  $[\Gamma \text{ (MeV)}]$  of  $P_c$  reported in Refs.  $[21-23]$ .

three pentaquark states, as listed in the right part of Table I. Comparing with their results of 2015, the main features of these new data are (1)  $P_c(4312)$  could be a new pentaquark state near the  $\Sigma_c \bar{D}$  threshold; (2)  $P_c(4440)$  and  $P_c(4457)$  are two narrow states which could not be resolved in the 2015 determination of  $P_c(4450)$ ; and (3)  $P_c(4380)$ , with about 200 MeV width according to the 2015 analysis, could be a very broad state, and is not given mass and width in this analysis. It is important to note that these three narrow states are all just below the corresponding anticharmed meson-charmed baryon threshold, and hence the simplest interpretation is that they are made of meson-baryon components, as suggested in Refs. [\[69–82\]](#page-12-0).

The nucleon resonances with hidden charm, called  $N_{c\bar{c}}^*$ from now on in this paper, can also be investigated by using the electromagnetic production of  $J/\psi$  from the nucleon, such as  $e + p \rightarrow e' + J/\psi + p$  studied in Refs. [\[11,12\]](#page-11-0). The prediction of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  cross section within the coupled-channel model of Ref. [\[12\]](#page-11-0) was then made in Ref. [\[83\]](#page-12-0) by using the vector meson dominance (VMD) model to generate vector (*V*) mesons,  $\rho$ ,  $\omega$ , and  $J/\psi$ , from photons. A few more predictions of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  had been made [\[84–90\]](#page-12-0) within the meson-baryon coupled-channel model since 2015. The differences among these works are in their choices of  $N_{c\bar{c}}^*$  model, vector mesons included in the VMD, and the background amplitudes which could be calculated from Pomeron exchange or two-gluon and threegluon exchange models.

In parallel to these theoretical efforts, an experi-ment [\[91,92\]](#page-12-0) [JLab(E12-16-007)] on  $\gamma p \rightarrow J/\psi p$  near threshold at the Thomas Jefferson National Accelerator Facility (JLab) was approved in 2016, and the data from this effort will soon become available. A separate effort at JLab using the GlueX detector [\[93\]](#page-12-0) recently published [\[94\]](#page-12-0) their measured total cross sections of  $\gamma p \rightarrow J/\psi p$ . The main purpose of this work is to provide information for examining whether the  $N_{c\bar{c}}^*$  predicted by the available meson-baryon coupledchannel models can be observed in the data from these two experiments.

To proceed, it is necessary to first recognize that  $P_c$  states reported by the LHCb Collaboration are from the measurements of the *J*/ $\psi$ -*p* invariant mass distribution of the  $\Lambda_c \rightarrow$  $K + J/\psi + p$  decay. Thus the information one can use to test the available  $N_{c\bar{c}}^*$  models consists of the total widths and masses of the reported  $P_c$  states. The spins and parities of these states cannot be determined since a partial-wave analysis of  $\Lambda_c \to K + J/\psi + p$  decays requires detailed angular distribution data, not just the invariant mass distributions. Accordingly, one cannot determine the partial decay

width for each possible meson-baryon channel of  $N_{c\bar{c}}^* \rightarrow$  $J/\psi N$ ,  $\rho N$ ,  $\omega N$ ,  $\bar{D}\Lambda_c$ ,  $\bar{D}^*\Lambda_c$ ,  $\bar{D}\Sigma_c$ ,  $\bar{D}^*\Sigma_c$ , ... . Here we also mention that two of the resonance peaks reported by the LHCb Collaboration are near the threshold of the  $\bar{D}^{(*)}\Sigma_c$ channel and thus the identification of resonances in this region must account for the cusp effect in a analysis constrained by three-body unitarity. The importance of three-body unitarity in analyzing the three-body decays of heavy mesons was demonstrated [\[95\]](#page-12-0) recently, but is not considered in the analysis by the LHCb Collaboration. Therefore, no attempt will be made here to revise the considered meson-baryon models to reproduce the resonance peaks of the LHCb data. Instead, we will only consider the available models which have predicted  $N_{c\bar{c}}^*$  with masses within the range of the LHCb data. Furthermore, there is no new theoretical prediction of the partial decay widths, which are needed in our investigations, for the  $N_{c\bar{c}}^*$  states after the new LHCb results published in 2019. Thus, in the current work, we will only employ the theoretical models based on the old LHCb data published in 2015. By using the spins, parities, and partial decay widths from those models, we can then use the VMD to predict the amplitudes of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$ . Here we notice that the VMD coupling constant  $g_{\gamma, V} = e m_V^2 / f_V$  for the  $\gamma \to V$  transitions for  $V = \rho$ ,  $\omega$ ,  $J/\psi$  are conventionally determined from the decay widths of  $V \to \gamma \to e^+e^-$  with  $q^2 = m_V^2$  of the intermediate  $\gamma$ . In the situation of  $\gamma p \to V p \to N_{c\bar{c}}^*$ , we have  $q^2 = 0$ , i.e., the intermediate vector is far off mass shell, and thus the VMD parameter  $g_{\gamma,V}$  must be modified to account for this  $q^2$  dependence. Ideally, this  $q^2$  dependence should be calculated from a QCD model as done in Ref. [\[96\]](#page-12-0). Here we will treat it as a phenomenological part of our calculation by introducing an off-shell form factor  $F_V = \Lambda_V^4 / [\Lambda_V^4 + (q^2 - q^2)$  $(m_V^2)^2$ ] with  $\Lambda_V$  determined by the available total cross section data, as will be explained in Sec. [III.](#page-7-0) We also make sure that the parametrization of VMD is gauge invariant when the off-shell form factor is included.

To predict the cross sections of  $\gamma p \rightarrow J/\psi p$ , it is necessary to include the nonresonant amplitudes due to the gluon-exchange mechanisms. In this work, we use the model of Donnachie and Landshoff [\[7\]](#page-11-0) within which the gluonexchange mechanism is phenomenologically parametrized as Pomeron exchange within the Regge phenomenology of high energy reactions. By fitting the total cross section data up to very high energy  $W = 300$  GeV, the Pomeron parameters are well determined and can be used to define the nonresonant amplitudes in the near-threshold region of our interest in this paper. Our approach is therefore different from the approaches using the models of two-gluon and three-gluon exchange of Refs. [\[97\]](#page-12-0), as will be discussed later.

<span id="page-2-0"></span>For additional studies of  $N_{c\bar{c}}^*$  excitations, we also explored other meson photoproduction processes which do not have Pomeron-exchange mechanisms. We found that experiments on  $\gamma p \to \bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$  could be useful. With the same VMD model and the same coupled-channel models of  $N_{c\bar{c}}^*$ , we calculated the resonant amplitudes for the  $\gamma p \rightarrow V p \rightarrow$  $N_{c\bar{c}}^* \to \bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$  processes. By adding the nonresonant amplitudes due to the exchange of  $\bar{D}^{*0}$  ( $\bar{D}^{0}$ ), we then predicted the cross sections of  $\gamma p \to \bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$  for additional experimental tests of the available meson-baryon coupledchannel models of  $N_{c\bar{c}}^*$ .

The paper is organized as follows. In Sec.  $II$ , we give formulas for calculating the cross sections of  $\gamma p \rightarrow J/\psi p$ , and we present formulas for calculating the Pomeron-exchange amplitude and the  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  resonant amplitudes. In Sec. [III,](#page-7-0) we present our results for  $\gamma p \rightarrow J/\psi p$ . The results for  $\gamma p \to \bar{D}^0 \Lambda_c$ ,  $\bar{D}^{*0} \Lambda_c$  are given in Sec. [IV.](#page-8-0) The discussion and summary will be given in the last section.

# **II. CROSS SECTION FORMULA FOR VECTOR MESON PHOTOPRODUCTION REACTION ON THE NUCLEON**

We consider the photoproduction of a meson-baryon (*MB*) system:  $\gamma(q) + p(p) \rightarrow M(q') + B(p')$ . In the center-of-mass system, the four-momenta of these particles can be defined as

$$
q = (k, k),
$$
  
\n
$$
q' = (E_M(k'), \vec{k}'),
$$
  
\n
$$
p = (E_N(k), -\vec{k}),
$$
  
\n
$$
p' = (E_B(k), -\vec{k}'),
$$
  
\n
$$
P = q + q' = p + p' = (W, \vec{0}),
$$

where  $k(k')$  is the length of three-momenta  $k(k')$ ,  $E_a(k) =$ where k (k') is the length of three-momenta k (k')  $\sqrt{m_a^2 + k^2}$  is the energy of a particle with mass is  $\sqrt{m_a^2 + k^2}$  is the energy of a particle with mass  $m_a$ , and *W* is the invariant mass of the system. For a given *W* and angle  $(\Omega)$  between *k* and *k'*, all of the above kinematic variables are determined by  $W = k + E_N(k) = E_M(k') + E_B(k')$ . The differential cross section can then be written as

$$
\frac{d\sigma}{d\Omega} = \frac{(2\pi)^4}{k^2} \rho_{\gamma N}(k) \rho_{MB}(k')
$$
  
 
$$
\times \frac{1}{4} \sum_{\lambda_{\gamma}, \lambda_M} \sum_{m_s, m'_s} |\langle \vec{k}' \lambda'_M m'_s | T(W) | \vec{k} \lambda_{\gamma} m_s \rangle|^2, \quad (1)
$$

where  $\lambda'_M$  ( $\lambda_\gamma$ ) is the helicity of the meson *M* (photon  $\gamma$ ),  $m_s(m'_s)$  is the *z* component of the spin of initial proton *p* (final baryon  $B$ ). The phase space factors in Eq.  $(1)$  are

$$
\rho_{\gamma N}(k) = \frac{k^2 E_N(k)}{W}, \n\rho_{MB}(k') = \frac{k' E_M(k') E_N(k')}{W}.
$$
\n(2)

