Application of an *ab initio S* matrix to data analysis of transfer reactions to the continuum populating ^{11}Be

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Recently, the bound and continuum spectrum of ^{11}Be has been calculated within the *ab initio* no-core shell model with continuum (NCSMC) method, with the parity inversion in the ground state successfully reproduced. The continuum spectrum obtained is in agreement with known experimental levels. The *S* matrix contained in the NCSMC continuum wave functions of the n + ^{10}Be system is used in this work for the first time in a transfer-to-the-continuum (TC) reaction calculation. The TC approach is applied to study the excitation energy spectrum of ^{11}Be measured in the ^{9}Be(^{18}O, ^{16}O)^{11}Be reaction at 84 MeV. Previously known levels are confirmed and theoretical and experimental evidence for a $9/2^+$ state at $E_x = 5.8$ MeV is given, whose configuration is thought to be $^{10}\mathrm{Be}(2^+) + \mathrm{n}(d_{5/2})$.

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I. INTRODUCTION

This work presents the first application of an *ab initio* structure model for continuum states [1] to the calculation of a transfer reaction to resonant states [2–6]. Thanks to the *S* matrix obtained within the no-core shell model with continuum (NCSMC) method [1], the traditional reduction of the complicated many-body problem to a one-body problem via a phenomenological optical potential to treat the continuum neutron-target final state is not necessary, as previously done for ^{10}Be [6,7]. We show how various states of an exotic nucleus such as ^{11}Be are built up by the addition of one or two neutrons to a ^{9}Be target.

The case of a ^{9}Be target is particularly interesting because one can access the ^{10}Be and ^{11}Be residual nuclei, which have been the subject of a large number of studies by the nuclear physics community in the past years. In particular, ^{11}Be is a weakly bound nucleus with a very rich and somehow surprising phenomenology [8,9]. Some of the interesting characteristics are, for example, the role of correlations between core excited states and neutron orbitals [1,10–12], neutron-neutron pairing, and even molecular clustering [13]. They are all relevant for the discussion in the present work. Thus by comparing one- and two-neutron transfer results, as we have consistently done for some time now, one can understand how correlations arise in an exotic nucleus.

In our investigation we use one- and two-neutron transfer reactions induced by an ^{18}O projectile. In this way states in neutron-rich nuclei are populated starting from stable targets [6,14–21]. The observed selectivity of ^{18}O-induced transfer reactions allows for a consistent exploration of both single-particle features via the (^{18}O, ^{17}O) reaction and two-neutron correlations via the (^{18}O, ^{16}O) reaction. In particular, we argue that the (^{18}O, ^{16}O) reaction proceeds mainly via two mechanisms: (i) a two-step single-neutron transfer, where the two neutrons are independently transferred to accessible orbitals in the target field, and (ii) a one-step transfer of a correlated pair of neutrons, populating mainly two-neutron configurations in the states of the residual nucleus. In the following we use a theoretical model corresponding to mechanism (i).

Here we would like to stress the fact that in the part of the ^{11}Be spectrum that we aim to describe in this work, one neutron is transferred to a bound state while the other is transferred to the continuum, thus the independence of the two mechanisms is better justified. At the moment there is no reaction model using a two-neutron correlated final state with one neutron bound and the other unbound.

The state of the art on this subject is different for (t, p) reactions populating bound states in the target. It can be summarized with the findings in Refs. [22,23], which agree with all previous literature. In the first paper it is stated that "the simultaneous and non-orthogonal contributions [of the two neutrons to transfer] are in anti-phase, so that the contributions corresponding to the coherent superposition of these two amplitudes tend to cancel. The calculated total cross section thus essentially coincides with the successive process." On the other hand, Ref. [23] shows that the two-step, successive transfer also occurs in a correlated fashion. If we were to use the model in Ref. [23], for example, the different

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possible quantum mechanical paths associated with the population of different intermediate states of ^{10}Be + ^{17}O, in our case, would be summed coherently. Also, the different terms (successive transfer, simultaneous transfer, and nonorthogonality contribution) would represent different amplitude contributions to the second-order process, to be added coherently to get the second-order cross section. Their relative importance in a coupled-channel model such as that discussed in Ref. [23] depends on the arbitrary choice of the reaction representation (prior-prior, post-post, prior-post, or post-prior). However, in our case because we treat transfer between heavy ions we can use a semiclassical model in which the post and prior representations coincide. Furthermore, the transfer to the continuum (TC) provides analytical probabilities for energy spectra in the continuum in heavy-ion reactions. Results for (p, t) or (t, p) to bound states can give only partial guidance to this work, due to the presence of a heavy core in the projectile and target and of a neutron final continuum state in our case.

