Role of baryon number conservation in measurements of fluctuations

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I discuss the role and impact of net-baryon number conservation in measurements of net-proton fluctuations in heavy-ion collisions. I show that the magnitude of the fluctuations is entirely determined by the strength of two particle correlations. At LHC and top RHIC energy, this implies the fluctuations are proportional to the integral of the balance function (BF), $B^{p\bar{p}}$ of protons and antiprotons, while in the context of the RHIC beam energy scan (BES), one must also account for correlations of stopped protons. The integral of $B^{p\bar{p}}$ measured in a 4π detector depends on the relative cross sections of processes yielding $p\bar{p}$ and those balancing the proton baryon number via the production of other antibaryons. The accepted integral of $B^{p\bar{p}}$ further depends on the shape and width of the BF relative to the width of the acceptance. The magnitude of the measured second-order cumulant of net-proton fluctuations thus has much less to do with QCD susceptibilities than with the creation/transport of baryons and antibaryons in heavy-ion collisions, and most particularly the impact of radial flow on the width of the BF. I thus advocate that net-proton fluctuations should be studied by means of differential BF measurements rather than with integral correlators. I also derive an expression of net-baryon fluctuations in terms of integrals of balance functions of identified baryon pairs and argue that measurements of such balance functions would enable a better understanding of the collision system expansion dynamics, the hadronization chemistry, and an experimental assessment of the strength of net-baryon fluctuations.

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I. INTRODUCTION

Lattice QCD (LQCD) calculations with physical quark masses suggest that at RHIC top energy and LHC energy, the matter produced in heavy-ion collisions consists of a state of matter known as quark gluon plasma (QGP) [1,2]. LQCD also indicates that for vanishing baryon chemical potential (μ_B), the transition from the QGP to a hadron gas phase (HGP) is of crossover type [3], while at large baryon chemical potential, it should be of first order. This implies the existence of a critical point (CP). Theoretical considerations further suggest that within the vicinity of the CP, one should expect sizable changes in the matter's correlation length and that divergent net-charge (ΔQ), net-strangeness (ΔS), or net-baryon (ΔB) fluctuations should occur [4]. Away from the CP, in the crossover region, some trace of critical behavior might also remain [3,5]. There is thus a strong interest in mapping the magnitude of ΔQ, $\Delta S, \Delta B$ with μ_B and temperature (T). This can be accomplished, in principle, by measuring second-, third-, and fourth-order cumulants of these quantities as a function of beam energy ($\sqrt{s_{\rm NN}}$). However, a number of caveats must be considered. First, LQCD predicts the magnitude of ΔB fluctuations in a finite coordinate space volume, V, but, experimentally, in heavy-ion collisions, these are measured based on a specific volume, Ω, in momentum space. It is *ab initio* unclear how charge transport (e.g., flow, diffusion, etc.), within the QGP produced in heavy-ion collisions map V onto Ω and how this mapping will affect the fluctuations observed in momentum space [6]. Second, ΔB fluctuations in V are not globally constrained by net-baryon number conservation while those in Ω intrinsically are [7]. Third, it is not obvious that a measurement of proton vs. antiproton fluctuations is sufficient to make a statement about baryon number fluctuations. What is indeed the effect of the unobserved baryons, i.e., antineutron ($\bar{n}$), anti-lambda ($\bar{\Lambda}$), etc.? A host of other questions may also be considered, including whether the produced system has time to thermalize globally and whether, consequently, it is meaningful to invoke the notion of susceptibility.

In this paper, I first focus the discussion on fluctuations of conserved charges, more specifically the net-proton number ΔN_p, and examine the impact of baryon number conservation on measurements of the second cumulant $\kappa_2(\Delta N_p)$. I next consider the effects of a partial measurement of baryon fluctuations based on fluctuations of the net-proton number. Finally, I extend the discussion and consider fluctuations of net baryons in terms of contributions from identified pairs of baryons and antibaryons.

In the context of the grand canonical ensemble (GCE), fluctuations of ΔB are related to the reduced susceptibility $\hat{\chi}_2^B$ according to [7,8]

$$\hat{\chi}_2^B = \frac{1}{VT^3}\kappa_2(\Delta B), \tag{1}$$

where V is the volume of the system, T its temperature, and $\kappa_2(\Delta B)$, the second-order cumulant of ΔB. The second-order cumulants amounts to the variance and is calculated

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according to

$$\kappa_2(\Delta B) = \langle \Delta B^2 \rangle - \langle \Delta B \rangle^2, \tag{2}$$

where $\langle \Delta B \rangle$ and $\langle \Delta B^2 \rangle$ are the first and second moments, measured over an ensemble of events, of the net-baryon number $\Delta B = N_B - N_{\bar{B}}$. The variables N_B and $N_{\bar{B}}$ represent multiplicities of baryon and antibaryons, respectively, within the volume V in a particular instance of the system (collision). Averages are computed over all possible instances of the system. Within the GCE, the susceptibility $\hat{\chi}_2^B$ is calculated as the second derivative of the reduced thermodynamic pressure $\hat{p} = p/T^4$ with respect to the reduced baryon chemical potential $\hat{\mu}_B \equiv \mu_B/T$

$$\hat{\chi}_2^B = \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_B^2}. \tag{3}$$

Higher cumulants, $\kappa_n(\Delta B)$, $n \geqslant 3$, of the net-baryon number ΔB are likewise related to higher-order susceptibilities corresponding to nth derivatives of the pressure. Because these susceptibilities have a finite dependence on the volume of the system, which is relatively ill defined in the context of nucleus-nucleus collisions, it is customary to consider ratios of the cumulants $\kappa_n(\Delta B)$ by κ_2 to eliminate this dependence. Higher cumulants are deemed of great interest because of their higher-power dependence on the correlation length ξ, which should diverge in the vicinity of the CP [9].

Measurements of second-, third-, and fourth-order cumulants of ΔQ, ΔS, and ΔB have been conducted at RHIC, in particular, in the context of the first beam energy scan (BES I) [10,11]. Cross cumulants have also been reported [12]. While second, third, and fourth order of ΔQ and ΔS are observed to have either modest or monotonic dependence on the beam energy, the third and fourth cumulant of the net-proton number exhibit nonmonotonic behaviors vs. $\sqrt{s_{\rm NN}}$, with what appears to be a statistically significant minimum near $\sqrt{s_{\rm NN}} = 20$ GeV. Interestingly, this energy is also the locus of a minimum in the magnitude of directed flow, v_1, observed in Au-Au collisions vs. beam energy [13]. The existence of these two minima at the same energy has been interpreted as an indicator of the presence of the CP in this vicinity [14]. However, the observed nonmonotonic behavior and minimum have received a variety of other interpretations [15]. Indeed, several caveats may impact the interpretation of the existing results, as well as those of future experiments. Primary among these are concerns associated with the role of baryon number conservation.