The reaction amplitude is written as

$$
\langle k' \lambda'_{M} m'_{s} | T(W) | k \lambda_{\gamma} m_{s} \rangle
$$
  
= 
$$
\frac{1}{(2\pi)^{3}} \sqrt{\frac{m_{N} m_{B}}{4k E_{N}(k) E_{M}(k') E_{B}(k')} \epsilon_{\nu}(q, \lambda_{\gamma})}
$$
  

$$
\times [j^{\nu}_{\lambda'_{M}, m'_{s}, m_{s}}(q', p', q, p)],
$$
 (3)



FIG. 1. Gluon-exchange mechanism of  $\gamma N \rightarrow J/\psi + N$ .

where  $\epsilon_{\nu}(q, \lambda_{\gamma})$  is the polarization vector of photon, and  $j^{\nu}_{\lambda'_{M},m'_{s},m_{s}}(q, p, q', p')$  is a Lorentz covariant current matrix element. For the vector meson photoproduction  $\gamma p \rightarrow J/\psi p$  $(M = J/\psi$  and  $B = N$ ) process, the current matrix element can be written as

$$
j^{\nu}_{\lambda'_{J/\psi}, m'_s, m_s}(q, p, q', p')
$$
  
=  $\bar{u}_p(p', m'_s) \epsilon^*_{\mu}(q', \lambda'_{J/\psi}) \mathcal{M}^{\mu\nu}(q, p, q', p') u_p(p, m_s),$  (4)

where  $u_a(p, m_s)$  is the spinor of the baryon *a* [with the normalization  $\bar{u}_a(p, m_s)u_a(p, m'_s) = \delta_{m_s, m'_s}$ ] and  $\epsilon_v(q, \lambda'_J/\psi)$ is the polarization vector of  $J/\psi$ . The current matrix element must satisfy the gauge invariance condition  $j^{\nu}q_{\nu} = 0$ .

In this work, we assume that the  $J/\psi$  photoproduction amplitudes  $\mathcal{M}^{\mu\nu}(q, p, q', p')$  of Eq. (4) can be written as

$$
\mathcal{M}^{\mu\nu}(q, p, q', p') = \mathcal{M}_{\mathbb{P}}^{\mu\nu}(q, p, q', p') + \mathcal{M}_{N^*}^{\mu\nu}(q, p, q', p'),
$$
\n(5)

where  $\mathcal{M}_{\mathbb{P}}^{\mu\nu}(q, p, q', p')$  is the Pomeron-exchange amplitude of Donnachie and Landshoff, and  $\mathcal{M}_{N*}^{\mu\nu}(q, p, q', p')$  is the  $\gamma N \to N_{c\bar{c}}^* \to J/\psi N$  amplitude. In the following, we will describe the calculations of these two amplitudes.

#### **A. Pomeron-exchange mechanism**

It is well recognized that the photoproduction of  $J/\psi$  from the nucleon is mainly due to a gluon-exchange mechanism, such as the leading two-gluon exchange mechanism illustrated in Fig.  $1(a)$ . It is also known that Pomeron exchange is an essential element in Regge phenomenology. Within the model of Donnachie and Landshoff (DL) [\[7\]](#page-11-0), it is assumed that the Pomeron  $(\mathbb{P})$  can be identified with gluons and the Pomeron-exchange mechanism can be parametrized in terms of Pomeron-quark coupling constant  $\beta_q$  and appropriate form factors at the  $\mathbb{P} J/\psi \to J/\psi$  and  $\mathbb{P} N \to N$  vertices. The DL model is illustrated in Fig.  $1(b)$ . Following a study of nonperturbative two-gluon exchanges [\[98\]](#page-12-0), they further adopted the Pomeron-photon analogy that the Pomeron can be treated as a  $C = +1$  isoscalar photon to parametrize the quark-Pomeron vertex. Thus the  $\mathbb{P} N \to N$  vertex can be expressed in term of the isoscalar electromagnetic form factor of the nucleon. Following Ref. [\[99\]](#page-12-0), the Pomeron-exchange amplitude in Eq.  $(5)$  is written as

$$
\mathcal{M}_{\mathbb{P}}^{\mu\nu}(q, p, q', p') = G_{\mathbb{P}}(s, t) \mathcal{T}_{\mathbb{P}}^{\mu\nu}(q, p, q', p') \qquad (6)
$$

with

$$
\mathcal{T}_{\mathbb{P}}^{\mu\nu}(q, p, q', p')
$$
  
=  $i12 \frac{eM_V^2}{f_V} \beta_{q_V} F_V(t) \beta_{u/d} F_1(t) [\oint g^{\mu\nu} - q^{\mu} \gamma^{\nu}],$  (7)

<span id="page-3-0"></span>

FIG. 2. Total cross sections of photoproduction of  $J/\psi$  on the proton target. The solid curve is from the DL model of Pomeron exchange. The dotted (dash-dotted) curves are from the two-gluon  $exchange$  (two-gluon  $+$  three-gluon exchange) model. The experimental data can be found in Ref. [\[9\]](#page-11-0), except the blue open squares are from Ref. [\[93\]](#page-12-0).

where  $\beta_{qv}$  ( $\beta_{u/d}$ ) defines the coupling of the Pomeron with the quark *qV* (*u* or *d*) in the vector meson *V* (nucleon *N*). Here we have introduced the form factor for the Pomeron-vector meson vertex as

$$
F_V(t) = \frac{1}{M_V^2 - t} \left( \frac{2\mu_0^2}{2\mu_0^2 + M_V^2 - t} \right),\tag{8}
$$

where  $t = (p - p')^2$ . By using the Pomeron-photon analogy mentioned above, the form factor for the Pomeron-nucleon vertex is defined by the isoscalar electromagnetic form factor of the nucleon as

$$
F_1(t) = \frac{4M_N^2 - 2.8t}{(4M_N^2 - t)(1 - t/0.71)^2}.
$$
\n(9)

Here *t* is in units of GeV<sup>2</sup> and  $M_N$  is the proton mass. Note that the factor  $\frac{eM_V^2}{f_V}$  in Eq. [\(7\)](#page-2-0) implies a relation between the DL model and the VMD.

The crucial ingredient of the Regge phenomenology is the propagator  $G_{\mathbb{P}}$  for the Pomeron in Eq. [\(6\)](#page-2-0). It is of the following form:

$$
G_{\mathbb{P}} = \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1} \exp\left\{-\frac{i\pi}{2}[\alpha_P(t)-1]\right\},\qquad(10)
$$

where  $s = (q+p)^2 = W^2$ ,  $\alpha_P(t) = \alpha_0 + \alpha_P' t$ . By fitting the data of  $\rho^0$ ,  $\omega$ , and  $\phi$  photo-production [\[99\]](#page-12-0), the parameters of the model have been determined:  $\mu_0 = 1.1 \text{ GeV}^2$ ,  $\beta_{u/d} =$ 2.07 GeV<sup>-1</sup>,  $\beta_s = 1.6 \text{ GeV}^{-1}$ ,  $\alpha_0 = 1.08$ , and  $\alpha'_P = 1/s_0 = 1/s_0$ 0.25 GeV<sup>-2</sup>. In our previous paper [\[9\]](#page-11-0), we found that, with the same  $\mu_0^2$ ,  $\beta_{u/d}$ , and  $\alpha'_P$ , the  $J/\psi$  photoproduction data can be fitted by setting  $\beta_c = 0.84 \text{ GeV}^{-1}$  and choosing a larger  $\alpha_0 = 1.25$ . In the left side of Fig. 2, the results (black solid curves) from the constructed Pomeron-exchange model are compared with all of the the total cross section data of  $\gamma p \rightarrow J/\psi p$  up to invariant mass  $W = 300$  GeV. Here we note that the two-gluon (dotted curves) and three-gluon (dot-dashed curves) exchange models, with the parameters given in Refs. [\[97\]](#page-12-0), cannot describe the data above about  $W =$ 10 GeV. The new data from the JLab GlueX Collaboration are considerably larger in magnitude than the previous data, as



FIG. 3. The total cross section of  $\gamma p \rightarrow J/\psi + p$  via Pomeron exchange with the center-of-mass energy *W* in the near-threshold energy region. The experimental data are from Refs. [\[100–102\]](#page-12-0).

can be seen more clearly in the right side of Fig. 2. While these data can be better described by the two-gluon  $+$  three-gluon exchange model, as also shown by the GlueX Collaboration, they need further confirmation from separate experiments at JLab. Thus our study of  $N_{c\bar{c}}^*$  starts with Fig. 3 in which the data before 2018 are compared with the results calculated by using the Pomeron-exchange model.

# **B.** Excitation of  $N_{c\bar{c}}^*$  resonances

We focus on the  $N_{c\bar{c}}^*$  predicted by the meson-baryon coupled-channel models with the parameters constrained by the SU(4) symmetry and the fit to the meson-baryon reaction data. Alternatively,  $N_{c\bar{c}}^*$  can be predicted by constituent quark models or nonperturbative QCD models. These are, however, not considered in this work.

