Nucleon-pair picture of low-lying states in semi-magic and open-shell nuclei

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In this paper we demonstrate that a simple nucleon-pair picture, previously investigated for semi-magic even-even nuclei, survives in both odd-mass semi-magic nuclei and a few neutron-rich open-shell nuclei. This is exemplified by low-lying states of the semi-magic nuclei $^{43-48}$Ca and three open-shell nuclei ^{132}Cd, ^{131}Ag, and ^{130}Pd, calculated with effective interactions. Not only the generalized-seniority-zero, -one and -two states, but also states with larger seniority numbers are shown to be well represented by one-dimensional nucleon-pair wave functions. Very interestingly, such one-dimensional nucleon-pair wave functions are readily adopted to be those with the lowest expectation energies among all nucleon-pair basis states. This provides us with a simple approach to study yrast states of semi-magic nuclei as well as yrast states of open-shell nuclei with only a few valence nucleons.

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I. INTRODUCTION

Atomic nuclei are complicated many-body systems composed of protons and neutrons. Due to the short range and attractive character of effective interactions between nucleons, pairing correlation [1,2] plays a crucial role in low-lying states of atomic nuclei, and gives rise to a number of simple patterns in nuclear systems. A very simple notation of pairing correlation is the (generalized) seniority [3–6], which is the number of nucleons not coupled to (collective) spin-zero pairs, namely, S pairs, in many-body basis states. The generalized seniority scheme [4–6] is very useful in interpreting regularities of low-lying structures in semi-magic nuclei. There have been extensive studies on the generalized seniority scheme from different perspectives, see e.g., recent works of Refs. [7–9]. Unpaired nucleons can be further coupled into pairs of nonzero good spins in the nucleon-pair approximation [2,10–12] of the shell model.

Applications of the generalized seniority scheme and the nucleon-pair approximation to low-lying states of semi-magic nuclei have attracted much attention. For example, Lei *et al.* calculated the overlaps between wave functions based on nucleon-pair approximation and those of exact shell-model calculations for ^{46}Ca [13] with a phenomenological shell-model Hamiltonian and for $^{43-46}$Ca [14] with the GXPF1A interaction [15]; Luo and Caprio *et al.* studied low-lying states of $^{42-58}$Ca and some other nuclei in the generalized seniority scheme with seniority number $\nu \leqslant 4$ [16,17]. For heavier semi-magic nuclei, there have been studies in regard to the magnetic moments of the 2_1^+ states and the $B(E2, 2_1^+ \to 0_1^+)$ values throughout the even-even $^{102-130}$Sn isotopes, using the nucleon-pair approximation [18,19] and the generalized seniority scheme [20,21]. In addition, low-lying states of neutron-rich ^{134,136,138}Sn were shown to be well described by the generalized seniority scheme [22]. Low-lying isomers were observed experimentally in ^{136,138}Sn, as well as in the $N = 82$ isotones ^{130}Cd and ^{128}Pd, and interpreted in terms of the seniority structures within the shell-model framework [23–25].

For open-shell nuclei, there have been a number of studies for transitional nuclei in the $A \sim 130$ region [26–34], where collective behaviors (e.g., the back-bending phenomenon in even-even transitional nuclei [28,33,34] and doublet bands of odd-odd transitional nuclei [32]) were interpreted from the perspective of collective nucleon pairs. For $N \sim Z$ open-shell nuclei, there have been intensive studies interpreting the structures of low-lying states in terms of isoscalar proton-neutron pairs; see, e.g., Refs. [35–40]. For neutron-rich nuclei, it is expected that the pairing correlation between like nucleons plays an even more pronounced role than that in stable nuclei. Recently the 2_1^+ state of the very neutron-rich ^{132}Cd nucleus in the southeast region of ^{132}Sn has been observed experimentally, and a dominant neutron excitation was suggested for this excited state [41].