The total baryon number of an isolated system is a conserved quantity. This implies that the net-baryon number of all particles produced in a given A-A collision should add to the sum of the baryon numbers of the incoming nuclei. However, fluctuations of the net-baryon number, ΔB, will be observed when measuring baryon production in a fiducial acceptance limited to central rapidities. This much is true. Furthermore, it is generally assumed that the measured magnitude of $\kappa_n(\Delta B)$ will inform us about about the susceptibilities χ_n^B. It is argued, in particular, that great care has to be given to the choice of the width of the rapidity acceptance used in measurements of $\kappa_n(\Delta B)$: too narrow an acceptance should lead to Poisson fluctuations of ΔB while too wide an acceptance should greatly suppress the fluctuations because the net-baryon number of the entire system must be conserved. Moreover, it is often stated that for an acceptance of about one to two units of rapidity, such as those of the STAR and ALICE experiments, the effect of baryon number conservation should be negligible and only small corrections need to be applied to interpret $\kappa_n(\Delta B)$ measurements in terms of susceptibilities. Unfortunately, these assertions are factually incorrect as I will demonstrate in this paper: at LHC and top RHIC energies, the nontrivial part of the cumulant $\kappa_2(\Delta B)$ is entirely determined by baryon number conservation and the width of the experimental acceptance, while at lower energies of the RHIC Beam Energy Scan (BES), one must account for fluctuations in the proton yield associated with baryon stopping and collision geometry. The good news, however, is that local baryon number conservation applies both in infinite static matter and within a system (heavy-ion collision) undergoing fast longitudinal and radial expansion. The only important consideration then is how radial and longitudinal expansion affect the fraction of conserved baryons focused within the experimental acceptance, on average. While such a fraction cannot be measured directly by means of cumulants, it can be assessed and extrapolated, in principle, from measurements of balance functions. It is my goal, in this paper, to demonstrate that second cumulants of the net-baryon number are intrinsically and entirely determined by baryon number conservation, radial flow, and the width of the acceptance. I further show that while integral correlators, such as $\kappa_2(\Delta B)$, are sensitive to radial flow, they do not allow easy discrimination between effects of radial flow and the width of the acceptance in transverse momentum, $p_{\rm T}$, and pseudorapidity, η. However, differential correlation functions in the form of balance function (BF) offer a much better method to assess the interplay between finite acceptance, radial flow, and baryon number conservation.

In order to demonstrate these assertions, I first need to express the second-order cumulant of net-baryon (proton) fluctuations, measured within a specific acceptance, in terms of second-order (pair) factorial cumulants. I will then show that these are related to the $\nu_{\rm dyn}$ correlation observable, which in turn, is proportional to the integral, within the same acceptance, of the baryon balance function. I will show how the integral of the balance function is determined by the hadrochemistry of the collision system and that the shape and width of the balance function are largely determined by longitudinal and radial flow.

This paper is divided as follows. Section II defines moment, cumulant, factorial moment, factorial cumulant, and balance function notations used in the remainder of the paper. The Poisson limit of fluctuations and the relation between κ_2 and the $\nu_{\rm dyn}$ correlator are discussed in Sec. III. The connection between $\nu_{\rm dyn}$ and the balance function, and the role of baryon number conservation at LHC and top RHIC energy are discussed in Sec. IV, while the impact of baryon stopping and a net excess of baryons in the fiducial volume of the measurement are addressed in Sec. V. Section VI extends the discussion of net-baryon fluctuations in terms of a sum of balance functions of identified baryon and antibaryon pairs. Conclusions are summarized in Sec. VII.

II. DEFINITIONS AND NOTATIONS

A. Integral correlators

For simplicity, all particles of interest (e.g., protons and antiprotons) are assumed to be measured in the same fiducial momentum acceptance Ω. Measured multiplicities of species α and β, in a given event, are denoted N_α and N_β, respectively. Antibaryons are indicated with overbar symbols, e.g., $N_{\bar{\alpha}}$ denotes the multiplicity of antiparticles of species α. For instance, proton and antiproton multiplicities are denoted N_p and $N_{\bar{p}}$, where as the net-proton number is written $\Delta N_p = N_p - N_{\bar{p}}$.

Theoretically, the fluctuations may be described in terms of a joint probability $P(N_\alpha, N_\beta|\Omega, C)$ determined by the acceptance Ω and the centrality C of the heavy-ion collisions of interest. Experimentally, fluctuations may be characterized in terms of moments of multiplicities calculated as event ensemble averages denoted $\langle O\rangle$. First and second moments of multiplicities N_α and N_β are defined according to

$$m_1^\alpha = \langle N_\alpha\rangle = \sum_{i=0}^{\infty} N_\alpha P(N_\alpha, N_\beta|\Omega, C), \tag{4}$$

$$m_2^{\alpha,\beta} = \langle N_\alpha N_\beta\rangle = \sum_{i=0}^{\infty} N_\alpha N_\beta P(N_\alpha, N_\beta|\Omega, C). \tag{5}$$

Cumulants of multiplicities N_α and N_β are written

$$\kappa_1^\alpha = m_1^\alpha, \tag{6}$$

$$\kappa_2^{\alpha,\beta} = m_2^{\alpha,\beta} - m_1^\alpha m_1^\beta. \tag{7}$$

The cumulants $\kappa_2^{\alpha,\alpha}$ and $\kappa_2^{\alpha,\beta}$, with $\beta \neq \alpha$, correspond to the variance of N_α and the covariance of N_α and N_β, respectively.

Experimentally, particle losses associated with the detection and event reconstruction modify these moments and cumulants. Corrections for such losses are most straightforward when carried out for single particles and pairs of particles. It is thus convenient to introduce factorial moments of the multiplicities N_α and N_β as

$$f_1^\alpha = \langle N_\alpha\rangle = m_1^\alpha, \tag{8}$$

$$f_2^{\alpha,\beta} = \langle N_\alpha N_\beta - \delta_{\alpha,\beta} N_\alpha\rangle = m_2^{\alpha,\beta} - \delta_{\alpha,\beta} m_1^\alpha. \tag{9}$$

Given factorial moments of measured multiplicities n_α and n_β, corrected factorial moments are obtained as

$$f_1^\alpha = \tilde{f}_1^\alpha/\varepsilon_\alpha, \tag{10}$$

$$f_2^{\alpha,\beta} = \tilde{f}_2^{\alpha,\beta}/(\varepsilon_\alpha\varepsilon_\beta), \tag{11}$$

where $\tilde{f}_1^\alpha$ and $\tilde{f}_2^{\alpha,\beta}$ represent raw (or uncorrected) factorial moments, while ε_α and ε_β are detection efficiencies for particle species α and β, respectively. Note that best experimental precision may require one accounts for dependences of these quantities on the transverse momentum, the azimuth angle, and the pseudorapidity of the particles [16,17]. By construction, these factorial moments are determined by the single and pair densities of produced particles according to

$$f_1^\alpha = \int_\Omega \rho_1^\alpha(\vec{p})d^3p, \tag{12}$$

$$f_2^{\alpha,\beta} = \int_\Omega \rho_2^{\alpha,\beta}(\vec{p}_1, \vec{p}_2)d^3p_1 d^3p_2, \tag{13}$$

where $\rho_1^\alpha(\vec{p})$ is the single particle density of particle species α, and $\rho_2^{\alpha,\beta}(\vec{p}_1, \vec{p}_2)$ is the pair-density of particle species α and β.