In Table  $II$ , we list the predictions from most, if not all, of the coupled-channel models of  $N_{c\bar{c}}^*$  in the literature. The relative importance of the predicted  $N_{c\bar{c}}^*$  in determining  $\gamma p \rightarrow J/\psi p$  can be estimated by using a well known relation between the total cross section  $\sigma^{(\text{tot})}$  at resonance energy  $W =$ *M<sub>R</sub>* and the partial decay widths  $\Gamma_{N_{cc}^*$ , *J*/ψ *p*,  $\sigma$  *N*<sup>\*</sup><sub>*cc*</sub>  $\rightarrow$  *J*/ψ *p*,  $\Gamma_{N_{cc}^*,\gamma p}$  of  $N_{c\bar{c}}^* \to \gamma p$ , and the total width  $\Gamma_{N^*,c\bar{c}}^{(\text{tot})}$ .

$$
\sigma^{\text{(tot)}}(W = M_R) = \frac{2J + 1}{4} \frac{4\pi}{q_R^2} \frac{\Gamma_{N_{cc}^*, J/\psi} p \Gamma_{N_{cc}^*, \gamma p}}{\left[\Gamma_{N_{cc}^*}^{\text{(tot)}}\right]^2} \qquad (11)
$$

where *J* is the spin of  $N_{c\bar{c}}^*$ , and  $q_R$  is defined by the resonance mass by  $M_R = \sqrt{M_N^2 + q_R^2} + q_R$ . We note here that, except for the model by Lin *et al.* [\[45\]](#page-11-0), the decay width  $\Gamma_{N_{cc}^*, \gamma p}$  to the  $\gamma p$  channel is not predicted by the models listed in Table [II.](#page-4-0) Thus the only way we can use these models is to use the VMD model to describe the excitation of  $N_{c\bar{c}}^*$  as the  $\gamma N \rightarrow$  $V N \rightarrow N_{c\bar{c}}^*$  mechanism with  $V = \rho, \omega J/\psi$ , as illustrated in Fig. [4.](#page-4-0)

In Table [II,](#page-4-0) we also see that the predicted  $N_{c\bar{c}}^*$  mainly decay into channels associated with the  $\bar{D}$  meson and charmed  $\Sigma_c$ baryons, specified as the main channel (MC) in the table. However, the available energy at JLab is not high enough to investigate the  $\gamma p \rightarrow \overline{D} \Sigma_c$  process. Instead the experiment

<span id="page-4-0"></span>TABLE II. The mass [*m* (MeV)], total width [ $\Gamma$  (MeV)], partial-decay widths to  $\gamma p$  [ $\Gamma_{p\gamma}$  (keV)], and the partial dacay widths [ $\Gamma_{MB}$  (MeV)] for  $MB = J/\psi N$ ,  $\rho N$ ,  $\omega N$ ,  $\bar{D}\Lambda_c$ , and  $\bar{D}^* \Lambda_c$  predicted by the available theoretical models are listed. MC denotes the main coupled channel predicted by the model of each reference given in the last column. The results for Nos. 19 and 20 are from the experimental data. The masses in Nos. 9, 10, 13, and 18 used in Ref. [\[42\]](#page-11-0) are from the experimental data. All other listed masses are from the theoretical models.

No.	${\cal J}^P$	m	$\Gamma$	$\Gamma_{J/\psi N}$	$\Gamma_{\rho N}$	$\Gamma_{\omega N}$	$\Gamma_{\bar{D}\Lambda_c}$	$\Gamma_{\bar{D}^*\Lambda_c}$	$\Gamma_{p\gamma}$	MC	Ref.
$\mathbf{1}$	$rac{1}{2}$	4262	35.6	10.3			0.01			$\bar{D}\Sigma_c$	$[17]$
$\sqrt{2}$		4308	7.1	1.2			0.02	1.4		$\bar{D}\Sigma_c$	$[103]$
3		4412	47.3	19.2	$3.2\,$	10.4				$\bar{D}^* \Sigma_c$	[11, 12]
$\overline{4}$		4410	58.9	52.5			0.8	0.7		$\bar{D}^* \Sigma_c$	$[17]$
5		4460	6.2	3.9			1.0	0.3		$\bar{D}^* \Sigma_c$	$[103]$
6		4481	57.8	14.3			1.02	0.3		$\bar{D}^* \Sigma_c^*$	$[17]$
7	$rac{3}{2}$	4334	38.8	38.0				$0.8\,$		$\bar{D}\Sigma_c^*$	$[17]$
$\,8\,$		4375	2.4	1.5				0.9		$\bar{D}\Sigma_c^*$	$[103]$
9		4380	144.3	3.8	1.4	5.3	1.2	131.3	0.7	$\bar{D}\Sigma_c^*$	$[42]$
10		4380	69.9	16.6	0.15	0.6	17.0	35.3		$\bar{D}^* \Sigma_c$	$[42]$
11		4412	47.3	19.2	3.2	10.4				$\bar{D}^* \Sigma_c$	[11, 12]
12		4417	8.2	4.6				3.1		$\bar{D}^* \Sigma_c$	$[17]$
13		4450	139.8	16.3	0.14	0.5	41.4	72.3		$\bar{D}^* \Sigma_c$	$[42]$
14		4450	21.7	0.03			1.4	6.8		$\bar{D}^* \Sigma_c$	$[48]$
15		4450	16.2	11			0.6	4.2		$\Psi' N$	$[48]$
16		4453	1.8	1.5				0.3		$\bar{D}\Sigma_c^*$	$[103]$
17		4481	34.7	32.8				1.2		$\bar{D}^* \Sigma_c^*$	$[17]$
18	$rac{5}{2}$	4450	46.4	4.0	0.3	$0.3\,$	18.8	20.5	1.13	$\bar{D}^* \Sigma_c$	$[42]$
19	$\frac{3}{2}^{-}$ , $\frac{5}{2}^{+}$	$4380^{\pm 8}_{\pm 29}$	$205^{\pm 18}_{\pm 86}$							Expt.	[21, 22]
20		4450 $\pm$ <sup>2</sup>	$39^{+5}_{+19}$							Expt.	[21, 22]

on the process  $\gamma p \rightarrow \bar{D}\Lambda_c$  may be possible. Thus we will also consider the  $\gamma p \rightarrow \bar{D}\Lambda_c$  reaction, which does not have a Pomeron-exchange mechanism. This can be studied using the models which also provide partial decay widths of  $N_{c\bar{c}}^* \rightarrow$  $\bar{D}\Lambda_c$ , as also shown in Table II.

To proceed, we recall that the VMD is defined by the following Lagrangian:

$$
L_{\text{VMD}}(x) = \frac{em_V^2}{f_V} A_\mu(x) \phi_V^\mu(x),\tag{12}
$$

where  $m_V$  is the mass of the vector meson *V*, and  $A_\mu$  and  $\phi_V^\mu$ are the field operators for the photon and vector meson, respectively. The width of  $V \rightarrow e^+e^-$  can then be calculated by

$$
\Gamma_{V \to e^+e^-} = \frac{1}{3} \alpha^2 m_V \frac{4\pi}{f_V^2}.
$$
\n(13)

By using the data of  $\Gamma_{V\to e^+e^-}$ , the decay constants of Eq. (12) can be determined:  $f_{\rho} = 5.33$ ,  $f_{\omega} = 15.2$ ,  $f_{\phi} = 13.4$ , and



FIG. 4. The diagram for  $\gamma p \rightarrow N_{c\bar{c}} \rightarrow J/\psi p$  with the VMD by  $\rho$ ,  $\omega$ , and *J*/ $\psi$  coupled with  $\gamma$ .

 $f_{J/\psi} = 11.2$ . For our later discussions, we here note that these coupling constants are determined at the photon four-momentum  $q^2 = m_V^2$ . Thus, in order to use the Lagrangian (12) in other processes with real photon  $q^2 = 0$ , a model must be used to account for the off-shell effects on these coupling constants. In our calculations, we thus will set

$$
\frac{1}{f_V} \to \frac{1}{f_V} F_V(q^2),\tag{14}
$$

$$
F_V(m_V^2) = 1. \tag{15}
$$

Ideally  $F_V(q^2)$  should be calculated from the quark-loop mechanism  $V \rightarrow q\bar{q} \rightarrow \gamma(q^2)$  within a nonperturbative QCD model. Here, we will determine it phenomenologically, as will be specified later.

With VMD, the  $\gamma N \to N_{c\bar{c}}^*$  amplitude can be calculated from  $\gamma N \to V_{VMD}N \to N_{c\bar{c}}^*$ , where  $V_{VMD} = \rho, \omega, J/\psi$ , and  $V_{VMD}N \rightarrow N_{c\bar{c}}^{*}$  is calculated from the considered mesonbaryon coupled-channel models of  $N_{c\bar{c}}^*$ . The full amplitude  $\gamma N \to N_{c\bar{c}}^* \to VN$  can then calculated by using  $N_{c\bar{c}}^* \to VN$ generated from the same coupled-channel models of  $N_{c\bar{c}}^*$ . In the following subsections, we will give formulas for calculating these amplitudes. With the calculated  $\Gamma_{N_{cc}^*,\gamma p}$  and the predicted widths  $\Gamma_{N_{cc}^*$ , *J*/ $\psi$  *p* and  $\Gamma_{N_{cc}^*}^{\text{(tot)}}$  listed in Table II, we then can use Eq. [\(11\)](#page-3-0) to estimate the predicted  $\sigma^{(\text{tot})}$  for each model and then select only the cases where the estimated  $\sigma^{\text{(tot)}}$  are close to the available data to make predictions.

<span id="page-5-0"></span>TABLE I[II.](#page-4-0) Coupling constants  $g_{2JV}$  and  $\tilde{g}_{2JV}$  are determined from fitting the partial decay widths listed in Table II. The number shown in the first column indicates that the results for the model are from using the widths given for the same number specified in Table [II.](#page-4-0)  $\Gamma_{\gamma p}$  are the partial decay widths calculated from  $\tilde{g}_{2JV}$  within VMD, as explained in the text. Thus the resulting values of  $\Gamma_{\gamma p}$  for models No. 9 and No. 18 are different from the values obtained in Ref. [\[42\]](#page-11-0) which used a different mechanism, as explained in the text.  $\sigma^{\text{(tot)}}$  is the total cross section of  $\gamma p \rightarrow J/\psi p$  calculated from using Eq. (20) by choosing  $\Lambda_V = 0.55$  GeV for the off-shell form factor  $F_V(q^2)$ .