Also for the (^{18}O, ^{16}O) two-neutron transfer reactions the 2^+ excited state at 1.98 MeV in ^{18}O plays a key role in the coupling scheme that would require a coupled-channel theoretical treatment, which, at the moment, is still missing [15]. On the other hand, the successful results of a semiclassical, two-step treatment of the ^{208}Pb(^{16}O, ^{18}O)^{206}Pb reaction [24], have been used for a long time as a justification [25] for neglecting interference effects and simultaneous pair transfer. In our case this is particularly true, as we do not study the angular distribution but energy spectra.

Using the one-neutron transfer reaction we have been able in the past to obtain both bound and continuum states of ^{10}Be [6]. A very accurate description of the latter was achieved thanks to a previous work in which two different n + ^{9}Be optical potentials, fitted to scattering data over a very large energy range [7], were compared. These potentials allowed some of us to calculate an *S* matrix, which is necessary for the correct definition of a continuum nucleon-nucleus state. This in turn is one of the ingredients of the transfer-to-the-continuum model [2–4], which allows us to reproduce the excitation energy spectrum of the target [5,6]. The ^{9}Be(^{18}O, ^{17}O)^{10}Be experiment and relative theoretical description [6] constituted the first step in the study of the ^{9}Be(^{18}O, ^{16}O)^{11}Be reaction that we discuss in this paper. Two-neutron transfer experiments on ^{9}Be have been performed previously at higher incident energies and with different projectiles such as tritium [26–30], ^{6}He [31], ^{13}C [32,33], and ^{16}O [34–36]. From the experimental point of view the novelty in our case resides in the fact that for a heavy projectile the core spectator situation is realized, and thanks to the large relative angular momentum of the projectile and target, coupled with the initial-state angular momentum, high spin states in the final nucleus can be reached [37]. From the theoretical point of view we achieve here a unification of structure and reaction formalisms via the use of an *ab initio S* matrix.

II. THE EXPERIMENT

The ^{18}O^{6+} beam at 84 MeV incident energy was accelerated by the Tandem Van de Graaff facility of the Istituto Nazionale di Fisica Nucleare–Laboratori Nazionali del Sud. A

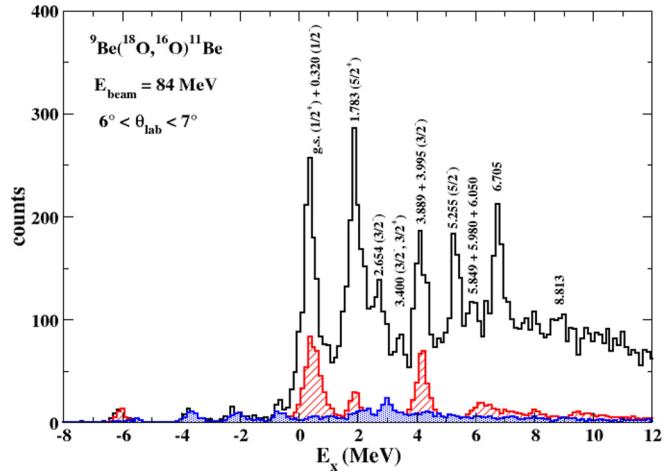

FIG. 1. Inclusive excitation energy spectrum of the ^{9}Be(^{18}O, ^{16}O)^{11}Be reaction at 84 MeV incident energy and $6^\circ < \theta_{\rm lab} < 7^\circ$. The background coming from C and O contaminations in the target is shown as red and blue hatched areas, respectively.