Recently, some of the present authors have shown [42,43] that low-lying states of even-even semi-magic $N = 82$ isotones and Sn isotopes are well described by one-dimensional, collective-nucleon-pair wave functions. Such a one-dimensional pattern in the nucleon-pair configurations

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coincides with the generalized seniority scheme for states with generalized-seniority-zero and -two, while it is unexpected for states with larger seniority numbers and higher spins. It is therefore the purpose of this article to study whether or not this simple picture of one-dimensional nucleon-pair wave functions survives in more general situations other than low-lying states of even-even semi-magic nuclei. We demonstrate that this picture is also well applicable to low-lying states of odd-A semi-magic nuclei as well as a few open-shell nuclei. This applicability is exemplified by three odd-A Ca isotopes, ^{43,45,47}Ca, and their even-even neighbors, ^{44,46,48}Ca, with the effective interaction GXPF1A [15], and three neutron-rich open-shell nuclei, ^{132}Cd, ^{131}Ag, and ^{130}Pd, with the effective interaction proposed recently for the southeast region of ^{132}Sn in Ref. [44]. Very interestingly, we show that the approximated wave functions of these low-lying states are readily constructed without resorting to diagonalization of the shell-model Hamiltonian.

This paper is organized as follows. In Sec. II we give a brief introduction to our theoretical framework, including our nucleon-pair basis states and the shell-model Hamiltonian adopted in this paper. In Sec. III we present our numerical results based on the simple one-dimensional nucleon-pair wave function and a detailed comparison between results of this simple picture and exact shell-model results, for both even-A and odd-A semi-magic nuclei $^{43-48}$Ca and open-shell nuclei ^{132}Cd, ^{131}Ag, and ^{130}Pd. Finally, we summarize our main conclusions in Sec. IV of this paper.

II. THEORETICAL FRAMEWORK

In this paper our study is performed within the framework of the nucleon-pair approximation (NPA) of the shell model [2,10–12], in which the basis states are constructed by collective nucleon pairs with various spins. In this section we present a brief introduction to the basis states in the NPA and the shell-model Hamiltonian adopted in this paper.

A. Collective-pair basis states

We begin with the definition of nucleon-pair operators. A collective nucleon pair with spin r is defined by

$$A^{r\dagger} \equiv A^{r\dagger}_{\mu} = \sum_{ab} y(abr) A^{r\dagger}(ab),$$
$$A^{r\dagger}(ab) \equiv A^{r\dagger}_{\mu}(ab) = (a^{\dagger} \times b^{\dagger})^{r}_{\mu}. \tag{1}$$

Here we denote the creation operator of the a orbit associated with quantum numbers n_a, l_a, j_a, m_a by using $a^{\dagger} \equiv a^{\dagger}_{j_a m_a}$, and that of the b orbit by using $b^{\dagger} \equiv a^{\dagger}_{j_b m_b}$. $A^{r\dagger}(ab)$ is a noncollective pair, and $(a^{\dagger} \times b^{\dagger})^{r}_{\mu} = \sum_{m_a m_b} C^{r\mu}_{j_a m_a j_b m_b} a^{\dagger} b^{\dagger}$ where $C^{r\mu}_{j_a m_a j_b m_b}$ is the Clebsch-Gordan coefficient. The collective pair creation operator $A^{r\dagger}$ is given by the linear combination of various noncollective pairs all with spin r. $y(abr)$ is the structure coefficient which is optimized with the shell-model Hamiltonian.

For a system with $2N$ valence nucleons, the pair basis states are constructed by coupling N nucleon pairs successively,

$$[(A^{r_1\dagger} \times A^{r_2\dagger})^{(J_2)} \times \cdots \times A^{r_N\dagger}]^{(J_N)}|0\rangle; \tag{2}$$

and for a system with $(2N+1)$ valence nucleons, the pair basis states are constructed by coupling the N nucleon pairs successively to the odd nucleon, namely,

$$\{[(a^{\dagger}_{j} \times A^{r_1\dagger})^{(J_1)} \times A^{r_2\dagger}]^{(J_2)} \times \cdots \times A^{r_N\dagger}\}^{(J_N)}|0\rangle. \tag{3}$$

Here $a^{\dagger}_{j}$ denotes the operator that creates a nucleon in the j orbit.