No.	${\cal J}^P$	m	$\Gamma_{\text{tot}}$	$g_{2JV}$ $J/\psi p$	$\tilde{g}_{2JV}$ $J/\psi p$	$\tilde{g}_{2JV}$ $\rho p$	$\tilde{g}_{2JV}$ $\omega p$	$\Gamma_{p\gamma}$ (kev)	$\sigma^{\text{(tot)}}$ (nb)	MC	Ref.
-1	$rac{1}{2}$	4262	35.6	0.39	0.32			$3.9 \times 10^{-5}$	$1.9 \times 10^{-4}$	$\bar{D}\Sigma_c$	$[17]$
$\overline{2}$		4308	7.1	0.13	0.11			$4.5 \times 10^{-6}$	$5.9 \times 10^{-9}$	$\bar{D}\Sigma_c$	$[103]$
3		4412	47.3	0.46	0.38	0.078	0.14	1.14	5.4	$\bar{D}^* \Sigma_c$	[11, 12]
$\overline{4}$		4410	58.9	0.75	0.62			$1.5 \times 10^{-4}$	$1.3 \times 10^{-3}$	$\bar{D}^* \Sigma_c$	$[17]$
5		4460	6.2	0.20	0.16			$1.1 \times 10^{-5}$	$6.2 \times 10^{-4}$	$\bar{D}^* \Sigma_c$	$[103]$
6		4481	57.8	0.37	0.31			$3.8 \times 10^{-5}$	$8.8 \times 10^{-5}$	$\bar{D}^* \Sigma_c^*$	$[17]$
$\overline{7}$	$rac{3}{2}$	4334	38.8	1.19	0.98			$1.3 \times 10^{-4}$	$3.7 \times 10^{-3}$	$\bar{D}\Sigma_c^*$	$[17]$
8		4375	2.4	0.23	0.19			$4.6 \times 10^{-6}$	$1.4 \times 10^{-3}$	$\bar{D}\Sigma_c^*$	$[103]$
9		4380	144.3	0.36	0.30	0.090	0.17	0.53	0.11	$\bar{D}^* \Sigma_c$	$[42]$
10		4380	69.9	0.75	0.62	0.039	0.059	0.060	0.23	$\bar{D}^* \Sigma_c$	[11, 12]
11		4412	47.3	0.79	0.65	0.14	0.24	1.1	10.8	$\bar{D}^* \Sigma_c$	$[17]$
12		4417	8.2	0.39	0.32			$1.4 \times 10^{-5}$	$1.0 \times 10^{-3}$	$\bar{D}^* \Sigma_c$	$[42]$
13		4450	139.8	0.71	0.58	0.028	0.053	0.054	0.048	$\bar{D}^* \Sigma_c$	$[42]$
14		4450	21.7	0.030	0.025			$8.4 \times 10^{-8}$	$5.8 \times 10^{-9}$	$\bar{D}^* \Sigma_c$	$[48]$
15		4450	16.2	0.58	0.48			$3.1 \times 10^{-5}$	$1.4 \times 10^{-3}$	$\Psi' N$	$[48]$
16		4453	1.8	0.21	0.18			$4.2 \times 10^{-6}$	$2.2 \times 10^{-3}$	$\bar{D}\Sigma_c^*$	$[103]$
17		4481	34.7	0.98	0.81			$8.8 \times 10^{-5}$	$2.6 \times 10^{-3}$	$\bar{D}^* \Sigma_c^*$	$[17]$
18	$rac{5}{2}$	4450	46.4	0.35	0.27	0.016	0.016	$8.3 \times 10^{-2}$	0.25	$\bar{D}^* \Sigma_c$	$[42]$

# *1. The*  $N_{c\bar{c}}^* \rightarrow NV$  *transition amplitudes*

Following the formulation of Refs. [\[104,105\]](#page-12-0), the  $N_{c\bar{c}}^*(J^P, P) \to N(p_N) + V(p_V)$  transitions for spin-parity  $J^P = \frac{1}{2}^-$ ,  $\frac{3}{2}^-$ , and  $\frac{5}{2}^+$  can be written as

$$
\mathcal{M}_{N^*(\frac{1}{2}^-)NV}(P; p_V p_N) = \bar{u}_N(p_N)\gamma_5\tilde{\gamma}_\mu u_{N^*}(P)\epsilon_v^*(p_V)\bigg[g_{1V}g^{\mu\nu} - f_{1V}\bigg(\frac{3}{2}\frac{\tilde{r}^\mu \tilde{r}^\nu}{\tilde{r}^2} - \frac{1}{2}\tilde{g}_{N^*}^{\mu\nu}\bigg)\bigg],
$$
\n
$$
\mathcal{M}_{N^*(\frac{3}{2}^-)NV}(P; p_V p_N) = \bar{u}_N(p_N)u_{N^*}\mu(P)\epsilon_v^*(p_V)\bigg[g_{3V}g^{\mu\nu} - f_{3V}\bigg(\frac{3}{2}\frac{\tilde{r}^\mu \tilde{r}^\nu}{\tilde{r}^2} - \frac{1}{2}\tilde{g}_{N^*}^{\mu\nu}\bigg)\bigg]
$$
\n
$$
+ h_{3V}\epsilon_{\mu\nu\lambda\delta}\bar{u}_N\gamma_5\big(\tilde{\gamma}^\mu g_\alpha^\beta + \tilde{\gamma}_\alpha g^{\mu\beta}\big)u_{N^*}\beta\epsilon_v^*\big(\frac{\tilde{r}^\alpha \tilde{r}^\lambda}{\tilde{r}^2} - \frac{1}{3}\tilde{g}_{N^*}^{\alpha\lambda}\bigg)\tilde{P}^\delta,
$$
\n
$$
\mathcal{M}_{N^*(\frac{5}{2}^+)NV}(P; p_V p_N) = \bar{u}_N(p_N)u_{N^*}\mu_\nu(P)\epsilon_u^*(p_V)\bigg[\frac{g_{5V}}{m_N}g^{\alpha\mu}\tilde{r}^\nu - \frac{f_{5V}}{m_N}\bigg(\frac{5}{3}\frac{\tilde{r}^\mu \tilde{r}^\nu \tilde{r}^\alpha}{\tilde{r}^2} - \frac{1}{3}\bigg(\tilde{g}_{N^*}^{\mu\nu}\tilde{r}^\alpha + \tilde{g}_{N^*}^{\nu\alpha}\tilde{r}^\mu + \tilde{g}_{N^*}^{\alpha\mu}\tilde{r}^\nu\bigg)\bigg]
$$
\n(17)

$$
\mathcal{M}_{N^*(\frac{5}{2}^+)NV}(P; p_V p_N) = \bar{u}_N(p_N)u_{N^*\mu\nu}(P)\epsilon_\alpha^*(p_V)\left[\frac{g_{SV}}{m_N}g^{\alpha\mu}\tilde{r}^\nu - \frac{f_{SV}}{m_N}\left(\frac{5}{3}\frac{\tilde{r}^\mu\tilde{r}^\nu\tilde{r}^\alpha}{\tilde{r}^2} - \frac{1}{3}\left(\tilde{g}_{N^*}^{\mu\tilde{r}}\tilde{r}^\alpha + \tilde{g}_{N^*}^{\nu\alpha}\tilde{r}^\mu + \tilde{g}_{N^*}^{\nu\mu}\tilde{r}^\nu\right)\right)\right] + \frac{h_{SV}}{m_N}\epsilon_{\mu\nu\lambda\delta}\bar{u}_N\gamma_5(\tilde{r}^\mu g_{\xi\alpha}g_{\sigma\beta} + \tilde{\gamma}_{\xi}g_{\sigma\beta}g_{\mu\beta} + \tilde{\gamma}_{\sigma}g_{\mu\beta}g_{\xi\beta})u_{N^*}^{\alpha\beta}\epsilon_\nu^*\mu\left(\frac{\tilde{r}^\xi\tilde{r}^\lambda\tilde{r}^\sigma}{\tilde{r}^2} - \frac{1}{3}\left(\tilde{g}_{N^*}^{\xi\sigma}\tilde{r}^\lambda + \tilde{g}_{N^*}^{\sigma\lambda}\tilde{r}^\xi + \tilde{g}_{N^*}^{\lambda\xi}\tilde{r}^\sigma\right)\right)\hat{P}^\delta,
$$
\n(18)

where

$$
r^{\mu} = p_N^{\mu} - p_V^{mu}, \quad \tilde{g}_{N^*}^{\mu\nu} = g^{\mu\nu} - \frac{P^{\mu} P^{\nu}}{W^2}, \quad (19)
$$

$$
\tilde{r}^{\nu} = r_{\mu} \tilde{g}_{N^*}^{\mu \nu}, \quad \tilde{\gamma}^{\nu} = \gamma_{\mu} \tilde{g}_{N^*}^{\mu \nu}, \tag{20}
$$

$$
\hat{P}_{\mu} = P_{\mu}/W. \tag{21}
$$

The terms with coupling constants  $f_{2J}$  and  $h_{2J}$  are the contributions from higher partial waves. For simplicity, we neglect these terms and set  $f_{2JV} = h_{2JV} = 0$ . We thus can use

the partial decay width  $\Gamma_{N_{cc}^*,NV}$  listed in Table [II](#page-4-0) to determine the parameter  $g_{2JV}$  by the following formula:

$$
\Gamma_{N_{c\bar{c}}^*NV} = \frac{|p|}{8\pi m_{N_{c\bar{c}}}^2} \left| \mathcal{M}_{N_{c\bar{c}}^*NV} \right|^2, \tag{22}
$$

where  $|p|$  is on-shell momentum of the final state vector in the rest frame of  $N_{c\bar{c}}^*$ .

The determined  $g_{2JV}$  for  $V = J/\psi$  are listed in the fifth column of Table III.

