Coexistence and 2*n* transfer in even Zr nuclei

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In a simple coexistence model, I have examined 2*n* transfer ratios of 0^+ states in even Zr nuclei. Results indicate that mixing is small in ^{96}Zr, but larger and nearly equal in ^{92,94}Zr, with ^{90}Zr slightly larger. The ^{96}Zr result is consistent with earlier analyses of *E*2 strengths.

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I. INTRODUCTION

Some time ago, Carchidi and co-workers developed a generalized coexistence model [1–3] for use in analyzing 2*n* transfer to 0^+ states in a chain of isotopes. The model was first applied to results of (t, p) and (p, t) reactions involving $^{68-78}$Ge [4]. It was later found to be useful in understanding proton [5] and alpha [6] transfer and *E*2 strengths [7,8].

Briefly, the model assumes that the ground state (g.s.) and the first excited 0^+ state are linear combinations of two basis states whose structure changes slowly with *A*. Then,

$$|\text{g.s.(A)}\rangle = a_A g_A + b_A e_A, \quad |\text{exc. } 0^+(A)\rangle = -b_A g_A + a_A e_A.$$

Strengths for 2*n* transfer were denoted as (using Ge as an example)

$$T_A{}^2 = \sigma[^{\text{A}}\text{Ge}(t,p)\,^{\text{A+2}}\text{Ge(exc. } 0^+)]/\sigma[^{\text{A}}\text{Ge}(t,p)\,^{\text{A+2}}\text{Ge(g.s.)}],$$

and

$$P_A{}^2 = \sigma[^{\text{A+2}}\text{Ge}(p,t)\,^{\text{A}}\text{Ge(exc. } 0^+)]/\sigma[^{\text{A+2}}\text{Ge}(p,t)\,^{\text{A}}\text{Ge(g.s.)}].$$

Note that the subscript is the lighter of target and residual nucleus in both pickup and stripping.

Results were described in terms of a dimensionless parameter *R* of order unity, which represents the ratio of 2*n* transfer amplitudes between basis states:

$R = \langle e|2n \text{ transfer } |e\rangle/\langle g|2n \text{ transfer } |g\rangle$, which is taken to be independent of *A*. The situation is depicted schematically in Fig. 1. In the simple model limit (SML), *R* is 1.0, and *r* is zero. The "smoothness" assumption manifests itself in the fact that *R* and *r* are taken to be independent of *A*, and it is expected that f_A will vary reasonably smoothly with *A*. No additional assumptions are made about the structure of the basis states.

II. ANALYSIS AND RESULTS

Here, I apply this model to (t, p) and (p, t) reactions among even Zr nuclei. Energies of the first excited 0^+ states are plotted in Fig. 2. The g.s. and excited 0^+ have long been considered as linear combinations of $(p_{1/2})^2$ and $(g_{9/2})^2$ configurations in the proton space [9,10]. The lightest stable isotope, ^{90}Zr, corresponds to the $N = 50$ shell closure and has been rather well described in terms of the shell model [11]. The heaviest stable isotope, ^{96}Zr, corresponds to a partial closure of the $N = 56$ subshell. Inelastic scattering from ^{94}Zr and ^{92}Zr was analyzed in terms of proton excitations involving $p_{1/2}$ and $g_{9/2}$ plus neutron excitations involving $d_{5/2}$ and $d_{3/2}$ [12]. Heavier Zr nuclei are known to be strongly deformed [13].

The *A* dependence of the absolute (t, p) cross sections [9] is illustrated in Fig. 3. For A = 90, 92, and 94, these are summed cross sections over the angular range 10–70°. For $A = 96$, only the cross section at 39° is given, so I have used the 39° data for the other nuclei to normalize the point for $A = 96$. In a standard shell-model description, the increased yield for the excited 0^+ state for 94 → 96 is an indication of a partial subshell closure at $A = 96$ for the neutrons. Relevant 2*n* transfer ratios are listed in Table I [9,14–17]. A slight *Q* value dependence has been removed with the aid of distorted-wave calculations [18]. The amplitudes T_A and P_A are obtained by taking square roots of the cross-section ratios. The absolute values of these are plotted in Fig. 4. In the simplest version of the model, T_A and P_A were required to have opposite signs for every *A*. A brief inspection indicates that no such solution exists for the Zr nuclei. This failure could be related to the approximate neutron subshell closures for ^{90}Zr and ^{96}Zr.