self-supporting 200 ± 10 μg/cm^2 thick ^{9}Be target was used. Supplementary runs with a 50 ± 3 μg/cm^2 self-supporting ^{12}C target and a 260 ± 10 μg/cm^2 WO_3 target were recorded in order to estimate the background in the energy spectra from ^{12}C and ^{16}O impurities in the ^{9}Be target. The MAGNEX magnetic spectrometer [38] was used to momentum analyze the ^{16}O ejectiles, detected by its focal plane detector [39]. The angular range between 3° and 10° in the laboratory reference frame was explored. Details about the particle identification and the trajectory reconstruction techniques used for the reduction of the MAGNEX data can be found in Refs. [40–43]. An overall energy and angular resolution of about 500 keV (FWHM) and 0.3° were obtained. The absolute cross section was also estimated according to Ref. [41], with a total error of about 10% induced by the uncertainties in the target thickness and beam current integration. An example of an energy spectrum for the (^{18}O, ^{16}O) reaction with a ^{9}Be target, in a limited angular range, is shown in Fig. 1, where the background contributions are also shown. The angle-integrated absolute cross-section spectrum obtained after background subtraction is shown in Fig. 2.

III. MODEL-INDEPENDENT ANALYSIS OF THE ENERGY SPECTRUM

A number of peaks signal the population of bound and resonant states of ^{11}Be in this transfer reaction. In our data the $1/2^+$ ground state and the $1/2^-$ first excited state at 0.320 MeV are not resolved. Above the one-neutron separation energy $S_{\rm n} = 0.502$ MeV, a strong excitation of the $5/2^+$ state at 1.783 MeV is observed, while the states at 2.654 and 3.400 MeV are less populated. Interestingly these are the only states observed in one-neutron transfer reactions in the past [44–46], showing a pattern similar to that observed here. This could indicate that, for these states, the ^{10}Be$_{\rm g.s.}$ is preferentially populated as an intermediate system in the first

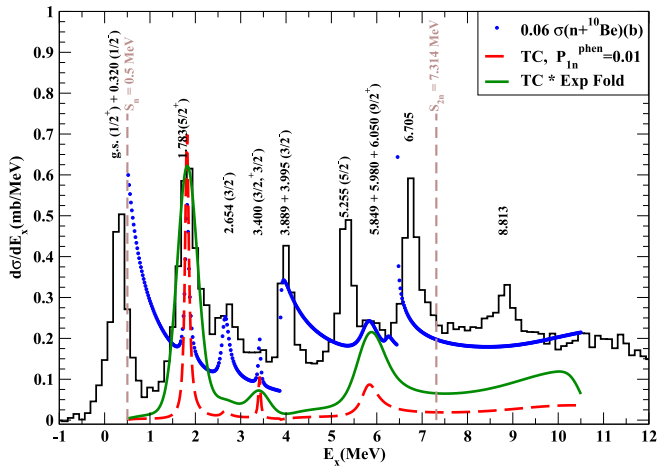

FIG. 2. ^{11}Be inclusive excitation-energy spectrum from the ^{9}Be(^{18}O, ^{16}O)^{11}Be reaction at 84 MeV incident energy, integrated in the total measured angular range $3^\circ < \theta_{\rm lab} < 10^\circ$. Black histogram: experimental data after background subtraction. Dashed red curve: total 2n transfer calculations resulting from the use of the S matrix from Ref. [1]. Solid green curve: the red curve calculation folded with the experimental resolution and renormalized by a factor of 3.2. Dotted blue curve: theoretical free n + ^{10}Be cross section, in barns; see text for details.

step of the reaction and a second neutron is then transferred to accessible single-particle orbitals.