# *2. The*  $N_{c\bar{c}}^* \to N\gamma$  *transition amplitudes*

<span id="page-6-0"></span>As illustrated in Fig. [4,](#page-4-0) we assume that the  $N_{c\bar{c}}^* \to N\gamma$ transition amplitudes can be calculated by the  $\gamma \rightarrow V$  transition defined by the VMD Lagrangian [\(12\)](#page-4-0), the propagator of *V*, and the  $VN \rightarrow N_{c\bar{c}}^*$  amplitudes defined in Eqs. [\(16\)](#page-5-0)– [\(21\)](#page-5-0). Since we can determine the parameters by using only one value of  $\Gamma_{N_{cc,\gamma N}^*}$  predicted by a  $N_{c\bar{c}}^*$  model, we need to

make a simplification. Here we also need to make sure that the simplified amplitudes are gauge invariant. We find that this can be accomplished by setting  $h_{2JV} = 0$ , like what we have chosen in determining  $\Gamma_{N_{c\bar{c}}^*} \to NV$ , but we need to keep the  $f_{2JV}$  term and set  $g_{2JV} = f_{2JV} = \tilde{g}_{2JV}$ . For example, the amplitude of  $N^*(\frac{1}{2}^-) \to N\rho \to N\gamma$  with the simplification  $h_{2JV} = 0$  is

$$
\mathcal{M}_{N^*(\frac{1}{2}^-) \to N\rho \to N\gamma}(P; q = p_\rho p_N) = \frac{ie}{f_\rho} \frac{-m_\rho^2}{q^2 - m_\rho^2 + i\Gamma_\rho m_\rho} \mathcal{M}^\nu(P; p_\rho p_N) \epsilon_\gamma^{*\nu}(q)
$$
(23)

with

$$
\mathcal{M}^{\nu}(P;q=p_{\rho} p_N) = \overline{u}_{N^*}(P)\gamma_5\tilde{\gamma}_{\mu}u_N(p_N)\bigg[g_{1\rho}g^{\mu\nu'} - f_{1\rho}\bigg(\frac{3}{2}\frac{\tilde{r}^{\mu}\tilde{r}^{\nu}}{\tilde{r}^2} - \frac{1}{2}\tilde{g}_{N^*}^{\mu\nu'}\bigg)\bigg]\tilde{g}_{\rho\nu'\nu}(q). \tag{24}
$$

Obviously this amplitude will be gauge invariant if  $\mathcal{M}^{\nu}q_{\nu} = 0$ . However, it is straightforward to show that  $\mathcal{M}^{\nu}q_{\nu} \sim (g_{1\rho} - g_{2\rho})$  $f_{1\rho}$   $\neq$  0. Therefore a simple way to have a gauge invariant amplitude is to set  $g_{2J}$   $\psi = f_{2J}$   $\psi = \tilde{g}_{2J}$ . This is part of the phenomenology and needs to be improved in the future. For our present limited and exploratory purpose, this simplification is sufficient.

By using Eqs. [\(16\)](#page-5-0)–[\(18\)](#page-5-0) and setting  $g_{2JV} = f_{2JV} = \tilde{g}_{2JV}$  and  $h_{2JV} = 0$ , we can then use Eq. [\(22\)](#page-5-0) to determine  $\tilde{g}_{2JV}$  by using the partial decay withs listed in Table [II.](#page-4-0) The resulting  $\tilde{g}_{2J}$  *y* are listed in the columns 6–8 of Table [III.](#page-5-0) Including the off-shell form factor  $F_V(q^2)$  according to Eq. [\(14\)](#page-4-0), we then get the following expressions for the  $N_{c\bar{c}}^* \to VN \to \gamma N$  transition amplitudes:

$$
\mathcal{M}_{N^*(\frac{1}{2}^-)\to NV\to Ny}(P;q\,p_N)=\frac{ie}{f_V}\frac{-m_V^2\tilde{g}_V}{-m_V^2+i\Gamma_Vm_V}\bar{u}_N(p_N)\gamma_5\tilde{\gamma}_\mu u_{N^*}(P)\bigg[\epsilon_{\gamma\,\nu}^*\bigg(g^{\mu\nu}-\frac{3}{2}\frac{\tilde{r}^\mu\tilde{r}^\nu}{\tilde{r}^2}+\frac{1}{2}\tilde{g}_{N^*}^{\mu\nu}\bigg)F_V(q^2)\bigg],\tag{25}
$$

$$
\mathcal{M}_{N^*(\frac{3}{2}^-)\to N V\to N \gamma}(P; q \, p_N) = \frac{ie}{f_V} \frac{-m_V^2 \tilde{g}_{3V}}{-m_V^2 + i\Gamma_V m_V} \bar{u}_N(p_N) u_{N^*\mu}(P) \bigg[ \epsilon_{\gamma \nu}^* \bigg( g^{\mu \nu} - \frac{3}{2} \frac{\tilde{r}^\mu \tilde{r}^\nu}{\tilde{r}^2} + \frac{1}{2} \tilde{g}_{N^*}^{\mu \nu} \bigg) F_V(q^2) \bigg],\tag{26}
$$

$$
\mathcal{M}_{N^*(\frac{5}{2}^+) \to NV \to N\gamma}(P; q \, p_N) = \frac{ie}{f_V} \frac{-m_V^2 \tilde{g}_{SV}/m_N}{-m_V^2 + i\Gamma_V m_V} \bar{u}_N(p_N) u_{N^*\mu\nu}(P) \epsilon_{\gamma \alpha}^* F_V(q^2) \left(g^{\alpha \mu} \tilde{r}^{\nu} - \frac{5}{3} \frac{\tilde{r}^{\mu} \tilde{r}^{\nu} \tilde{r}^{\alpha}}{\tilde{r}^2} + \frac{1}{3} \left(\tilde{g}_{N^*}^{\mu\nu} \tilde{r}^{\alpha} + \tilde{g}_{N^*}^{\nu\alpha} \tilde{r}^{\mu} + \tilde{g}_{N^*}^{\alpha \mu} \tilde{r}^{\nu}\right)\right),
$$
\n(27)

where  $p_V = q$  is used to evaluate  $\tilde{r}^{\nu}$  and  $\tilde{g}_{N^*}^{\alpha\mu}$  according to Eqs. [\(19\)](#page-5-0) and [\(20\)](#page-5-0). For the off-shell form factors, we assume

$$
F_V(q) = \frac{\Lambda_V^4}{\Lambda_V^4 + (q^2 - m_V^2)^2}.
$$
 (28)

Here we should emphasize that the above phenomenological form for the off-shell form factor is very unsatisfactory. One can see that, for a small value of  $\Lambda_V \sim m_\rho \sim m_\omega$ , the contribution from  $J/\Psi$  to  $F_V(q^2 = 0)$  is strongly suppressed. The  $J/\Psi$  contribution becomes significant only for large  $\Lambda_V \sim m_{J/\psi}$ . This feature is rather uncertain because it has no relation to the internal structure of the considered three vector mesons. Because of this limitation,  $F_V(q^2)$  resulting from our fit to the very specific total cross section data of  $\gamma p \rightarrow J/\Psi p$  can only be used with caution to apply VMD to investigate other photoproduction processes. Clearly our naive phenomenological approach needs to be investigated in the near future by using a QCD model. For example, one can try to relate the off-shell form factor  $F_V(q^2)$  to the QCD model calculations of the quark loop  $\gamma^* \to q\bar{q} \to$ *V* by extending the approach developed in Refs. [\[96,106\]](#page-12-0). This, however, is nontrivial and is beyond the scope of this work.

With the determined  $\tilde{g}_V$  listed in Table [III](#page-5-0) and a given choice of the cutoff  $\Lambda_V$ , we can use the  $N^* \to NV \to Ny$ amplitudes given in Eqs.  $(25)-(27)$  to calculate the decay width of  $N^* \to N\gamma$  within VMD:

$$
\Gamma_{N^* \to N\gamma} = \frac{1}{8\pi} \frac{k}{m_{N^*}^2} \left| \sum_{V} M_{N^* \to N V \to N\gamma} \right|^2.
$$
 (29)

The cutoff  $\Lambda_V$  is a parameter of the model. In Table [III](#page-5-0) we list the calculated  $\Gamma_{N^* \to N\gamma}$  for each model by setting  $\Lambda_V = 0.55$  GeV (the dependence on the value of  $\Lambda_V$  will be discussed in the next section). By using the partial decay widths listed in Tables [II](#page-4-0) and [III](#page-5-0) we can use Eq.  $(11)$  to estimate the total cross section  $\sigma^{(\text{tot})}$  of  $\gamma p \rightarrow J/\psi p$  at the resonance positions, as also given in the ninth column of Table [III.](#page-5-0)

# *3. The amplitude of*  $\gamma p \rightarrow N_{c\bar{c}}^* \rightarrow J/\psi p$

The amplitude  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  is shown in Fig. [4.](#page-4-0) By using the definition of vertexes of  $N^* \to NV$  as shown Eqs. [\(16\)](#page-5-0)–[\(18\)](#page-5-0) and  $N^* \rightarrow NV \rightarrow N\gamma$  as shown Eqs. (25)– (27), we can write the amplitude  $\mathcal{M}_{N^*}^{\mu\nu}(q, p, q', p')$ , which is