Factorial moments (corrected for efficiency losses) are combined to obtain factorial cumulants according to

$$F_1^\alpha = f_1^\alpha = \kappa_1^\alpha = m_1^\alpha, \tag{14}$$

$$\begin{aligned} F_2^{\alpha,\beta} &= f_2^{\alpha,\beta} - f_1^\alpha f_1^\beta, \\ &= m_2^{\alpha,\beta} - \delta_{\alpha,\beta} m_1^\alpha - m_1^\alpha m_1^\beta. \end{aligned} \tag{15}$$

Factorial cumulants $F_2^{\alpha,\beta}$ are, by construction, true measures of pair correlations: they vanish identically in the absence of particle correlations and take finite values, either negative or positive, in the presence of such correlations. However, null $F_2^{\alpha,\beta}$ values are not a sufficient condition to conclude measured particles are uncorrelated. Using the above definitions of first- and second-order factorial cumulants, one verifies second-order cumulants may be written

$$\kappa_2^{\alpha,\beta} = \delta_{\alpha,\beta} F_1^\alpha + F_2^{\alpha,\beta}. \tag{16}$$

It is convenient to introduce normalized factorial cumulants defined according to

$$R_2^{\alpha,\beta} \equiv \frac{f_2^{\alpha,\beta}}{f_1^\alpha f_1^\beta} - 1 = \frac{F_2^{\alpha,\beta}}{F_1^\alpha F_1^\beta}, \tag{17}$$

as well as the following linear combination of normalized two-cumulants:

$$\nu_{\rm dyn}^{\alpha,\beta} = R_2^{\alpha,\alpha} + R_2^{\beta,\beta} - 2R_2^{\alpha,\beta}, \tag{18}$$

where $\alpha \neq \beta$ represent two distinct types of particles. The correlator $\nu_{\rm dyn}$ was originally introduced to search for the suppression of net-charge fluctuations in heavy-ion collisions [18–22]. It is of practical interest because it is experimentally robust, impervious to statistical fluctuations, and singles out dynamical fluctuations involved in particle production [20]. Its use has since been extended to study fluctuations of the relative yields of several types of particle species at RHIC and LHC energies [23–26].

B. Balance functions

General balance functions (BFs) are differential correlations functions that contrast the strength of like-sign (in the context of this paper, same baryon number) and unlike-sign (opposite charge or opposite baryon number) particles correlations [27,28]. General balance functions for pairs of species α and β are nominally defined according to

$$B^{\alpha,\bar{\beta}}(\Delta y) = \frac{1}{2}\left[\frac{\rho_2^{\alpha,\bar{\beta}}(\Delta y)}{\rho_1^\alpha} - \frac{\rho_2^{\alpha,\beta}(\Delta y)}{\rho_1^\alpha} + \frac{\rho_2^{\bar{\alpha},\beta}(\Delta y)}{\rho_1^{\bar{\alpha}}} - \frac{\rho_2^{\bar{\alpha},\bar{\beta}}(\Delta y)}{\rho_1^{\bar{\alpha}}}\right], \tag{19}$$

in which labels without (e.g., α, β) and with (e.g., $\bar{\alpha}, \bar{\beta}$) an overbar indicate baryons and antibaryons, respectively. Expressions ρ_1^{α} and $\rho_2^{\alpha,\beta}(\Delta y)$ denote single-particle and pair densities of baryons (antibaryons), respectively. Particle α is considered as the trigger or given particle, while particle β is regarded as the associate. The ratios $\rho_2^{\alpha,\beta}(\Delta y)/\rho_1^{\alpha}$ are conditional densities expressing the number of particles of species β at a separation Δy from a particle of species α. In the context of this work, it is useful to calculate the BF according to

$$B^{\alpha,\bar{\beta}}(\Delta y) = \tfrac{1}{2}\left[D_2^{\alpha,\bar{\beta}}(\Delta y) + D_2^{\bar{\alpha},\beta}(\Delta y)\right], \tag{20}$$

in which $D_2^{\alpha,\bar{\beta}}(\Delta y)$ and $D_2^{\bar{\alpha},\beta}(\Delta y)$ represent differences of conditional densities defined as

$$D_2^{\alpha,\bar{\beta}}(\Delta y) = \rho_1^{\bar{\beta}} R_2^{\alpha,\bar{\beta}}(\Delta y) - \rho_1^{\beta} R_2^{\alpha,\beta}(\Delta y), \tag{21}$$

$$D_2^{\bar{\alpha},\beta}(\Delta y) = \rho_1^{\beta} R_2^{\bar{\alpha},\beta}(\Delta y) - \rho_1^{\bar{\beta}} R_2^{\bar{\alpha},\bar{\beta}}(\Delta y), \tag{22}$$

with normalized two-particle normalized cumulants

$$R_2^{\alpha,\beta}(\Delta y) = \frac{\rho_2^{\alpha,\beta}(\Delta y)}{\rho_1^{\alpha} \otimes \rho_1^{\beta}(\Delta y)} - 1 = \frac{F_2^{\alpha,\beta}(\Delta y)}{F_1^{\alpha} \otimes F_1^{\beta}(\Delta y)}. \tag{23}$$

The correlators $F_2^{\alpha,\beta}(\Delta y) = \rho_2^{\alpha,\beta}(\Delta y) - \rho_1^{\alpha} \otimes \rho_1^{\beta}(\Delta y)$ are differential factorial cumulants with an explicit dependence on the pair separation Δy.

III. MOMENTS OF NET PROTON DISTRIBUTION AND SKELLAM LIMIT

The net-proton number is defined as $\Delta N_p \equiv N_p - N_{\bar{p}}$. One straightforwardly verifies that its first and second cumulants are

$$\kappa_1(\Delta N_p) = \kappa_1^{p} - \kappa_1^{\bar{p}}, \tag{24}$$

$$\kappa_2(\Delta N_p) = \kappa_2^{p,p} + \kappa_2^{\bar{p},\bar{p}} - 2\kappa_2^{p,\bar{p}}, \tag{25}$$

where the first and second cumulants of proton and antiproton multiplicities, denoted by the indices p and $\bar{p}$, respectively, are defined according to Eqs. (6) and (7). These may alternatively be written

$$\kappa_1(\Delta N_p) = F_1^{p} - F_1^{\bar{p}}, \tag{26}$$

$$\kappa_2(\Delta N_p) = F_1^{p} + F_1^{\bar{p}} + F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}. \tag{27}$$

One finds that the second cumulant of the net-proton number involves two parts, the first being determined by the average multiplicities of protons and antiprotons and a more interesting part driven by two-particle correlations.