Beyond the ^{10}Be core excitation threshold we observe a peak at 3.9 MeV, where we cannot separate the transitions to the known states at 3.889 MeV ($5/2^-$) and 3.955 MeV ($3/2^-$). Only in Ref. [26] were the two states separated in a two-neutron transfer reaction, showing that they are both strongly excited by such a probe. Intense transitions are observed for the $5/2^-$ state at 5.255 MeV and the state at 6.705 MeV, resembling the situation observed in all the reported two-neutron transfer reaction data [32–34]. These states are not populated in single-neutron transfer or in the ^{11}B(^{7}Li,^{7}Be)^{11}Be charge exchange reaction [47,48], suggesting a selective response of ^{11}Be to two-neutron transfer operators, likely populating $^{9}\mathrm{Be} \otimes (sd)^2$ configurations. Studies of β decay from the $3/2^-$ ^{11}Li parent nucleus show transitions to the $3/2^-$ state at 3.955 MeV and the $5/2^-$ state at 5.255 MeV of ^{11}Be. This does not contradict the above argument about the $^{9}\mathrm{Be} \otimes (sd)^2$ configurations for these states, since β decay can also occur within the ^{9}Li core of the ^{11}Li parent nucleus. In addition, β decay does not populate the state at 6.705 MeV, which would indicate either a different parity or a spin higher than that achievable by the allowed Fermi and Gamow-Teller operators. In Refs. [32–34] this state is described as belonging to a rotational band built on the $3/2^-$ state at 3.955 MeV, together with the 5.255-MeV state, with a $7/2^-$ spin assignment.

Also debated are the states at 5.8 and 8.813 MeV, which are both present in all reported two-neutron transfer experiments, even though the centroid energy and width are slightly different in the various studies. The former is not observed in β decay or in single-neutron transfer, but it is likely the same as observed in the ^{11}B(^{7}Li,^{7}Be)^{11}Be charge exchange reaction at 6.05 MeV (FWHM, 300 keV) [47]. Due to the large momentum transfer, such a reaction can easily excite states with a high spin, not populated in β decay. In Ref. [34] the neutron decay of this state to the first excited 2^+ state of ^{10}Be suggests a core-excited configuration. Recently a $9/2^+$ spin parity for a state with a $[^{10}\mathrm{Be}(2_1^+) \otimes (1d)^{5/2}]^{9/2+}$ stretched configuration in the $^{10}\mathrm{Be}(2_1^+)$ excited state was predicted in the *ab initio* NCSMC approach [1], which is in perfect agreement with the present finding and our interpretation of previous literature. For the state at 8.813 MeV, β-decay studies [49,50] assign a $3/2^-$ spin parity, conflicting with the assumption of a high-spin member of the rotational band built on the $3/2^-$ state at 3.955 MeV proposed in Refs. [32–34]. Our data, despite confirming the population of this state in two-neutron transfer reactions, do not add much to the previous debate.

IV. THEORETICAL APPROACH

From the theoretical point of view, the description of the n + ^{10}Be part of the spectrum above $S_{\rm n}$ is very challenging because no experimental data exist on the free n + ^{10}Be scattering. However, the recent *ab initio* calculation of the ^{11}Be spectrum [1] over a wide energy range allows for a description of both bound and continuum states, the continuum part providing an S matrix. Thus we can apply the TC method overcoming the knowledge of the optical potential thanks to the NCSMC S matrix. One particularly interesting aspect is that the n + ^{10}Be component of the spectrum has contributions from bound states of ^{10}Be.

A. Reaction model

We imagine the reaction going through two independent steps as anticipated in Sec. I and indicated as mechanism (i). In the first step the reaction ^{9}Be(^{18}O, ^{17}O)^{10}Be takes place and neutron n1 is transferred from ^{18}O to populate the bound states of ^{10}Be.

We then imagine that the second neutron, n2, is transferred from ^{17}O to ^{10}Be, and since we just want to describe the continuum part of the spectrum (above $E_x = S_{\rm n}$) we use the following formula for the two-neutron transfer description:

$$\frac{d\sigma_{2n}}{d\varepsilon_{f2}} = C^2S\, P_{\rm n1}^{\rm phen}(R_s) \int_{b_{\rm min}}^{b_{\rm max}} d\mathbf{b_c} |S_{\rm ct}(b_c)|^2 \frac{dP_{\rm n2}(b_c)}{d\varepsilon_{f2}}, \quad (1)$$