In our nucleon-pair states studied in this paper, S pairs play the key role, due to the attractive and short-range nature of effective nucleon-nucleon interactions. We define

$$S^{\dagger} = \sum_{j} y(jj0)(a^{\dagger}_{j} \times a^{\dagger}_{j})^{(0)} = \sum_{j} y(jj0) S^{\dagger}_{j}. \tag{4}$$

For a given system with $2N$ identical valence nucleons, the structure coefficients $y(jj0)$ are determined variationally to minimize the energy expectation of the S-pair condensate state [45], i.e., to solve the following equation

$$\delta \frac{\langle S^N|H|S^N\rangle}{\langle S^N|S^N\rangle} = 0. \tag{5}$$

We consider next a non-S pair with spin r (r is not equal to zero). We diagonalize the shell-model Hamiltonian in the $(S^{\dagger})^{(N-1)} A^{r\dagger}(j_1 j_2)$ space with j_1, j_2 running over all the single-particle orbits, namely in the space with a generalized-seniority number $\nu = 2$. The lowest-energy wave function is written in the form

$$(S^{\dagger})^{(N-1)} \sum_{j_1 j_2} c(j_1 j_2) A^{r\dagger}(j_1 j_2), \tag{6}$$

and we assume $A^{r\dagger} = \sum_{j_1 j_2} y(j_1 j_2 r) A^{r\dagger}(j_1 j_2)$ with $y(j_1 j_2 r) = c(j_1 j_2)$; the structure coefficients for the second collective pair with the same spin and parity correspond to the second-lowest-energy wave function, and so on. For the system of $(2N+1)$ particles, we use the same structure coefficients as those of $2N$ particles. For open-shell nuclei, the structures of proton pairs and those of neutron pairs are determined separately and as above.

B. Shell-model Hamiltonian

The shell-model Hamiltonian with an effective interaction is given by

$$H = \sum_{j} \varepsilon_j N_j + \sum_{j_1 \leqslant j_2} \sum_{j_3 \leqslant j_4} \sum_{JM} \sum_{TM_T} \times \frac{V_{JT}(j_1 j_2 j_3 j_4)}{\sqrt{(1+\delta_{j_1 j_2})(1+\delta_{j_3 j_4})}} A^{JT\dagger}_{MM_T}(j_1 j_2) A^{JT}_{MM_T}(j_3 j_4). \tag{7}$$

Here $N_j = \sum_{m\tau} a^{\dagger}_{jm\frac{1}{2}\tau} a_{jm\frac{1}{2}\tau}$; the pair creation operator with both good spin J and good isospin T is given by

$$A^{JT\dagger}_{MM_T}(j_1 j_2) = \sum_{m_1 m_2} \sum_{\tau_1 \tau_2} C^{JM}_{j_1 m_1 j_2 m_2} C^{TM_T}_{\frac{1}{2}\tau_1 \frac{1}{2}\tau_2} a^{\dagger}_{j_1 m_1 \frac{1}{2}\tau_1} a^{\dagger}_{j_2 m_2 \frac{1}{2}\tau_2}.$$

The two-body effective interaction $V_{JT}(j_1 j_2 j_3 j_4)$ can be either derived microscopically from the realistic nuclear force [46–49], or obtained by fitting to experimental data in a shell region [15,50–52].