<span id="page-7-0"></span>defined in Eq. [\(4\)](#page-2-0):

$$
\mathcal{M}^{\mu\nu}_{N^*(\frac{1}{2}^-)}(q, p, q', p') = \sum_{V=J/\psi, \rho, \omega} g_{1J/\psi} \gamma_5 \tilde{\gamma}_\alpha \tilde{g}^{\alpha\mu}(q) \frac{q+p+m_{N^*_{c\bar{c}}}}{W^2 - m_{N^*_{c\bar{c}}}} + i\Gamma_{N^*_{c\bar{c}}} m_{N^*_{c\bar{c}}} F_V(0) \frac{ie}{f_V} \frac{-m_V^2 \tilde{g}_{1V}}{-m_V^2 + i\Gamma_V m_V} \gamma_5 \tilde{\gamma}_\beta \left( g^{\beta\nu} - \frac{3}{2} \frac{\tilde{r}^\beta \tilde{r}^\nu}{\tilde{r}^2} + \frac{1}{2} \tilde{g}^{\beta\nu}_{N^*} \right),
$$
\n(30)

$$
\mathcal{M}^{\mu\nu}_{N^*(\frac{3}{2}^-)}(q, p, q', p') = \sum_{V=J/\psi, \rho, \omega} g_{3J/\psi} g^{\mu\alpha} \frac{\left(q + p + m_{N^*_{c\bar{c}}}\right) P_{\alpha\beta}^{\frac{3}{2}}(p+q)}{W^2 - m_{N^*_{c\bar{c}}^*}^2 + i\Gamma_{N^*_{c\bar{c}}} m_{N^*_{c\bar{c}}}} F_V(0) \frac{ie}{f_V} \frac{-m_V^2 \tilde{g}_{3V}}{-m_V^2 + i\Gamma_V m_V} \left(g^{\beta\nu} - \frac{3}{2} \frac{\tilde{r}^\beta \tilde{r}^\nu}{\tilde{r}^2} + \frac{1}{2} \tilde{g}_{N^*}^{\beta\nu}\right), \quad (31)
$$

$$
\mathcal{M}^{\mu\nu}_{N^*(\frac{5}{2}^+)}(q, p, q', p') = \sum_{V=J/\psi, \rho, \omega} \frac{85J/\psi}{m_N} g^{\mu\alpha} \tilde{r}^{\beta} \frac{(q+p+m_{N_{cc}^*}) P_{\alpha\beta\alpha'\beta'}^{\frac{5}{2}}(p+q)}{W^2 - m_{N_{cc}^*}^2 + i\Gamma_{N_{cc}^*} m_{N_{cc}^*}} F_V(0) \frac{i e}{f_V} \frac{-m_V^2 \tilde{g}_{SV}/m_N}{-m_V^2 + i\Gamma_V m_V} \times \left( g^{\nu\alpha'} \tilde{r}^{\beta'} - \frac{5}{3} \frac{\tilde{r}^{\nu} \tilde{r}^{\alpha'} \tilde{r}^{\beta'}}{\tilde{r}^2} + \frac{1}{3} \left( \tilde{g}_{N^*}^{\nu\alpha'} \tilde{r}^{\beta'} + \tilde{g}_{N^*}^{\nu\beta'} \tilde{r}^{\alpha'} + \tilde{g}_{N^*}^{\alpha'\beta'} \tilde{r}^{\nu} \right) \right),
$$
\n(32)

where  $P_{\alpha\beta}^{\frac{3}{2}}(p)$  and  $P_{\alpha\beta\alpha'\beta'}^{\frac{5}{2}}(p)$  are the Lorentz structure functions of propagators of 3/2 and 5/2 particles, respectively. Their formulas are  $[105]$ 

$$
P_{\alpha\beta}^{\frac{3}{2}}(p) = -g_{\alpha\beta} + \frac{1}{3}\gamma_{\mu}\gamma_{\nu} + \frac{2}{3}\frac{p_{\mu}p_{\nu}}{m_{N^*}^2} + \frac{1}{3m_{N^*}}(\gamma_{\mu}p_{\nu} - \gamma_{\nu}p_{\mu}),
$$
\n(33)

$$
P^{\frac{5}{2}\alpha\beta\alpha'\beta'}(p) = \frac{1}{2} \left( \tilde{g}_{N^*}^{\alpha\alpha'} \tilde{g}_{N^*}^{\beta\beta'} + \tilde{g}_{N^*}^{\alpha\beta'} \tilde{g}_{N^*}^{\beta\alpha'} \right) - \frac{1}{5} \tilde{g}_{N^*}^{\alpha\beta} \tilde{g}_{N^*}^{\alpha'\beta'} - \frac{1}{10} \left( \tilde{\gamma}^{\alpha} \tilde{\gamma}^{\alpha'} \tilde{g}_{N^*}^{\beta\beta'} + \tilde{\gamma}^{\alpha} \tilde{\gamma}^{\beta'} \tilde{g}_{N^*}^{\beta\alpha'} + \tilde{\gamma}^{\beta} \tilde{\gamma}^{\alpha'} \tilde{g}_{N^*}^{\alpha\beta'} + \tilde{\gamma}^{\beta} \tilde{\gamma}^{\beta'} \tilde{g}_{N^*}^{\alpha\alpha'} \right). \tag{34}
$$

# **III.** PREDICTIONS FOR  $\gamma p \rightarrow J/\psi p$

In this section, we will first use the available total cross section data to fix the cutoff parameter  $\Lambda_V$  of the off-shell form factor Eq. [\(28\)](#page-6-0) of the  $\gamma p \to N_{c\bar{c}}^*$  amplitude. We then make predictions for using differential cross sections for identifying the  $N_{c\bar{c}}^*$  from future experimental data.

#### **A. Total cross section**

From Fig. [3,](#page-3-0) we see that the available data for  $\gamma p \rightarrow$  $J/\psi$  *p* in the near-threshold region are below about 0.8 nb and have some structure which may be due to the experimental uncertainties, but may be due to the  $N_{c\bar{c}}^*$  excitations. In this section we will make predictions for investigating the extent to which these available data can accommodate the  $N_{c\bar{c}}^*$  exci-tations predicted by the models listed in Tables [I](#page-1-0) and  $II$ . In particular, we are interested in the predictions of Ref. [\[42\]](#page-11-0), since this is the only model which predicts the partial decay width to the  $\gamma p$  channel for the  $\frac{3}{2}$ <sup>-</sup>(4380) and  $\frac{5}{2}$ <sup>+</sup>(4450) states. The triangular mechanism they used for  $\gamma N \to N_{c\bar{c}}^*(4380)$  is similar to our model based on VMD, but for  $N_{c\bar{c}}^*(4450)$  they are different by one magnitude order.

Our first step is to determine the cutoff parameter  $\Lambda_V$  of the off-shell form factor Eq. [\(28\)](#page-6-0). To compare with the results of Ref. [\[42\]](#page-11-0), we perform calculations including only  $\frac{3}{2}$  (4380) and  $\frac{5}{2}^{+}(4450)$  using the parameters (No. 9 and No. 18 of Ref. [\[42\]](#page-11-0)) listed in Table [III.](#page-5-0) We find that the calculated total cross sections can be close to the available data shown in Fig. [3](#page-3-0) if we choose the cutoff in the range of  $500 \le \Lambda_V \le 650$ MeV. In Fig.  $5(a)$ , we see that the choice  $\Lambda_V = 550$  MeV gives results within the uncertainties of the available data.

The structure of the solid curve at  $W \approx 4.35$  GeV is due to the interference between the Pomeron-exchange amplitude (dotted curve) and the resonant amplitude (long dashed curve). Furthermore, we also see that the resonant amplitude is dominated by the  $\frac{5}{2}$ <sup>+</sup>(4450) state, as shown in Fig. 5(b).

With the same cutoff  $\Lambda_V = 550$  MeV, we then calculate  $\Gamma_{N_{cc}^* \to \gamma N}$  for all states, as listed in Table [III.](#page-5-0) With the widths given in Table [III,](#page-5-0) we then estimate the total cross sections of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  by using Eq. [\(11\)](#page-3-0) for all models. We can see in the last column of Table [III](#page-5-0) that, except for the  $\frac{3}{2}$  (4380) and  $\frac{5}{2}^+(4450)$  states of Ref. [\[42\]](#page-11-0), all of the estimated total cross sections are either too large or too small compared



FIG. 5. The total cross sections of  $\gamma p \rightarrow J/\psi p$  plotted vs the invariant mass of  $\gamma p$ . (a) The red dotted and blue dashed curves are the contributions from the Pomeron exchange and  $N_{c\bar{c}}^*$  with  $J^p = 3/2^-$  and  $5/2^+$  with  $\Lambda_V = 550$  MeV, respectively. The black solid line is for the coherent summation of all the above three contributions. (b) The red dotted and blue dashed curves are the contributions of  $N_{c\bar{c}}^*$  with  $J^p = 3/2^-$  and  $5/2^+$  with  $\Lambda_V = 550$  MeV, respectively. The solid black line is the coherent summation of two  $N$ <sup>∗</sup> $c\bar{c}$  and it is the same as the blue dashed curve in panel (a) here. The experimental data are from Refs. [\[100,101\]](#page-12-0).

<span id="page-8-0"></span>TABLE IV. Fitting the partial decay widths of the states included in our predictions.  $\Gamma_{\gamma p}$  are partial decay widths calculated from  $\tilde{g}_V$ within VMD, as explained in the text.  $\sigma^{(tot)}$  is the total cross section of  $\gamma p \rightarrow J/\psi p$  calculated from using Eq. [\(20\)](#page-5-0) by choosing  $\Lambda_V =$ 0.55 GeV for the off-shell form factor  $F_V(q^2)$ .

		No. $J^P$ m $\Gamma_{\text{tot}}$ $\Gamma_{J/\psi_P}$ $\Gamma_{p\gamma}$ (kev) $\sigma^{\text{(tot)}}$ (nb) Ref.		
		9 $\frac{3}{2}$ 4380 144.3 3.8 0.53 0.11 This work		
		4380 144.3 3.8 0.70 0.15 [42]		
		$18 \frac{5}{2}$ 4450 46.4 4.0 0.083	0.25	This work
		4450 46.4 4.0 1.13 3.4 [42]		

with the value  $\approx 0.5$  nb of the available data shown in Fig. [5.](#page-7-0)

In Table IV, we compare our results for  $\Gamma_{\gamma p}$  and  $\sigma^{\text{(tot)}}$  of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  with those of Ref. [\[42\]](#page-11-0). Here we see that our result for the  $\frac{5}{2}^+$ (4450) state is much smaller than theirs. The differences between this work and Ref. [\[42\]](#page-11-0) are from using rather different mechanisms to evaluate  $\gamma N \to N_{c\bar{c}}^*$ . It is therefore useful to examine how our predictions depend on the parameters of our model based on VMD. We first examine the contribution from each of the intermediate vector mesons, illustrated in Fig. [4,](#page-4-0) to the calculated total cross sections of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$ . Our results from including the  $J = \frac{3}{2}$  (4380) and  $\frac{5}{2}$  + (4450) states in the calculation are shown in Fig. 6. Clearly the intermediate  $\rho$  gives the largest contribution, and  $J/\psi$  is negligible. This can be understood from the employed off-shell form factor [\(28\)](#page-6-0) which depends on the mass of the intermediate vector meson. This is also the reason why the cross sections predicted by the models without ρ*p* channel listed in Table [III](#page-5-0) are extremely small.