As stated above, in the absence of two-particle or higher-order particle correlations, the factorial moments $F_2^{\alpha,\beta}$ vanish. The Poisson limit of the second-order cumulant, often called Skellam, is thus simply

$$\kappa_2^{\mathrm{Skellam}}(\Delta N_p) = F_1^{p} + F_1^{\bar{p}}. \tag{28}$$

It is convenient to consider the ratio, $r_{\Delta N_p}$, of a measured cumulant $\kappa_2(\Delta N_p)$ and its Skellam limit. Using Eqs. (27) and (28), one gets

$$r_{\Delta N_p} \equiv \frac{\kappa_2(\Delta N_p)}{\kappa_2^{\mathrm{Skellam}}(\Delta N_p)} = 1 + \frac{F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}}{F_1^{p} + F_1^{\bar{p}}}. \tag{29}$$

This may also be written

$$r_{\Delta N_p} = 1 + \frac{\left(F_1^{p}\right)^2 R_2^{p,p} + \left(F_1^{\bar{p}}\right)^2 R_2^{\bar{p},\bar{p}} - 2F_1^{p}F_1^{\bar{p}}R_2^{p,\bar{p}}}{F_1^{p} + F_1^{\bar{p}}}, \tag{30}$$

where I inserted normalized factorial cumulants defined according to Eq. (17).

IV. LHC AND TOP RHIC ENERGY

At LHC and top RHIC energy, one has $\langle N_p\rangle \approx \langle N_{\bar{p}}\rangle$. The ratio $r_{\Delta N_p}$ is thus approximately

$$r_{\Delta N_p} = 1 + \frac{F_1^{p}}{2}\left[R_2^{p,p} + R_2^{\bar{p},\bar{p}} - 2R_2^{p,\bar{p}}\right], \tag{31}$$

$$= 1 + \frac{1}{4}\langle N_T\rangle \nu_{\mathrm{dyn}}^{p,\bar{p}}, \tag{32}$$

where $\langle N_T\rangle = \langle N_p\rangle + \langle N_{\bar{p}}\rangle$ is formally defined as

$$\langle N_T\rangle = \int_{\Omega} \rho_1^{p}(\vec{p})d^3p + \int_{\Omega} \rho_1^{\bar{p}}(\vec{p})d^3p. \tag{33}$$

But given the densities ρ_1^{p} and $\rho_1^{\bar{p}}$ are approximately constant at central rapidities, one can write $\langle N_T\rangle = dN_T/d\eta \times \Delta\eta$, where $\Delta\eta$ represents the longitudinal width of the experimental acceptance. The ratio $r_{\Delta N_p}$ may thus be written

$$r_{\Delta N_p} = 1 + \frac{1}{4}\Delta\eta \frac{dN_T}{d\eta} \nu_{\mathrm{dyn}}^{p,\bar{p}}. \tag{34}$$

As I discuss below, net-baryon number conservation implies that $\nu_{\mathrm{dyn}}^{p,\bar{p}}$ is negative with an absolute magnitude that depends on the width $\Delta\eta$ of the fiducial acceptance. Neglecting this dependence, one would expect the ratio $r_{\Delta N_p}$ to have a trivial, approximately linear, dependence on the width of the acceptance [29]:

$$r_{\Delta N_p} \approx 1 - a\Delta\eta, \tag{35}$$

where $a \equiv \frac{1}{4} dN_T/d\eta |\nu_{\mathrm{dyn}}^{p,\bar{p}}|$. However, the value of $\nu_{\mathrm{dyn}}^{p,\bar{p}}$ should itself depend on $\Delta\eta$. The above is thus likely to be a somewhat poor approximation of the actual dependence of $r_{\Delta N_p}$ on $\Delta\eta$. I show later in this section that the quality of the approximation depends on the actual shape of the balance function and the rapidity range of interest. Additionally, given $r_{\Delta N_p} \to 1$ in the limit $\Delta\eta \to 0$, one might also be tempted to conclude that fluctuations of the net-proton number are Poissonian (Skellam) in that limit. That is actually incorrect. The true measure of correlations is given by $\nu_{\mathrm{dyn}}^{p,\bar{p}}$, which is, in general, nonvanishing even in the limit $\Delta\eta \to 0$. This is the case, e.g., for a system producing pions via the decay of ρ_0 mesons (see, e.g., Eq. (81) of Ref. [20]), and for systems that can be described with a balance function, as I demonstrate in the following.

It is clearly of interest to assess how the value of $\nu_{\mathrm{dyn}}^{p,\bar{p}}$ may depend on the acceptance of the measurement. This is readily achieved with the introduction of balance functions defined in Eqs. (19) and (20). Using Eq. (20), one finds that proton-proton balance functions may be written

$$B^{p,p}(\Delta y) = \tfrac{1}{2}\big[\rho_1^{\bar{p}} R_2^{p,\bar{p}}(\Delta y) - \rho_1^{p} R_2^{p,p}(\Delta y) + \rho_1^{p} R_2^{\bar{p},p}(\Delta y) - \rho_1^{\bar{p}} R_2^{\bar{p},\bar{p}}(\Delta y)\big], \tag{36}$$

where ρ_1^{p} and $\rho_1^{\bar{p}}$ are single particle densities of protons and anti-protons, respectively, and $R_2^{p,p}(\Delta y)$, $R_2^{p,\bar{p}}(\Delta y)$, $R_2^{\bar{p},p}(\Delta y)$, and $R_2^{\bar{p},\bar{p}}(\Delta y)$ are normalized cumulants of pair densities. The variable $\Delta y = y_1 - y_2$ represents the difference between the rapidities of particles y_1 and y_2 of any given pair.

In A-A collisions and in the limit $\langle N_p\rangle = \langle N_{\bar{p}}\rangle$, one has $\rho_1^{\bar{p}} = \rho_1^{p}$ and $R_2^{\bar{p},p}(\Delta y) = R_2^{p,\bar{p}}(\Delta y)$. The BF simplifies to

$$B^{p,p}(\Delta y) = -\frac{\Delta\eta}{4}\frac{dN_T}{d\eta}\big\{R_2^{p,p}(\Delta y) + R_2^{\bar{p},\bar{p}}(\Delta y) - 2R_2^{p,\bar{p}}(\Delta y)\big\}. \tag{37}$$

Integration of $F_2^{\alpha,\beta}(\Delta y)$ across the Δy acceptance yields the integral factorial cumulant $F_2^{\alpha,\beta}$ defined by Eq. (15). The integral of the BF can thus be written

$$I_{p,\bar{p}}(\Omega) = -\tfrac{1}{4}\langle N_T\rangle \nu_{\rm dyn}^{p,\bar{p}}(\Omega). \tag{38}$$

Up to a sign, the integral of the BF is equal to the second term of Eq. (32). One can then write

$$1 - r_{\Delta N_p} = I_{p,\bar{p}}(\Omega). \tag{39}$$

One concludes that at high-energy, i.e., in the limit $\langle N_p\rangle = \langle N_{\bar{p}}\rangle$, the deviation of the Skellam ratio from unity is identically equal to the integral of the BF. It is thus useful to examine what determines the magnitude of this integral.