$P_{\rm n1}^{\rm phen}$, given by Eq. (3), is the bound-state transfer probability for neutron n1 from ^{18}O to ^{9}Be, at the strong absorption radius between core and target, defined as $R_s = 1.4(A_c^{1/3} + A_t^{1/3})$. Then in the next step, n2 is the second neutron which is transferred to a continuum state from ^{17}O and C^2S is its initial wave function spectroscopic factor. ε_{f2} is the continuum final-state energy of n2 and $|S_{\rm ct}(b_c)|^2$ has been calculated according to [51]. It is the elastic scattering probability between ^{16}O and ^{10}Be, the second-step core and target nuclei, respectively. Following Ref. [6], the correspondence between the measured scattering angle $3^\circ < \theta_{\rm lab} < 10^\circ$ and the impact parameter has been obtained via a classical trajectory calculation according to Ref. [52], providing $7 < b_c < 8$ fm to be used

TABLE I. Structure parameters of the valence neutron (see text). The ^{10}Be spectroscopic factors in the present work were obtained with the same chiral N^2LO_{SAT} NN + 3N interaction [53] as used in the ^{11}Be calculations. The C^2S and C_i values for ^{17}O have been used in the calculations of Eq. (1), while the others are given here for completeness.

State	S_n (MeV)	C_i ($\mathrm{fm}^{-\frac{1}{2}}$)	j	l	C^2S
$^{18}O_{g.s.}$	8.04	1.73	5/2	2	1.7 [15]
$^{17}O_{g.s.}$	4.145	0.69	5/2	2	0.945 [15]
$^{10}Be_{g.s.}$	6.812	2.3	3/2	1	2.13 [54], 2.6 [55], 2.73[a]
$^{10}Be(2_1^+)$	3.44		1/2	1	0.0226 [54], 0.274 [55],[b] 0.036[a]
$^{10}Be(2_1^+)$			3/2	1	0.268 [54], 0.274 [55],[b] 0.24[a]
$^{10}Be(2_2^+)$	0.854		1/2	1	0.616 [54], 0.421 [55],[b] 0.406[a]
$^{10}Be(2_2^+)$			3/2	1	0.13 [54], 0.421 [55],[b] 0.076[a]

[a]From the present work.
[b]Reference [55] does not distinguish the two j components.

in Eq. (1). P_{n1}^{phen} is extracted as described below, from the integrated experimental one-neutron transfer cross section in the ^{9}Be(^{18}O, ^{17}O)^{10}Be reaction whose data were presented in Ref. [6]. Above the threshold for the first excited state of ^{10}Be, P_{n1}^{phen} is obtained consistently using the parameters appropriate to take into account the excitation energy in ^{10}Be listed in Table I.

To take advantage of the fact that the experimental cross section for one-neutron transfer is already known and to minimize the dependence from the parameter's incertitude, we use the one-neutron transfer cross section between bound states formula from [56],

$$\sigma_{1n} = \pi \frac{(R_s - a_c)}{\eta} P_{n1}(R_s), \tag{2}$$

which is obtained when the core-target S matrix is calculated in the sharp cutoff approximation:

$$|S_{ct}(b_c)|^2 = 1 \quad \text{if} \quad b_c > R_s,$$
$$|S_{ct}(b_c)|^2 = 0 \quad \text{if} \quad b_c < R_s.$$

η is a kinematical parameter depending on the initial and final neutron separation energies and the energy of relative motion [56],

$$\eta = \sqrt{\gamma_i^2 + k_1^2},$$

where $\gamma_i^2 = 2mS_{i(1,2)}/\hbar^2$ and $S_{i(1,2)}$ is the bound-state initial separation energy of the first and second neutrons, respectively. $k_1^2 = (Q + 1/2mv^2)^2/(\hbar v)^2$, where $Q = \varepsilon_{i(1,2)} - \varepsilon_{f(1,2)}$ is the Q value. The $\varepsilon_{i(1,2)}$ are the negative initial binding energies of neutrons n1 and n2 in their bound states, while $\varepsilon_{f(1,2)}$ is the negative final energy of the first-step neutron in ^{10}Be and is the positive continuum energy of neutron n2 in ^{11}Be. The η value ranges from 0.68 to 0.88 fm^{-1} for the three bound ^{10}Be states considered here.