However, one set of solutions does exist within the overall framework of the model. It has all T_A and P_A positive, except

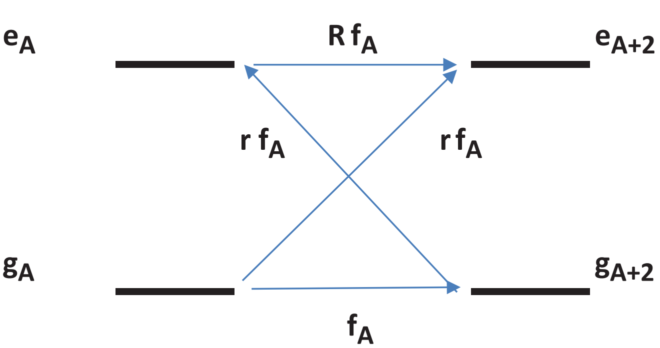

FIG. 1. Schematic depicting the 0^+ basis states and the 2n transfer amplitudes connecting them.

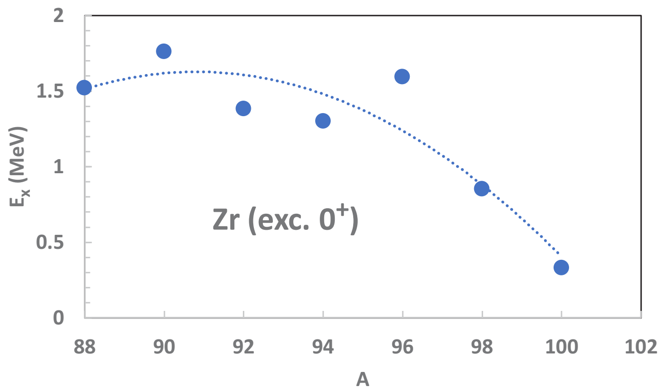

FIG. 2. Energies of first excited 0^+ states in Zr nuclei. The quadratic curve has no theoretical significance, but merely illustrates the trend and the fact that 0^+ energies are higher in ^{90,96}Zr.

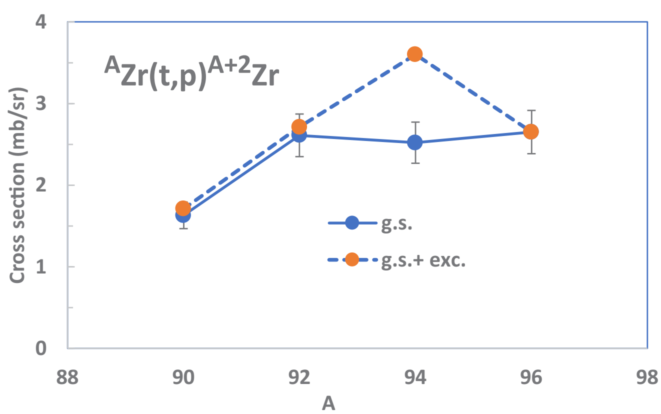

FIG. 3. The A dependence of the absolute (t,p) cross sections [9].

TABLE I. Ratios of cross sections σ(exc. 0^+)/σ(g.s.) in the reactions $^{A+2}$Zr$(p,t)^{A}$Zr and ^{A}Zr$(t,p)^{A+2}$Zr.

A	Ex (MeV)	Cross-section ratio (p,t) [14–17]	Cross-section ratio (t,p) [9]
88	1.520	0.110(10)	
90	1.761	0.013(1)	0.050(5)
92	1.389	0.035(3)	0.040(5)
94	1.295	0.039(4)	0.428(47)
96	1.594		$5(3) \times 10^{-4}$
98	0.854		

TABLE II. Amplitude ratios.