Neglecting the effect of incoming and stopped protons from incoming projectiles (I account for these in Sec. V), the shape and amplitude of the BF reflect how and where baryon-conserving balancing pairs of protons and antiprotons are created and transported in the aftermath of A-A collisions. If only an antiproton ($Q = -1, B = -1$) could balance the production of a proton ($Q = 1, B = 1$), then, by construction, the balance function would integrate to unity over the full phase space of particle production. However, baryon number conservation can be satisfied by the production of other antibaryons. An antibaryon of some kind must indeed accompany the production of a proton. The proton-baryon balance function may thus be written

$$B^{p,\bar{B}}(\Delta y) = B^{p,\bar{p}}(\Delta y) + B^{p,\bar{n}}(\Delta y) + B^{p,\bar{\Lambda}}(\Delta y) + \cdots = \sum_{\bar{\beta}} B^{p,\bar{\beta}}(\Delta y), \tag{40}$$

where the sum extends to all antibaryons that can balance the production of a proton. By construction, this balance function must integrate to unity over the full particle production phase space:

$$I_{p,\bar{B}}^{4\pi} = 1, \tag{41}$$

where 4π denotes that the integral is extending over all rapidities and transverse momenta. The production of pairs $p\bar{p}$, $p\bar{n}$, $p\bar{\Lambda}$, $p\bar{\Sigma}^-$, etc, have probabilities determined by their relative cross sections. These, in turn, must be equal to integrals of their respective balance functions. One can then write

$$1 \equiv I_{p,\bar{B}}^{4\pi} = I_{p,\bar{p}}^{4\pi} + I_{p,\bar{n}}^{4\pi} + I_{p,\bar{\Lambda}}^{4\pi} + \cdots = \sum_{\bar{\beta}} I_{p,\bar{\beta}}^{4\pi}, \tag{42}$$

where, once again, 4π denotes that the integrals are extending over all rapidities and transverse momenta, and $\sum_{\bar{\beta}}$ represents a sum over all antibaryons ($B = -1$). In this context, the functions $I_{p,\bar{\beta}}^{4\pi}$ can be considered as probabilities of the respective baryon number balancing processes determined by their cross sections. The $p\bar{p}$ balance function integral is one of many components of the full $p, \bar{B}$ BF. Its value is thus smaller than unity.

Experimentally, however, particles are measured within limited (pseudo)rapidity and transverse momentum ranges. The probabilities $I_{p,\bar{\beta}}^{4\pi}$, are thus not directly measurable. Extrapolation of BF integrals to the full rapidity and momentum ranges of particle production are nontrivial given they are highly dependent on their width and shape (pair separation profile). This is illustrated in Fig. 1, which presents examples of balance functions, with full $p_{\rm T}$ coverage, and their respective integrals for selected parameter values. Figures 1(a)–1(c) present balance function with Gaussian (G), double-Gaussian (DG), and exponential (E) dependence on the pair separation $\Delta y = y_1 - y_2$, respectively, and defined according to

$$B_{\rm G}(\Delta y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\Delta y^2}{2\sigma^2}\right), \tag{43}$$

$$B_{\rm DG}(\Delta y) = \frac{1.05}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\Delta y^2}{2\sigma^2}\right) - \frac{0.05}{\sqrt{2\pi}\sigma_{\rm N}}\exp\left(-\frac{\Delta y^2}{2\sigma_{\rm N}^2}\right), \tag{44}$$

$$B_{\rm E}(\Delta y) = \frac{1}{\tau}\exp\left(-\frac{|\Delta y|}{\tau}\right), \tag{45}$$

where σ is the rms width of the single-Gaussian rapidity, $\sigma_{\rm N} = 0.1$ corresponds to the rms width of the narrow Gaussian used here to model, e.g., baryon annihilation, and τ is used to model the rate of decay of the rapidity density. Figures 1(d)–1(f) present integrals of the Gaussian, double-Gaussian, and exponential balance function profiles as a function of the value of σ (τ) for a nominal acceptance $-1 < y < 1$. The examples shown clearly illustrate that the integral $I_{p,\bar{\beta}}$ depends on the shape and width of the balance function relative to the measurement acceptance. This is further illustrated in Figs. 1(g)–1(i), which display integrals $I_{p\bar{B}}(\Delta Y)$ of the BFs shown in Figs. 1(a)–1(c), as a function of $\Delta Y = y_{\rm max} - y_{\rm min}$ denoting the width of the single-particle acceptance $y_{\rm min} < y < y_{\rm max}$. One finds, indeed, that the rate at which the measured integral $I_{p,\bar{\beta}}$ converges to its 4π limit is dependent on the shape of the balance function as well as its width σ. Also note that the linear dependence on ΔY expected from Eq. (35) breaks down for these semirealistic balance functions, as clearly illustrated by the plots of $I_{p,\bar{B}}/\Delta Y$ vs. ΔY shown in the bottom row of Fig. 1. For most cases considered, the ratio $I_{p,\bar{B}}/\Delta Y$ varies with ΔY. The linear dependence embodied in Eq. (35) is thus