In the NPA, we decompose the shell-model Hamiltonian of Eq. (7) into the proton part, neutron part, and proton-neutron part, i.e.,

$$
\begin{aligned}
H &= \sum_{\sigma=\pi,\nu} H_\sigma + H_{\pi\nu}, H_\sigma = \sum_j \varepsilon_j n_{j\sigma} + \sum_{j_1\leqslant j_2}\sum_{j_3\leqslant j_4}\sum_J \frac{V_{JT=1}(j_1j_2j_3j_4)}{\sqrt{(1+\delta_{j_1j_2})(1+\delta_{j_3j_4})}} \hat{J}\big(A_\sigma^{J\dagger}(j_1j_2)\times \tilde{A}_\sigma^J(j_3j_4)\big)^{(0)},\\
H_{\pi\nu} &= -\sum_{j_1j_2}\sum_{j_3j_4}\sum_J V_J^{\pi\nu}(j_1j_2j_3j_4)\hat{J}\big(\big(a_{j_1\pi}^\dagger\times a_{j_2\nu}^\dagger\big)^J\times\big(\tilde{a}_{j_3\pi}\times\tilde{a}_{j_4\nu}\big)^J\big)^{(0)},\\
V_J^{\pi\nu}(j_1j_2j_3j_4) &= \frac{1}{2}[V_{JT=1}(j_1j_2j_3j_4)+V_{JT=0}(j_1j_2j_3j_4)]\sqrt{(1+\delta_{j_1j_2})(1+\delta_{j_3j_4})}.
\end{aligned}
\tag{8}
$$

Here $n_{j\sigma}=\sum_m a_{jm\sigma}^\dagger a_{jm\sigma}$, $\hat{J}=\sqrt{2J+1}$, and $\tilde{A}_\sigma^J$ is the time reversal operator of the pair destruction. To calculate matrix elements of the above $H_{\pi\nu}$ in the NPA, we further express $H_{\pi\nu}$ in terms of proton-neutron multipole-multipole interactions. Denoting $j_{1\pi}\equiv j_\pi$, $j_{2\nu}\equiv j_\nu$, and $j_{3\pi}\equiv j'_\pi$, $j_{4\nu}\equiv j'_\nu$, we have

$$
H_{\pi\nu} = \sum_{j_\pi j'_\pi}\sum_{j_\nu j'_\nu}\sum_k\left(\sum_J (-)^{J+j_\nu+j'_\pi}(2J+1)\begin{Bmatrix} j_\pi & j_\nu & J\\ j'_\nu & j'_\pi & k\end{Bmatrix} V_J^{\pi\nu}(j_\pi j_\nu j'_\pi j'_\nu)\right)(-)^k\hat{k}[Q^k(j_\pi j'_\pi)\times Q^k(j_\nu j'_\nu)]^{(0)}, \tag{9}
$$

where $Q^k(j_\sigma j'_\sigma)=(a_{j\sigma}^\dagger\times\tilde{a}_{j'\sigma})^k$.

For Ca isotopes, the Hamiltonian for valence neutrons is H_σ of Eq. (8) with $\sigma=\nu$, and we use the $T=1$ channel of the effective interaction GXPF1A [15] for these valence neutrons in the $0f1p$ shell. For open-shell nuclei ^{132}Cd, ^{131}Ag, and ^{130}Pd, we use the effective interaction proposed recently [44] for these very neutron-rich nuclei with valence protons in the 28–50 major shell and valence neutrons in the 82–126 major shell.

III. RESULTS AND DISCUSSIONS

We begin our discussion with the construction of our *one-dimensional nucleon-pair wave function* for yrast states. In the generalized seniority scheme, for seniority-zero ground states and seniority-two states, such wave functions are readily available, i.e., they are respectively the S-pair condensate state and the state of one broken pair coupled to $(N-1)$ S pairs [45]. If one goes to higher excited states involving two or more broken pairs, however, it is unknown *a priori* the magnitude of mixing between states with the same or different seniorities. Therefore it was quite unexpected that low-lying states of some even-even semi-magic nuclei, Sn isotopes and $N=82$ isotones, were found to be well represented by one-dimensional nucleon-pair wave functions in Refs. [42,43], where this simple picture was achieved through numerical experiments within the nucleon-pair approximation (NPA) of the shell model. The building blocks of our trial one-dimensional nucleon-pair wave functions are enumerated using lowest-energy collective nucleon pairs of all possible spins; therefore, except for states of generalized-seniority-zero and -two, many trial nucleon-pair basis states are enumerated to obtain the optimized, one-dimensional nucleon-pair wave function for a given yrast state. In Refs. [42,43], after diagonalizations of the shell-model Hamiltonian in both the NPA and full shell-model configurations, we obtain the NPA and shell-model wave functions, and the one-dimensional nucleon-pair wave function is adopted to be the pair basis state which overlaps best with the NPA wave function among all pair basis states included in the NPA configuration space.