#### **B. Differential cross sections**

In Fig. [5,](#page-7-0) we see that the feature of  $N_{c\bar{c}}^*$  excitation in the total cross section is not so pronounced because it interferes with the background from the Pomeron-exchange amplitude, which is very large in all energy regions. To extract the peak of  $N_{c\bar{c}}^*$ , we need to find other observables which are



FIG. 6. The cross sections of  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  plotted for different  $J^P$  of  $N_{c\bar{c}}^*$  vs the invariant mass of  $\gamma p$ . The orange dashed, blue dotted, and red dashed-dotted lines are the contributions purely from the VMD by  $\rho$ ,  $\omega$ , and  $J/\psi$  coupled with  $\gamma$ , respectively. The black solid lines are for the coherent summation of  $\rho$ ,  $\omega$ , and  $J/\psi$ contributions.



FIG. 7. The differential cross sections of  $\gamma p \rightarrow J/\psi p$  plotted vs the angle of the outgoing  $J/\psi$  at invariant mass of  $\gamma pW = 4.45$  GeV. The red dotted, blue dashed lines are the contributions purely from the Pomeron and  $N_{c\bar{c}}^*$  with  $\Lambda_V = 0.55$  GeV, respectively. The black solid line is for the coherent summation of all contributions.

not dominated by the Pomeron exchange. It is noticed that the Pomeron exchange is strongly suppressed with large *t* in Eq. [\(10\)](#page-3-0). In other words, the Pomeron exchange mainly contributes to the cross sections at forward angles. This is illustrated in Fig. 7. It is then clear that the resonance peaks will be easier to observe at large angles. This is illustrated in Fig. [8.](#page-9-0) At 60 $^{\circ}$ , the shoulder due to  $N_{c\bar{c}}^{*}(4380)$  shows up more clearly. However, the magnitudes of the differential cross sections decrease rather rapidly with angles. Thus the measurement around 30◦ may be optimal in examining the role of  $N_{c\bar{c}}^*$ .

# **IV.** PREDICTION ON  $\gamma p \to \bar{D}^0 \Lambda_c^+, \bar{D}^{*0} \Lambda_c^+$

It is important to note that the Pomeron-exchange amplitude is still dominant in determining the  $J/\psi$  production in the considered low energy region. Therefore it is interesting to test the VMD model of  $\gamma N \to N_{c\bar{c}}^*$  using other reactions which do not have the Pomeron-exchange mechanism and in the low energy region accessible to experiments measuring  $J/\psi$  production at JLab. With the  $N_{c\bar{c}}^*$  models No. 6, No. 9, and No. 18 selected from Table [II](#page-4-0) and listed in Table V, the reaction  $\gamma p \to \bar{D}^* {}^0 \Lambda_c^+, \bar{D}^0 \Lambda_c^+$  can be used for this purpose. In addition to calculating the  $\gamma p \to N_{c\bar{c}}^* \to \bar{D}^{*0} \Lambda_c^+$ ,  $\bar{D}^0 \Lambda_c^+$ amplitude, we also need to consider the meson-exchange mechanisms due to the  $\bar{D}^{*0} \rightarrow \bar{D}^0 \gamma$  process. We thus need to calculate the amplitudes of the two mechanisms shown in Fig. [9.](#page-9-0)

TABLE V. Partial decay widths use to calculate the coupling of  $g_{N^* \to \bar{D}\Lambda_c}$  and  $g_{N^* \to \bar{D}^* \Lambda_c}$ .

			No. $J^P$ m $\Gamma_{\bar{D}\Lambda_c}$ $g_{N^*\to\bar{D}\Lambda_c}$ $\Gamma_{\bar{D}^*\Lambda_c}$ $g_{N^*\to\bar{D}^*\Lambda_c}$ Ref.	
			6 $\frac{1}{2}$ 4481 1.02 0.40 0.3 0.043 [17]	
			9 $\frac{3}{2}$ 4380 1.2 1.29 131.3 1.90 [42]	
			18 $\frac{5}{2}$ 4450 18.8 13.39 20.5 2.18 [42]	

<span id="page-9-0"></span>

FIG. 8. The differential cross sections of  $\gamma p \rightarrow J/\psi p$  plotted vs the invariant mass of  $\gamma p$  at three fixed angles of outgoing  $J/\psi$ . The red dotted and blue dashed lines are the contributions purely from the Pomeron exchange and  $N_{c\bar{c}}^*$  with  $\Lambda_V = 0.55$  GeV, respectively. The black solid line is for the coherent summation of all contributions.

#### **A. meson-exchange amplitude**

The meson-exchange amplitudes shown in Fig. 9(b) can be calculated by using  $D^*D\gamma$ ,  $pD^{*0}\Lambda_c^+$ , and  $pD^0\Lambda_c^+$  vertices defined as follows:

$$
\mathcal{M}_{D^*D\gamma} = \frac{g_{D^{*0}D^0\gamma}}{\sqrt{m_{D^*}m_D}} \epsilon^{\mu\nu\alpha\beta} P_{\gamma\mu} \epsilon_{\gamma\nu} P_{D^*\alpha} \epsilon_{D^*\nu}, \qquad (35)
$$

$$
\mathcal{M}_{NDA_c^+} = g_{NDA_c^+} \bar{u}_{\Lambda_c^+} \gamma_5 u_N, \tag{36}
$$

$$
\mathcal{M}_{ND^*\Lambda_c^+} = g_{ND^*\Lambda_c^+} \bar{u}_{\Lambda_c^+} \gamma_\mu \epsilon_{D^*}^\mu u_N,\tag{37}
$$

where the coupling  $g_{D^{*0}D^{0}\gamma} = 1.07$  is calculated from the partial decay width of  $D^{*0} \to D^0 \gamma$  which is estimated from the measured ratio of widths  $\Gamma_{D^{*0}\to D^{0}\gamma}/\Gamma_{D^{*0}\to D^{0}\pi^{0}}$ , with  $\Gamma_{D^{*0}\to D^{0}\pi^0}$  obtained from the data of  $\Gamma_{D^{*+}\to D^{+}\pi^0}$  by using isospin. By using  $SU(4)$  symmetry  $[107]$ , the coupling constants in Eqs. (36) and (37) can be determined:  $g_{ND\Lambda_c^+}$  =  $-\frac{3\sqrt{3}}{5}g_{BBP}$  and  $g_{ND^*\Lambda_c^+} = -\sqrt{3}g_{BBV}$ , where  $g_{BBP} = 0.989$  and  $g_{BBV} = 3.25$ . Then the amplitude  $j_{D^* m'_s, m_s}^v(q, p, q', p')$ , de-fined in Eq. [\(3\)](#page-2-0), for  $\gamma p \to \bar{D}^0 \Lambda_c^+$  due to  $\bar{D}^{*0}$  exchange can be written as

$$
j_{D^*m'_s,m_s}^v(q,p,q',p') = \frac{g_{D^*D\gamma}g_{ND^*A_c^+}}{\sqrt{m_{D^*}m_D}} \epsilon^{\mu\nu\alpha\beta} \frac{q_{\mu}q'_{\alpha}\bar{u}_{A_c^+}(p',m'_s)\gamma_{\beta}\mu_p(p,m_s)}{(q'-q)^2 - m_{D^*}^2} F_{D^*}(q'-q). \tag{38}
$$

Similarly, the  $\bar{D}^0$ -exchange amplitude for  $\gamma p \to \bar{D}^*{}^0 \Lambda_c^+$  is

$$
j_{Dm'_s,m_s}^v(q,p,q',p') = \frac{g_{D^*D\gamma}g_{N D\Lambda_c^+}}{\sqrt{m_{D^*}m_D}} \epsilon^{\mu\nu\alpha\beta} \frac{q_{\mu}q'_{\alpha}\epsilon_{D^* \beta} \bar{u}_{\Lambda_c^+}(p',m'_s)\gamma_5 u_p(p,m_s)}{(q'-q)^2 - m_{D^*}^2} F_D(q'-q). \tag{39}
$$

# **B.** *N***<sup>∗</sup>** *cc***¯-excitation amplitudes**

The formulas for calculating the resonant amplitude  $\gamma N \to N_c^* \to \Lambda_c^+ \bar{D}^*$  are the same as Eqs. [\(30\)](#page-7-0)–[\(32\)](#page-7-0) except that the coupling constants  $g_{2J J/\psi}$  for  $J = 1/2, 3/2, 5/2$  are replaced by  $g_{N^* \to \bar{D}\Lambda_V}$  for each *J* listed in Table [V.](#page-8-0) For the  $\gamma N \to N_{c\bar{c}}^* \to \Lambda_c^+ \bar{D}$ , we define  $N_{c\bar{c}}^* \Lambda_c^+ \bar{D}$  vertices as follows:

$$
\mathcal{M}_{N^*(\frac{1}{2}^-)\Lambda_c^+\bar{D}} = g_1 \bar{u}_{\Lambda_c^+} u_{N^*},\tag{40}
$$

$$
\mathcal{M}_{N^*(\frac{3}{2}^-)\Lambda_c^+ \bar{D}} = \frac{g_3}{m_D^2} \bar{u}_{\Lambda_c^+} \gamma_5 \gamma_\mu u_{N^* \nu} \, p_{\bar{D}}^\mu \, p_{\bar{D}}^\nu,\tag{41}
$$

$$
\mathcal{M}_{N^*(\frac{5}{2}^+) \Lambda_c^+ \bar{D}} = \frac{g_5}{m_D^3} \bar{u}_{\Lambda_c^+} \gamma_5 \gamma_\mu u_{N^* \nu \lambda} \, p_D^\mu \, p_{\bar{D}}^\nu \, p_{\bar{D}}^\lambda, \tag{42}
$$

where  $p_{\bar{D}}$  is the four-momentum of  $\bar{D}$  meson. The coupling can be calculated from the partial decay widths listed in No. 6, No. 9, and No. 18 of Table [V.](#page-8-0) We then get  $g_1 = 0.40$ ,  $g_3 = 1.29$ , and  $g_5 = 13.39$ .