A	$\lvert P_A$ (exp.)$\rvert$	P_A (fit)	$\lvert T_A$ (exp.)$\rvert$	T_A (fit)
88	0.359(54)	±0.359		
90	0.113(5)	0.114	0.223(11)	0.238
92	0.187(8)	0.177	0.199(12)	0.191
94	0.196(10)	−0.196	0.654(36)	0.665
96			0.022(8)	±0.022

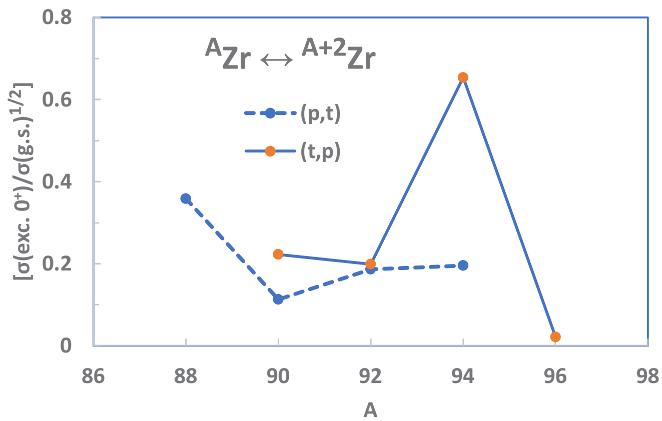

FIG. 4. Square roots of cross-section ratios from Table I.

for P_{94}. Experimental ratios are compared with those from this fit in Table II. Solutions exist for a limited range of the parameter R. For these solutions, the $g \leftrightarrow e$ ratio $r = \langle g|2n$ transfer $|e\rangle/\langle g|2n$ transfer $|g\rangle$ is not zero, but it is small—varying from 0 to 0.046 over the allowed range of R. Mixing amplitudes that result from the fit are plotted vs R in Fig. 5. They all have the common feature that mixing is small for ^{96}Zr and larger and nearly equal for ^{94,92}Zr, with ^{90}Zr larger still. One such solution is listed in Table III.

Coexistence and mixing in Zr nuclei [19–24] are topics of current interest [25–29]. In an earlier analysis of $E2$ strengths among low-lying 0^+ and 2^+ states in ^{96}Zr, the mixing parameter was determined to be 0.128(18) [26]. Note that the present mixing in ^{96}Zr, which was obtained from $2n$ transfer data alone, is close to this value. In fact, throughout the allowed range of R, b_{96} is consistent with various estimates from energies and $E2$ strengths [23,24], but not with a more recent value [27].

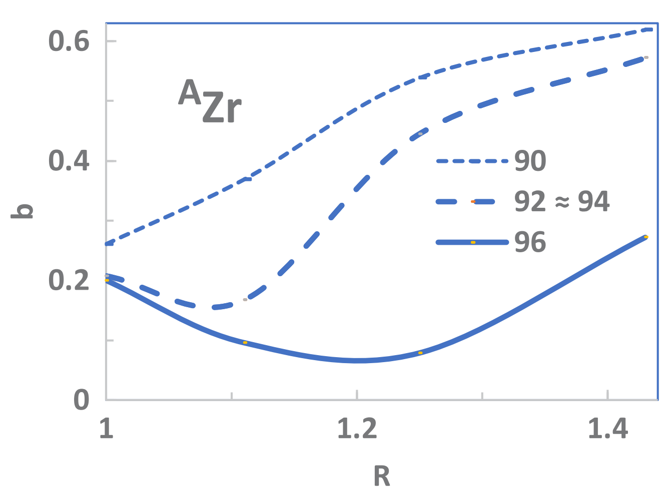

FIG. 5. Mixing amplitude b vs the dimensionless parameter R.

TABLE III. One set of mixing amplitudes from fit to 2*n* transfer in ^{A}Zr.

A	b
90	0.384
92	0.323
94	0.336
96	0.097

III. SUMMARY

I have applied a simple coexistence model to 2*n* transfer ratios involving the first two 0^+ states in even Zr nuclei. Results of the analysis are mixing amplitudes in a two-state model. The finding is that the mixing is small in ^{96}Zr, but larger and nearly equal in ^{92,94}Zr, with ^{90}Zr slightly larger. The ^{96}Zr result is consistent with earlier analyses of *E*2 strengths.