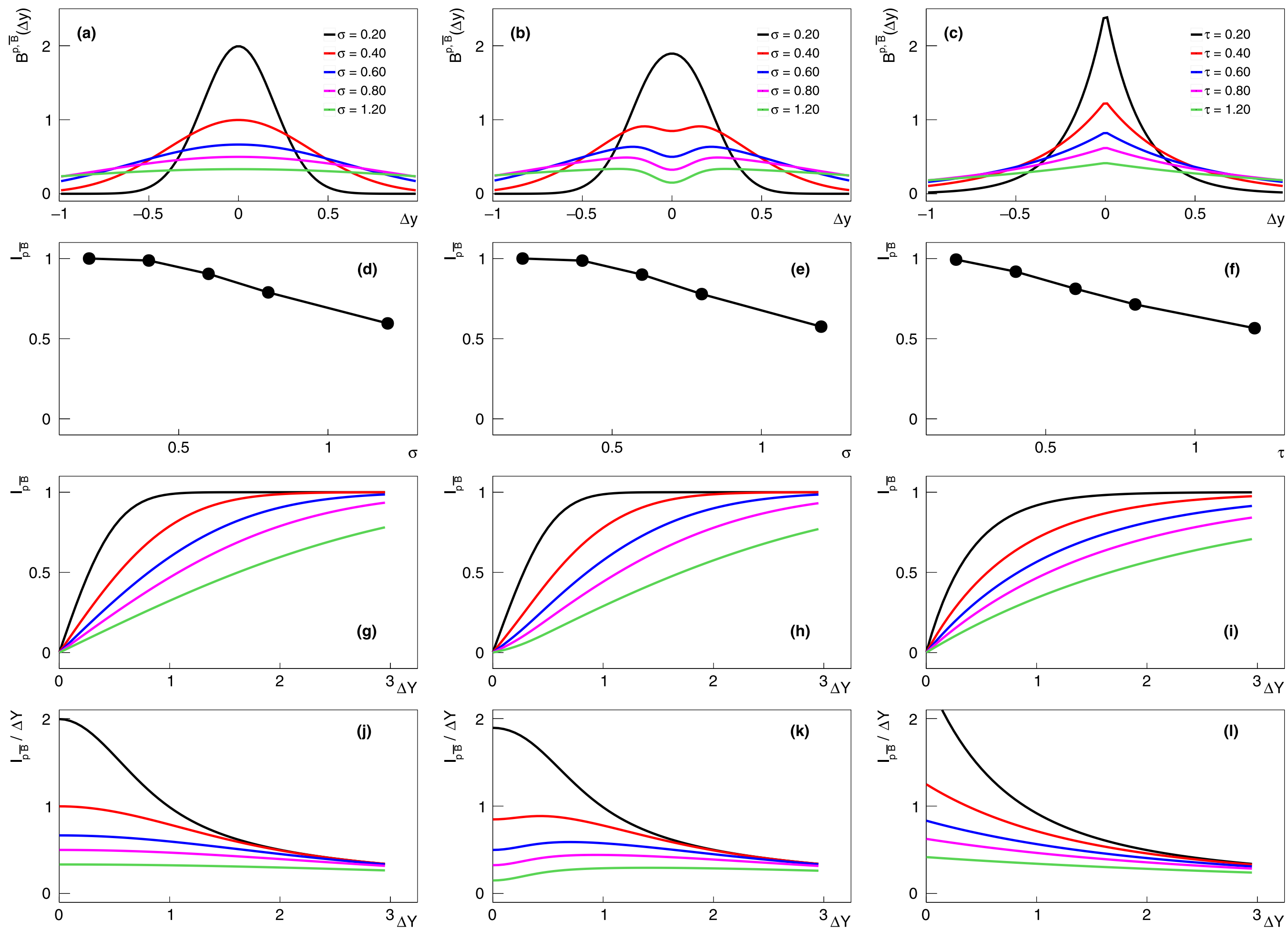

FIG. 1. Top row: (a) Gaussian, (b) double-Gaussian, and (c) exponential balance function models plotted as a function of the pair separation, $\Delta y = y_1 - y_2$, for selected parameter values; Upper middle row: Integrals of the (d) Gaussian, (e) double Gaussian, and (f) exponential balance functions vs. the rms width (σ) or mean (τ); Lower middle row: Integrals $I_{p\bar{B}}(\Delta Y)$ of the (g) Gaussian, (h) double Gaussian, and (i) exponential balance functions vs. the width of the acceptance (ΔY) for models and parameter values shown in the top row. Bottom row: Ratio $I_{p\bar{B}}(\Delta Y)/\Delta Y$ vs. ΔY.

indeed a poor approximation of the actual dependence of $I_{p,\bar{B}}$ on the width ΔY (or $\Delta\eta$) of the acceptance.

The above examples clearly illustrate that the integral of the BF is a function of its shape as well as the width ΔY of the experimental acceptance. Given the baryon number balancing of the proton may be achieved with several distinct antibaryon species, one must then consider the evolution of integrals $I_{p,\bar{\beta}}$ for all species $\bar{\beta}$ as a function of the measurement acceptance Ω, as illustrated schematically in Fig. 2.

Next recall that the integral of the balance function is proportional to $\nu_{\rm dyn}^{p,\bar{p}}$, which, as we saw in Eq. (39), is also proportional to $1 - r_{\Delta N_p}$. The magnitude of $\kappa_2(\Delta p)$, measured at high energy, is thus entirely determined by the integral of the balance function across the fiducial acceptance. The integral of the balance function, in turn, is determined by baryon number conservation and the chemistry of the collision, i.e., what fraction of protons are accompanied by an antiproton. If protons were balanced exclusively by antiprotons, the integral of the balance function over the entire phase space would yield

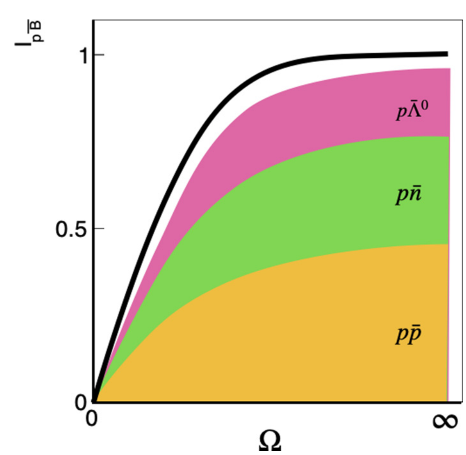

FIG. 2. Schematic dependence of the integral of balance functions $B^{p,\bar{B}}(\Delta y)$ vs. the width of the experimental acceptance Ω. The colored bands schematically illustrate contributions from distinct baryon number balancing antibaryons.

unity. With finite ranges in p_T and rapidity y, the integral is determined by the width of these ranges. The larger they are, the closer the integral gets to saturation (unity if only antiprotons balance protons). The measured values of $\kappa_2(\Delta p)$ at LHC and top RHIC energy are thus determined *ab initio* by baryon number conservation and the width of the balance function relative to that of the acceptance.

It is well established that the shape and width of the balance function of charge particles exhibit a significant narrowing with increasing collision centrality [25,30,31]. This narrowing is understood to result largely from radial flow and was successfully modeled with the blast wave model: the more central collisions are, the faster is the radial flow [32]. The value of $1 - r_{\Delta N_p}$ is thus determined in large part by the magnitude of radial flow and the width of the acceptance and much less by the full coverage integral $I^{4\pi}_{p,\bar{\beta}}$.