In this paper we report a new and particularly simple approach to obtain the optimized one-dimensional nucleon-pair wave function for yrast states. The first step is again to enumerate all possible nucleon-pair basis states (denoted as $|\tau\rangle$), in the form of Eq. (2) for a system of $2N$ valence nucleons and in the form of Eq. (3) for a system of $(2N+1)$ valence nucleons, both with $r_1\leqslant r_2\leqslant\cdots\leqslant r_N$. Then we calculate the expectation energy of each basis state, i.e.,

$$
E_\tau = \frac{\langle\tau|H|\tau\rangle}{\langle\tau|\tau\rangle}.
$$

These E_τ values are sorted from the smaller to the larger, $E_{\tau_1}\leqslant E_{\tau_2}\leqslant\cdots\leqslant E_{\tau_n}$, with n the number of all spin-J_N states constructed using considered pairs and allowed by the Pauli principle. We find, very interestingly, that $|\tau_1\rangle$, namely, the nucleon-pair basis state *which gives the lowest energy* of the Hamiltonian, well represents the shell-model wave function of corresponding yrast state.

In Fig. 1 we plot excitation energies of yrast states of $^{43-48}$Ca (both odd-mass and even-mass ones), including exact shell-model results (SM) and our E_{τ_1}, calculated with the same effective interaction, as well as experimental data. To construct pair basis states $|\tau_i\rangle$ with $i=1,\ldots,n$, we adopted lowest-energy pairs of spin-0,1,2,3,4,5,6. In Fig. 1 one sees that these three sets of excitation energies, i.e., the E_{τ_1}, the SM, and experimental results, are remarkably consistent with each other. The overlaps between the SM wave function and corresponding one-dimensional pair wave function $|\tau_1\rangle$ are plotted in Fig. 2, where one sees that these overlap values (except for a few exceptions) are larger than 0.9.

In Table I we present the one-dimensional pair wave function $|\tau_1\rangle$ for yrast states of ^{43,45,47}Ca with spin from 1/2 to 27/2. Normalization factors are not given for simplicity. We also present, in many cases, another one-dimensional pair wave function $|\tau'_1\rangle$ which has a very large overlap (listed in the parenthesis) with $|\tau_1\rangle$. One sees the pair wave functions $|\tau'_1\rangle$ form a set of states in which the lowest $f_{7/2}$ orbit is favored by the odd neutron and the spin of zero is favored by collective nucleon pairs. Thus, although the pair wave functions $|\tau_1\rangle$