R_s is again the strong absorption radius and a_c is the Coulomb length parameter. Then we derive from Eq. (2) the following phenomenological transfer probability at the strong absorption radius to be used to describe the two-neutron transfer in Eq. (1),

$$P_{n1}^{phen}(R_s) = \frac{\sigma_{1n}^{exp}}{\pi \frac{(R_s - a_c)}{\eta}}, \tag{3}$$

where σ_{1n}^{exp}= 2.1 mb ($^{10}Be_{g.s.}$), 2.2 mb ($^{10}Be_{2_1^+}$), and 3.2 mb ($^{10}Be_{2_2^+}$) are the experimental values from Ref. [6]. By adopting this formula we assume that all structure information, such as spectroscopic factors, of both projectile (^{18}O) and target (^{9}Be) are included in the measured σ_{1n}^{exp}.

Our approach is equivalent to the treatment of a two-step process via second-order perturbation theory [52,57], which is justified by the small first-step average probability ($P_{n1}^{phen} = 0.01$; cf. Fig. 2). Furthermore, the case of ^{10}Be, as an intermediate system in the transition from ^{9}Be to ^{11}Be, is quite peculiar and makes our assumption of incoherent summation still approximately valid. The key point is that the ^{10}Be level density is quite low in the energy region of interest, and in fact we only need to consider three states, i.e., $^{10}Be_{g.s.}$ and the excited states at 3.4 MeV ($^{10}Be_{2_1^+}$) and 6 MeV ($^{10}Be_{2_2^+}$). As a consequence, the $^{10}Be_{2_1^+}$ has an important role in ^{11}Be spectra from the inelastic thresholds of ($^{10}Be_{2_1^+}$) + n, located at about 3.9 MeV, while the $^{10}Be_{2_2^+}$ from the inelastic thresholds of ($^{10}Be_{2_2^+}$) + n, located at about 6.5 MeV. Thus for ^{11}Be states up to 3.9 MeV the role of ^{10}Be first and second excited states is expected to be weak. For states of ^{11}Be between 3.9 and 6.5 MeV one can have the contribution from $^{10}Be_{g.s.}$ and the first $^{10}Be_{2^+}$. However, the states of ^{11}Be in that energy range with a pronounced $^{10}Be_{g.s.}$ + n configuration are quite broad since they are far from the ($^{10}Be_{g.s.}$) + n threshold, located at S_n = 0.5 MeV. As an example, a simulated pure single-particle $d_{3/2}$ state at 4 MeV excitation energy would have a width as large as 3 MeV or more. Therefore the role of such configurations, so spread out in energy, is minor in the narrow region covered by the analyzed sharp resonances. In fact, the $9/2^+$ state we claim at 5.9 MeV is built with the ^{10}Be first excited state and not with the $^{10}Be_{g.s.}$, as the *ab initio* calculation of [1] correctly indicates. The same argument holds for ^{11}Be states above 6.5 MeV, where configurations with $^{10}Be_{g.s.}$ and the first $^{10}Be_{2^+}$ likely do not contribute significantly.

The second neutron, n2, transfer probability to the continuum states of ^{11}Be is given by

$$\frac{dP_{n2}(b_c)}{d\varepsilon_{f2}} = \frac{|C_i|^2}{2k_{f2}} \left[\frac{\hbar}{mv}\right] \frac{e^{-2\eta b_c}}{2\eta b_c} \Sigma_{j_f,\nu} (|1 - \bar{S}_{j_f,\nu}|^2 + 1 - |\bar{S}_{j_f,\nu}|^2)(2j_f + 1)(1 + R_{if}) M_{l_f l_i}. \tag{4}$$

Details of the TC method have been given in several previous publications; see, for example, Ref. [3], where the definitions of the parameters appearing in Eq. (4) can also be found.

In Eq. (4) for each continuum energy the sum is over all possible n + ^{10}Be total angular momenta. Above the thresholds for the first and second 2^+ excited states in ^{10}Be, for each angular momentum there is also an incoherent sum over all channels ν contributing to it. This is the same as when calculating total-reaction cross sections and the observable is the final nucleus excitation energy spectrum. The situation is

different (cf. Eq. (6) in Ref. [57]) when two-neutron transfer is discussed with the aim of calculating core angular distributions. In the latter case for each angle different channels can interfere and contribute coherently.