Nominally, if effects of radial flow were invariant with collision centrality, the multiplicity $\langle N_T\rangle_{AA}$ measured in A-A collisions would scale in proportion to its value in pp collisions $\langle N_T\rangle_{pp}$ according

$$\langle N_T\rangle_{AA} = \langle n_s\rangle\langle N_T\rangle_{pp}, \tag{46}$$

where $\langle n_s\rangle$ is the effective number of sources involved, on average, in a given A-A centrality range. In contrast, one also expects that, in the absence of rescattering of secondaries, that $\nu^{p,\bar{p}(AA)}_{\text{dyn}}$ measured in A-A should scale as

$$\nu^{p,\bar{p}(AA)}_{\text{dyn}} = \frac{1}{\langle n_s\rangle}\nu^{p,\bar{p}(pp)}_{\text{dyn}}, \tag{47}$$

relative to the value $\nu^{p,\bar{p}(pp)}_{\text{dyn}}$ measured in pp collisions [20]. Such scaling is in fact essentially observed in Au-Au and Pb-Pb collisions [21,22,24]. In this context, the ratio $r_{\Delta N_p}$ would then be invariant with A-A collision centrality. But the radial flow velocity is known to increase in more central collisions thereby leading to a narrowing of the balance function [30]. This consequently leads to an increase of the integral $I_{p,\bar{\beta}}$ within the experimental acceptance. The centrality dependence of $r_{\Delta N_p}$ will then be driven primarily by the evolution of radial flow with collision centrality and it might have essentially nothing to do with the chemistry of the system and its susceptibility $\hat{\chi}^B_2$.

The width of the net-charge balance function is also observed to increase monotonically with decreasing beam energy ($\sqrt{s_{\text{NN}}}$) [33]. This can be in part understood as a result of slower radial flow profile with decreasing beam energy. Should the $p\bar{p}$ balance function behave in a similar fashion, one would expect the integral $I_{p,\bar{p}}$ to reduce monotonically with decreasing beam energy because the fraction of the BF within the acceptance shrinks as its width increases. Once again, one expects the magnitude of $\kappa_2(\Delta p)$ to change with beam energy for reasons completely independent of the susceptibility $\hat{\chi}^B_2$.

However, the ratio $\langle N_{\bar{p}}\rangle/\langle N_p\rangle$ is also known to fall rapidly with decreasing beam energy. The $\langle N_p\rangle = \langle N_{\bar{p}}\rangle$ hypothesis used to derive Eqs. (34) and (39) is thus indeed strictly invalid at the low-energy end of the BES. One must thus examine the effect of baryon stopping on the fluctuations.

V. NET PROTONS FLUCTUATIONS IN THE PRESENCE OF NUCLEAR STOPPING

In order to model the effect of baryon stopping, I will assume, as in Ref. [34], that one can partition the measured protons into two subsets: the first, denoted i, corresponding to stopped protons, and the second, denoted p, corresponding to protons produced by $p\bar{B}$ pair creation. All antiprotons are assumed produced by pair production and I will neglect, for simplicity, the impact of annihilation.

I thus consider Eq. (29) with the following substitutions for the first- and second-order factorial cumulants of protons and antiprotons:

$$\begin{aligned}
F_1^p &\to F_1^i + F_1^p = \langle N_i\rangle + \langle N_p\rangle\\
F_1^{\bar{p}} &\to F_1^{\bar{p}} = \langle N_{\bar{p}}\rangle\\
F_2^{p,p} &\to F_2^{i,i} + F_2^{i,p} + F_2^{p,i} + F_2^{p,p}\\
F_2^{p,\bar{p}} &\to F_2^{i,\bar{p}} + F_2^{p,\bar{p}}\\
F_2^{\bar{p},p} &\to F_2^{\bar{p},i} + F_2^{\bar{p},p}\\
F_2^{\bar{p},\bar{p}} &\to F_2^{\bar{p},\bar{p}}.
\end{aligned} \tag{48}$$

In symmetric A-A collisions, one must have $F_2^{i,p} = F_2^{p,i}, F_2^{i,\bar{p}} = F_2^{\bar{p},i}$, and $F_2^{p,\bar{p}} = F_2^{\bar{p},p}$. Introducing $\langle N_T\rangle = \langle N_i\rangle + 2\langle N_p\rangle$ and $\xi = \langle N_i\rangle/\langle N_T\rangle = F_1^i/(F_1^i + 2F_1^p)$, one gets

$$r_{\Delta N_p} = 1 + \frac{F_2^{i,i} + 2F_2^{i,p} + F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{i,\bar{p}} - 2F_2^{p,\bar{p}}}{F_1^i + F_1^p + F_1^{\bar{p}}}, \tag{49}$$

$$= 1 + \xi^2\langle N_T\rangle R_2^{i,i} + \frac{1}{4}(1-\xi)^2\langle N_T\rangle\nu^{p,\bar{p}}_{\text{dyn}}, \tag{50}$$

where in the second line, I neglected effects of annihilation, which imply that $F_1^p = F_1^{\bar{p}}, F_2^{p,p} = F_2^{\bar{p},\bar{p}}$, and I assumed $F_2^{i,p} \approx F_2^{i,\bar{p}}$. The second term, proportional to $R_2^{i,i}$, is a measure of the correlation strength of stopped protons, while the third term, proportional to $\nu^{p,\bar{p}}_{\text{dyn}}$ corresponds to the pair creation component found in the high-energy limit, Eq. (34). Experimentally, it has been observed that nucleons from the projectile and target lose, on average, approximately two units of rapidity in nuclear collisions. At LHC and top RHIC energy, this leads to a vanishing net-baryon density in the central rapidity region but for decreasing $\sqrt{s_{\text{NN}}}$, and particularly at the low end of RHIC the beam energy scan, this yields a large net-proton excess at central rapidity. Given the production of $p\bar{p}$ pairs is a logarithmic function of $\sqrt{s_{\text{NN}}}$, one expects the term proportional to $R_2^{i,i}$ should largely dominate at the low end of the BES range while the term proportional to $\nu^{p,\bar{p}}_{\text{dyn}}$, driven by baryon number conservation, should dominate at LHC and top RHIC energy. Equation (50) thus tells us that the beam energy evolution of $r_{\Delta N_p} - 1$ should be determined by the interplay of baryon stopping and net-baryon conservation, the former and the latter dominating at low and high $\sqrt{s_{\text{NN}}}$, respectively. Given the strength and Δy dependence of $R_2^{i,i}(\Delta y)$ and $\nu^{p,\bar{p}}_{\text{dyn}}(\Delta y)$ are determined by different mechanisms, they will likely have distinct dependences on $\sqrt{s_{\text{NN}}}$. As the contribution of stopped baryons decreases with increasing $\sqrt{s_{\rm NN}}$, one thus anticipates that the balance function of created pairs p, $\bar{p}$, and thus $\nu_{\rm dyn}^{p,\bar{p}}(\Delta y)$, will dominate. The net-proton fluctuations $r_{\Delta N_p} - 1$ might then exhibit a rather complicated dependence on $\sqrt{s_{\rm NN}}$. Such a dependence, however, has little to do with the properties of nuclear matter near equilibrium and more to do with dynamic considerations including nuclear stopping power and radial flow resulting from large inside-out pressure gradients.