FIG. 9. Diagrams (a) for  $\gamma p \to N_{c\bar{c}} \to \bar{D}^0(\bar{D}^{*0})\Lambda_c^+$  with the VMD using  $\rho$ ,  $\omega$ , and  $J/\psi$  coupled with  $\gamma$ , and (b) for  $\gamma p \to \bar{D}^0(\bar{D}^{*0})\Lambda_c^+$  by exchanging  $D^0(D^{*0})$ .

With the above equations and  $\gamma p \to N_{c\bar{c}}^*$  given in Table [III,](#page-5-0) we can calculate the amplitude for  $\gamma p \to N_{c\bar{c}}^* \to \bar{D}^0 \Lambda_c^+$  and obtain the corresponding current matrix element  $j^{\nu}_{N^* m'_s, m_s}(q, p, q', p')$  [defined in Eq. [\(3\)](#page-2-0)] as

$$
j_{N^{*}(\frac{1}{2}^{-})m'_{s},m_{s}}^{v}(q, p, q', p') = \sum_{V=J/\psi,\rho,\omega} g_{1} \frac{\bar{u}_{\Lambda_{c}^{+}}(p', m'_{s})(q+p+m_{N_{cc}^{*}})u_{p}(p, m_{s})}{W^{2}-m_{N_{cc}^{*}}^{2}+i\Gamma_{N_{cc}^{*}}m_{N_{cc}^{*}}}F_{V}(0)\frac{ie}{f_{V}}\frac{-m_{V}^{2}\tilde{g}_{1V}}{-m_{V}^{2}+i\Gamma_{V}m_{V}}\gamma_{5}\tilde{\gamma}_{\beta}\left(g^{\beta\nu}-\frac{3}{2}\frac{\tilde{r}^{\beta}\tilde{r}^{\nu}}{\tilde{r}^{2}}+\frac{1}{2}\tilde{g}_{N^{*}}^{\beta\nu}\right),\tag{43}
$$

$$
j_{N^{*}(\frac{3}{2}^{-})m'_{s},m_{s}}^{v}(q, p, q', p') = \sum_{V=J/\psi,\rho,\omega} \frac{g_{3}}{m_{D}^{2}} \frac{\bar{u}_{\Lambda_{c}^{+}}(p', m'_{s})\gamma_{5}q'(q+p+m_{N_{cc}^{*}})P_{\alpha\beta}^{\frac{3}{2}}(p+q)u_{p}(p, m_{s})}{W^{2}-m_{N_{cc}^{*}}^{2}+i\Gamma_{N_{cc}^{*}}m_{N_{cc}^{*}}}
$$

$$
\times q^{\prime\alpha}F_{V}(0)\frac{ie}{f_{V}-m_{V}^{2}+i\Gamma_{V}m_{V}}\left(g^{\beta\nu}-\frac{3}{2}\frac{\tilde{r}^{\beta}\tilde{r}^{\nu}}{\tilde{r}^{2}}+\frac{1}{2}\tilde{g}_{N^{*}}^{\beta\nu}\right),\tag{44}
$$

$$
j_{N^{*}(\frac{5}{2}^{+})m'_{s},m_{s}}^{v}(q, p, q', p') = \sum_{V=J/\psi, \rho, \omega} \frac{g_{5}}{m_{D}^{3}} \frac{\bar{u}_{\Lambda_{c}^{+}}(p', m'_{s})\gamma_{5}q'(q+p+m_{N_{cc}^{*}})P_{\alpha\beta\alpha'\beta'}^{\frac{5}{2}}(p+q)u_{p}(p, m_{s})}{W^{2}-m_{N_{cc}^{*}}^{2}+i\Gamma_{N_{cc}^{*}}m_{N_{cc}^{*}}}\right)q'^{\alpha}q'^{\beta}F_{V}(0)\frac{ie}{f_{V}}\frac{-m_{V}^{2}\tilde{g}_{SV}/m_{N}}{-m_{V}^{2}+i\Gamma_{V}m_{V}} \times \left(g^{\nu\alpha'}\tilde{r}^{\beta'}-\frac{5}{3}\frac{\tilde{r}^{\nu}\tilde{r}^{\alpha'}\tilde{r}^{\beta'}}{\tilde{r}^{2}}+\frac{1}{3}\left(\tilde{g}_{N^{*}}^{\nu\alpha'}\tilde{r}^{\beta'}+\tilde{g}_{N^{*}}^{\nu\beta'}\tilde{r}^{\alpha'}+\tilde{g}_{N^{*}}^{\alpha'\beta'}\tilde{r}^{\nu}\right)\right),
$$
\n(45)

#### **C. Predictions of total cross sections**

The predicted total cross sections of  $\gamma p \to N_{c\bar{c}}^* \to$  $\bar{D}^0(\bar{D}^{*0})\Lambda_c^+$  are shown in Fig. 10. All calculations are done with cutoff  $\Lambda_V = 550$  MeV, as determined in the previous sections for  $J/\psi$  production. We first find that the meson-exchange contributions (pink dotted ) to the predicted total cross section of  $\gamma p \to N_{c\bar{c}}^* \to \bar{D}^0(\bar{D}^{*0})\Lambda_c^+$  are very weak. The contribution from  $N_{c\bar{c}}^*(\frac{3}{2}^-)$  (blue dashed) is larger than that of  $N_{c\bar{c}}^*(\frac{5}{2}^-)$  (orange dashed-dotted). Clearly, if the predicted cross section given in Fig. 10 can be measured, it will provide an additional test of the prediction of the  $N_{c\bar{c}}^*(\frac{3}{2}^-)$  state. Hopefully such measurements can be made in the near future as an additional test of our prediction on  $J/\psi$  production, presented in the previous section.



FIG. 10. The total cross sections of  $\gamma p \to \bar{D}^0 \Lambda_c^+$  (a) and  $\gamma p \to$  $\bar{D}^{*0}\Lambda_c^+$  (b) plotted vs the invariant mass of  $\gamma p$ . The pink dotted, blue dashed, and orange dotted-dashed lines are the contributions purely from the  $N^*_{c\bar{c}(\frac{3}{2})}$ ,  $N^*_{c\bar{c}(\frac{5}{2})}$ , and the background with  $\Lambda_V = 0.55$  GeV, respectively. The black solid thick lines are the coherent summations of all contributions with background  $\Lambda_V = 0.55$  GeV.

### **V. SUMMARY**

By using the predictions from the available meson-baryon coupled-channel models, we have investigated the excitations of nucleon resonances with hidden charm,  $N_{c\bar{c}}^*$ , in  $\gamma p$  reactions. For the  $\gamma p \rightarrow J/\psi p$  process, the Pomeron-exchange model of Donnachie and Landshoff, with the parameters determined from fitting the available total cross section data up to  $W = 300$  GeV, is used to calculate the nonresonant amplitudes. The resonant  $\gamma p \to N_{c\bar{c}}^* \to J/\psi p$  amplitudes are calculated by using (1) the partial decay widths predicted by the considered meson-baryon coupled-channel models to evaluate the  $N_{c\bar{c}}^* \to MB$  transition matrix elements, and (2) the vector meson dominance (VMD) model to evaluate  $\gamma p \to N_{c\bar{c}}^*$ as  $\gamma p \rightarrow V p \rightarrow N_{c\bar{c}}^*$  with  $V = \rho$ ,  $\omega$ ,  $J/\psi$ . The predictions from adding these two amplitudes then depend on an offshell form factor  $F_V(q^2) = \lambda^4/[\Lambda_V^4 + (q^2 - m_V^2)^2]$ , which is needed to account for the  $q^2$  dependence of the VMD model. We find that with  $\Lambda_V = 0.55$  GeV the predicted total cross sections of  $\gamma p \rightarrow J/\psi p$  are within the range of the available data in the energy region near the  $J/\psi$  production threshold. We then demonstrate that the  $N_{c\bar{c}}^*$  can be most easily identified in the differential cross sections at large angles where the contribution from Pomeron exchange becomes negligible.

With the same VMD model and the same coupled-channel model of  $N_{c\bar{c}}^*$ , we then predict the cross sections of  $\gamma p \to \bar{c}$  $\bar{D}^0 \Lambda_c^+ (\bar{D}^{*0} \Lambda_c^+)$ . We suggest that experiments on these reactions can be more effective to study  $N_{c\bar{c}}^*$  since their nonresonant amplitudes, due to the exchange of  $\bar{D}^{*0}$  ( $\bar{D}^{0}$ ), are found to be very weak.

The most unsatisfactory aspect of this work is the phenomenological determination of the off-shell form factor  $F_V(q^2)$ . It is determined by only using the data of total cross sections of  $\gamma p \rightarrow J/\psi p$  near the threshold, shown in Fig. [3.](#page-3-0) While our predictions could be used as a first step to determine whether the  $N_{c\bar{c}}^*$  predicted by the available meson-baryon coupled-channel models can be found in the new data from

<span id="page-11-0"></span>JLab, it is necessary to develop a more fundamental approach to also predict  $F_V(q^2)$  from QCD models. Obviously, such an improvement is necessary for using the  $q^2$  dependence of the  $J/\psi$  electroproduction cross section data to investigate nucleon resonances with hidden charm.

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