We have usually calculated $\bar{S}_{j_f}$ from an optical potential [3]. The same procedure cannot be applied to the system n + ^{10}Be because there are no data available. However, the *S* matrix calculated by some of us in Ref. [1] is perfectly suited to be used in Eq. (4), as it is given in terms of the n-^{10}Be continuum energy and angular momentum j_f appearing in Eq. (4) and it contains, at each energy, all possible inelastic channel contributions. Note that Eq. (4) contains two terms proportional to $|1 - \bar{S}_{j_f}|^2$ and $1 - |\bar{S}_{j_f}|^2$, giving the elastic and inelastic neutron breakup from the initial state in the projectile, respectively.

B. *Ab initio S* matrix

Some of us have investigated the structure of ^{11}Be by studying the ^{10}Be + n system within the NCSMC approach [1]. This approach [58] uses a basis expansion with two key components: one describing all nucleons close together, forming the ^{11}Be nucleus, and a second describing the separated neutron and ^{10}Be clusters. The former part utilizes a square-integrable harmonic-oscillator basis expansion treating all 11 nucleons on the same footing. The latter part factorizes the wave function into products of ^{10}Be and neutron components and their relative motion with proper bound-state or scattering boundary conditions. We compute the *S* matrix from the 11-body NCSMC calculations by employing the calculable *R*-matrix method [59,60] and matching the many-body ^{11}Be internal wave function with the asymptotic binary n-^{10}Be channels at about 18 fm, well beyond the range of the nuclear interaction. The chiral N^{2}LO$_{\rm SAT}$ two-nucleon (NN) and three-nucleon (3N) interaction [53] served as input. For the ^{10}Be cluster, in addition to the ground state, we also included the first and the second excited 2^+ states. The outcomes of the NCSMC calculations are the energies and wave functions of the bound states, here $1/2^+$ and $1/2^-$ in the correct order compared to experiment, as well as of the continuum states. The latter include the *S* matrix that we in turn apply in the present investigation of the ^{9}Be(^{18}O, ^{16}O)^{11}Be two-neutron transfer reaction. As shown in Figs. 2 and 3 and Table I in Ref. [1], the N^{2}LO$_{\rm SAT}$ interaction provides a quite reasonable description of the low-lying bound states and resonances of ^{11}Be. However, to achieve the spectroscopic accuracy needed in the present study, we turn to the NCSMC-pheno approach which allows us to reproduce the experimental thresholds and bound- and resonant-state energies as presented in the right part of Table I in Ref. [1]. The NCSMC calculations with the N^{2}LO$_{\rm SAT}$ interaction predict a low-lying $9/2^+$ resonance at 5.42 MeV and, with the NCSMC-pheno approach, at 5.59 MeV after adjusting the ^{10}Be thresholds to experiment. In this work we examine an experimental candidate for this resonance at 5.3 MeV. Note that these energies are given with respect to the n + ^{10}Be threshold. Consequently, we have performed a new NCSMC-pheno calculation to fit the calculated $9/2^+$ resonance position to that energy. The resulting *S* matrix is then used in the present study.