[1] M. Carchidi, H. T. Fortune, G. S. F. Stephans, and L. C. Bland, Phys. Rev. C **30**, 1293 (1984).
[2] M. Carchidi and H. T. Fortune, J. Math. Phys. **27**, 633 (1986).
[3] M. Carchidi and H. T. Fortune, Phys. Rev. C **38**, 1403 (1988).
[4] S. Mordechai, H. T. Fortune, M. Carchidi, and R. Gilman, Phys. Rev. C **29**, 1699 (1984).
[5] G. S. F. Stephans, H. T. Fortune, L. C. Bland, M. Carchidi, R. Gilman, G. P. Gilfoyle, and J. W. Sweet, Phys. Rev. C **35**, 2033 (1987).
[6] M. Carchidi and H. T. Fortune, Phys. Rev. C **31**, 853 (1985).
[7] H. T. Fortune and M. Carchidi, Phys. Rev. C **36**, 2584 (1987).
[8] M. Carchidi and H. T. Fortune, Phys. Rev. C **37**, 556 (1988).
[9] E. R. Flynn, J. G. Beery, and G. Blair, Nucl. Phys. A **218**, 285 (1974).
[10] B. M. Preedom, E. Newman, and J. C. Hiebert, Phys. Rev. **166**, 1156 (1968).
[11] B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev. **115**, 1627 (1959).
[12] M. M. Stautberg and J. J. Kraushaar, Phys. Rev. **151**, 969 (1966).
[13] E. Cheifetz, R. C. Jarad, S. G. Thompson, and J. B. Wilhelmy, Phys. Rev. Lett. **25**, 38 (1970).
[14] J. B. Ball, R. L. Auble, R. M. Drisko, and P. G. Roos, Phys. Rev. **177**, 1699 (1969).
[15] J. B. Ball and J. S. Larsen, Phys. Rev. Lett. **29**, 1014 (1972).
[16] J. B. Ball, R. L. Auble, and P. G. Roos, Phys. Rev. C **4**, 196 (1971).
[17] J. B. Ball, R. L. Auble, and G. Roos, Phys. Lett. B **29**, 172 (1969).
[18] M. Carchidi and Ph.D. dissertation, University of Pennsylvania, 1985.
[19] K. Heyde and J. L. Wood, Rev. Mod. Phys. **83**, 1467 (2011).
[20] C. Y. Wu, H. Hua, D. Cline, A. B. Hayes, R. Teng, R. M. Clark, P. Fallon, A. Goergen, A. O. Macchiavelli, and K. Vetter, Phys. Rev. C **70**, 064312 (2004).
[21] H. Mach, M. Moszyński, R. L. Gill, F. K. Wohn, J. A. Winger, J. C. Hill, G. Molnár, and K. Sistemich, Phys. Lett. B **230**, 21 (1989).
[22] C. Y. Wu, H. Hua, and D. Cline, Phys. Rev. C **68**, 034322 (2003).
[23] H. Mach, G. Molnar, S. W. Yates, R. L. Gill, A. Aprahamian, and R. A. Meyer, Phys. Rev. C **37**, 254 (1988).
[24] D. Hofer *et al.*, Nucl. Phys. A **551**, 173 (1993).
[25] A. Chakraborty *et al.*, Phys. Rev. Lett **110**, 022504 (2013).
[26] H. T. Fortune, Phys. Rev. C **95**, 054313 (2017).
[27] C. Kremer *et al.*, Phys. Rev. Lett. **117**, 172503 (2016).
[28] D. A. Sazonov, E. A. Kolganova, T. M. Shneidman, R. V. Jolos, N. Pietralla, and W. Witt, Phys. Rev. C **99**, 031304(R) (2019).
[29] W. Witt, N. Pietralla, V. Werner, and T. Beck, Eur. Phys. J A **55**, 79 (2019).