VI. NET BARYON FLUCTUATIONS

Equation (1) relates the baryon susceptibility $\hat{\chi}_2^B$ to the second cumulant of the net-baryon number ΔB. One must thus consider, at least in principle, the fluctuations of all baryons and antibaryons, $\Delta B = N_B - N_{\bar{B}}$, not only those of the net-proton number ΔN_p. Repeating the derivation presented in Sec. III for net-baryon fluctuations, one gets

$$r_{\Delta B} \equiv \frac{\kappa_2(\Delta N_B)}{\kappa_2^{\rm Skellam}(\Delta N_B)} = 1 + \frac{F_2^{B,B} + F_2^{\bar{B},\bar{B}} - 2F_2^{B,\bar{B}}}{F_1^B + F_1^{\bar{B}}}, \quad (51)$$

which, in the high-energy limit, yields

$$r_{\Delta B} - 1 = \tfrac{1}{4}\langle N_{TB}\rangle \nu_{\rm dyn}^{B,\bar{B}} = I_{B,\bar{B}}(\Omega), \quad (52)$$

where $\langle N_{TB}\rangle = \langle N_B\rangle + \langle N_{\bar{B}}\rangle$ and $I_{B,\bar{B}}(\Omega)$ is the integral of the baryon-baryon balance function $B^{B,\bar{B}}$.

In order to express $B^{B,\bar{B}}$ in terms of elementary balance functions $B^{\alpha,\bar{\beta}}$, first note that single- and two-baryon densities can be written

$$\rho_1^B = \sum_\alpha \rho_1^\alpha; \quad \rho_2^{B,B} = \sum_\alpha \sum_\beta \rho_2^{\alpha,\beta}, \quad (53)$$

where sums on α and β span all produced baryons. Similar expressions can be written for single and pair densities involving antibaryons. Defining the yield fractions

$$f_\alpha = \frac{\rho_1^\alpha}{\rho_1^B}; \quad f_{\bar{\alpha}} = \frac{\rho_1^{\bar{\alpha}}}{\rho_1^{\bar{B}}}, \quad (54)$$

such that $\sum_\alpha f_\alpha = 1$ and $\sum_{\bar{\alpha}} f_{\bar{\alpha}} = 1$, one finds that the baryon-baryon balance function $B^{B,\bar{B}}$ may be written

$$B^{B,\bar{B}}(\Delta y) = \frac{1}{2}\left[\sum_\alpha f_\alpha D_2^{\alpha,\bar{B}}(\Delta y) + \sum_{\bar{\alpha}} f_{\bar{\alpha}} D_2^{\bar{\alpha},B}(\Delta y)\right]. \quad (55)$$

In the high-energy limit, one has $f_\alpha = f_{\bar{\alpha}}$, and the above expression simplifies to

$$B^{B,\bar{B}}(\Delta y) = \sum_\alpha f_\alpha B^{\alpha,\bar{B}}(\Delta y), \quad (56)$$

where

$$B^{\alpha,\bar{B}}(\Delta y) = \sum_{\bar{\beta}} B^{\alpha,\bar{\beta}}(\Delta y). \quad (57)$$

Single-particle production yields measured in heavy-ion collisions are very well described in the context of thermal production models determined by a (chemical) freeze-out temperature as well as charge and strangeness chemical potentials. Within such models, one finds the baryon (antibaryon) production is dominated by the lowest mass states (e.g., proton, neutron). The baryon-baryon balance function, $B^{B,\bar{B}}(\Delta y)$, will thus be dominated by contributions from proton-baryon, $B^{p,\bar{B}}(\Delta y)$, neutron-baryon, $B^{n,\bar{B}}(\Delta y)$, balance functions, with weaker contributions from Λ baryon or heavier strange baryons and with negligible contributions from charm or bottom baryons. On general grounds, and neglecting electric charge (or isospin), one can expect $B^{p,\bar{B}}(\Delta y)$ and $B^{n,\bar{B}}(\Delta y)$ to feature similar strengths and dependence on Δy. However, balance functions involving strange baryons, in particular $B^{p,\bar{\Lambda}}(\Delta y)$ and $B^{p,\bar{\Sigma}}(\Delta y)$, might have a rather different dependence on Δy owing to the fact that s quarks may be produced at earlier times than u and d quarks, or be subjected to different transport mechanisms. Fortunately, measurements of $B^{p,\bar{\Lambda}}(\Delta y)$, $B^{\Lambda,\bar{\Lambda}}(\Delta y)$, and perhaps even $B^{p,\bar{\Sigma}}(\Delta y)$, are in principle possible. One can then anticipate, in the near future, being able to estimate the shape and strength of $B^{p,\bar{B}}(\Delta y)$ and $B^{B,\bar{B}}(\Delta y)$ based on measurements within the acceptance of ongoing experiments (e.g., ALICE).

VII. SUMMARY

I showed there is straightforward connection between the fluctuations of net-baryon number measured at central rapidities in A-A collisions in terms of second-order cumulants of the net-baryon number and the strength of two-particle correlations factorial cumulants. I further showed that in the high-energy limit, corresponding to a vanishing net-baryon number, fluctuations are entirely determined by the strength and width of the $p\bar{p}$ balance function relative to the width of the acceptance. By contrast, at low energy, the fluctuations of the net-baryon number are more likely dominated by proton-proton correlations resulting from nuclear stopping. Overall, one can expect the fluctuations to display a smooth evolution with $\sqrt{s_{\rm NN}}$ between these two extremes but nowhere can one expect the magnitude of the fluctuations to be trivially sensitive to the nuclear matter baryon susceptibility χ_2^B.

I here focused the discussion on second-order cumulants of the net-baryon number but it is clear that the same line of argument can be extended to higher cumulants. Measurements of fluctuations by STAR at RHIC have used the magnitude of the second-order cumulant of the net-baryon number as a reference to factor out the ill-defined notion of volume involved in relations between cumulants and susceptibilities. This would make sense if the susceptibilities determined the magnitude of the cumulants. But, as I have shown, the magnitude of $\kappa_2(\Delta p)$ is in fact determined largely by the width of the acceptance of the measurement relative to the width of the balance function at high energy and by proton-proton correlations associated with nuclear stopping at low energy. The use of $\kappa_2(\Delta p)$ thus does not provide a sound basis to cancel out volume effects and normalize the magnitude of higher cumulants.

All is not lost, however. Measurements of momentum-dependent balance functions may be used to quantitatively assess the role of both baryon number conservation and nuclear stopping, and henceforth obtain sensitivity to QCD matter susceptibilities. Additionally, measurements of balance functions of pairs $(p, \bar{p})$, $(p, \bar{\Lambda})$, $(\Lambda, \bar{\Lambda})$, and perhaps even $(p, \bar{\Sigma})$, are possible. Results from these measurements will inform our understanding of the system expansion dynamics, our knowledge of the hadronization chemistry, and enable, as per the discussion in Sec. VI, an assessment of the relative strength of their contributions to net-baryon fluctuations.

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