V. DISCUSSION OF THE RESULTS

In Fig. 2 we show the experimental inclusive energy spectrum of ^{11}Be together with the theoretical calculations (dashed red curve). This calculation does not contain any fitting parameter. On the other hand, the solid green curve is the same calculation folded with the experimental resolution renormalized by a factor of 3.2 to fit the data. The first three continuum states are well reproduced as far as the position is concerned, indicating that the TC model contains the correct dynamics and the *ab initio S*-matrix accurate structure information. Note that also the order of magnitude is reproduced reasonably well within the incertitude due to the description of the two-neutron transfer reaction mechanism. The calculated relative population of the three resonances at $E_x = 1.783$, 2.654, and 3.400 MeV with $J^\pi = 5/2^+$, $3/2^-$, and $3/2^+$, respectively, compares well with the experimental results for ^{10}Be$(d, p)^{11}$Be [44]. The state at 2.654 MeV is depleted in our model calculation because of the unfavorable one-neutron transfer reaction matching, as it is in the data from [44]. However, it is well seen in the present data as well as in all previous two-neutron transfer experiments [34], confirming for this state a relevant 2n+^{9}Be configuration and a population via the simultaneous 2n-transfer reaction mechanism, type (ii), discussed in Sec. I. On the other hand the calculated peak at $E_x = 3.400$ MeV has such a small width ($\approx$0.02 MeV) from [1] that its presence has little physical significance and in fact it is hardly visible as a structure in the experiment. The transfer calculation probably misses the maximum value for the same reason. Note that in the figures we indicate ($3/2^+$, $3/2^-$) for this state because from the literature it is often quoted as $3/2^-$, while from the present *S*-matrix calculations it appears to be $3/2^+$. As far as the absolute total cross sections are concerned we believe that due to the experimental and theoretical incertitudes the only comparable values are those for the 1.783 MeV $d_{5/2}$ state, for which we get $\sigma_{\rm exp} = 359 \pm 35(\mu b)$ and $\sigma_{\rm th} = 175(\mu b)$. However, one might argue that due to the indistinguishability of the two neutrons, Eq. (1) should contain a factor of 2, which would give an almost-perfect agreement between theory and experiment. On the other hand, as we have discussed before our transfer mechanism model is oversimplified and thus we conclude that the present absolute theoretical cross sections are reliable only at the order-of-magnitude level.

The cross section of the resonance at 5.8 MeV is reasonably well reproduced by our model, thus strengthening the interpretation of a $9/2^+$ stretched configuration, discussed above and in Ref. [1]. It is the only state that can have a definite single-particle nature in the high-energy part of the spectrum, because it has a high spin which does not mix with underlying components of low angular momenta. It can be seen only in a reaction with heavy ions where the matching conditions allow the reaching of a high resonance energy and high angular momentum. This state can be reproduced by the TC, which contains explicitly the spin couplings between initial and final states besides the angular momentum couplings. Therefore the present work provides the first evidence for the existence of such a state. We believe our attribution is quite firm and well justified and is unique so far in the literature. From the point of view of the structure part, we can see the

$9/2^+$ because it is built on the 2_1^+ state of ^{10}Be that we include in our NCSMC calculation and thus it comes about as a $9/2^+$ from the *S*-matrix calculation.

Finally, for comparison, the free n + ^{10}Be cross section (elastic plus inelastic), calculated with the *ab initio S* matrix and rescaled by a factor of 0.06, is shown by the blue dots in Fig. 2. For this cross section the *y*-axis scale should be read in units of barns as indicated in the caption. Obviously this is another observable with respect to the transfer cross section shown in the figure. Note that the magnitude of this cross section is consistent with that of the n + ^{9}Be data and calculation in Ref. [7]. This is a proof of the accuracy of the *ab initio* model in providing the magnitude of the neutron-core couplings (interaction). The three resonance states coupled to the ^{10}Be$_{\text{g.s.}}$ indeed appear at the same positions as in the data and they scale as the simple $1/k^2$ law. Above the first and second 2^+ excited-state thresholds, respectively, the shown free cross section is the sum of the tails of the cross sections from lower neutron energies, in accordance with the hypothesis that ^{10}Be excited states are populated by the first neutron.

VI. CONCLUSIONS

In conclusion, we have presented new experimental data on the ^{9}Be(^{18}O, ^{16}O)^{11}Be two-neutron transfer reaction at 84 MeV, providing the absolute cross-section energy spectra. An *ab initio S* matrix coupled for the first time with the TC method is able to reproduce the position and widths of the n + ^{10}Be components in the experimental excitation energy spectrum of the exotic nucleus ^{11}Be and to predict the order of magnitude of the absolute cross sections. We have given evidence, both theoretical and experimental, of the presence of a $9/2^+$ state at $E_x = 5.8$ MeV with a $[^{10}\text{Be}(2_1^+) \otimes (1d)^{5/2}]^{9/2+}$ configuration.

It is very encouraging to see that very elaborate *ab initio* structure calculations for continuum states can provide the correct ingredients to be easily incorporated in a reaction model which is simple and whose accuracy is within the present state of the art of the literature. We have also shown that exotic nuclei can be successfully studied at stable beam facilities providing interesting complementary information to that obtained with radioactive ion beams.

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