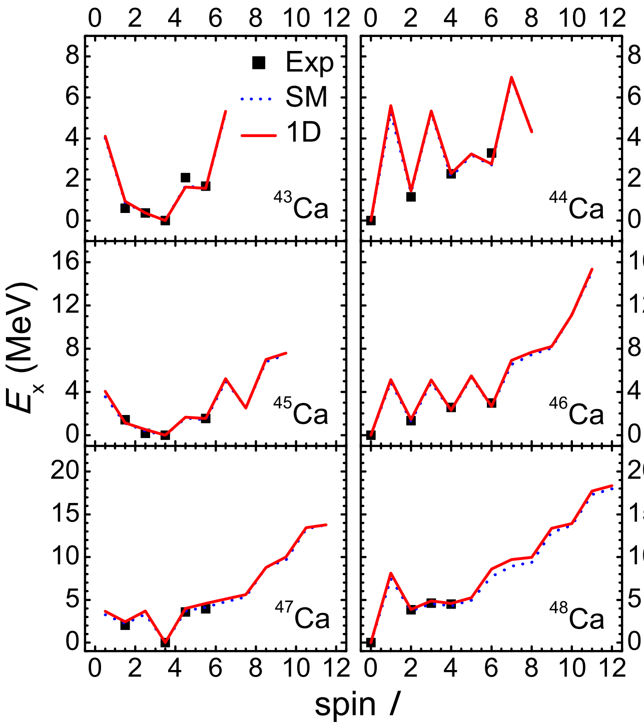

FIG. 1. Excitation energies (denoted as E_x, in units of MeV) vs the spin of the state, for negative-parity yrast states of odd-mass ^{43,45,47}Ca and positive-parity yrast states of even-even ^{44,46,48}Ca. The experimental data are taken from [53] and denoted by "Exp", and the results obtained in the shell-model space and those given by the one-dimensional pair structures are denoted by "SM" and "1D", respectively. The two-body effective interaction GXPF1A together with the single-particle energies is taken from Ref. [15].

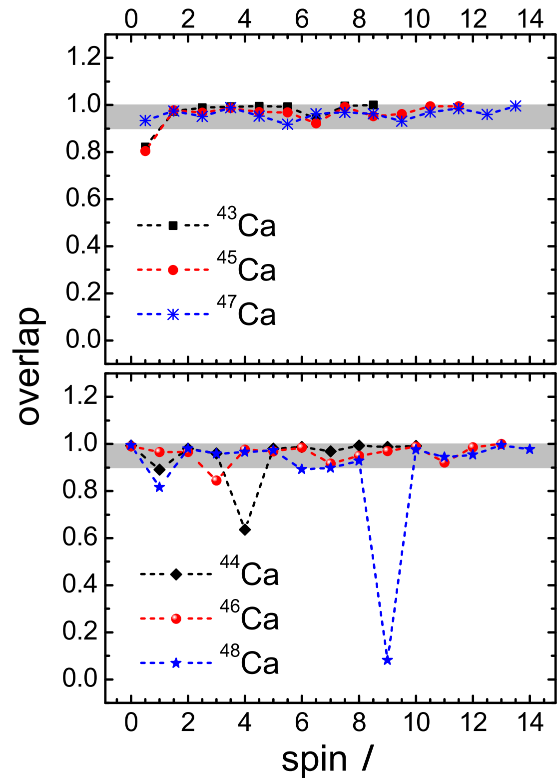

FIG. 2. Overlaps between one-dimensional nucleon-pair wave functions and corresponding shell-model wave functions. We consider lowest-energy pairs of spin-0,1,2,3,4,5,6. We adopt the pair basis states with the lowest values of $\frac{\langle\tau|H|\tau\rangle}{\langle\tau|\tau\rangle}$ to be the one-dimensional pair wave functions for these yrast states.

look irregular, they essentially respect the lowest-seniority scheme.

It has been well known that two nucleon-pair basis states of even-even nuclei, constructed by pairs of different spins, are in general nonorthogonal. Here we also see examples in Table I that two nucleon-pair basis states with the odd nucleon in different orbits can overlap very well with each other. This seemingly surprising phenomenon is originated from the fact that we use collective pairs. We exemplify this using the $\frac{19}{2}{}^-_1$ state of ^{45}Ca and the $\frac{3}{2}{}^-_1$ state of ^{47}Ca. For the $\frac{19}{2}{}^-_1$ state of ^{45}Ca, $|p_{\frac{3}{2}}GG;\frac{11}{2}\rangle$ is actually well represented by $|\alpha\rangle = |p_{\frac{3}{2}}G(f_{\frac{7}{2}}f_{\frac{7}{2}})G(f_{\frac{7}{2}}f_{\frac{7}{2}});\frac{11}{2}\rangle$ with an overlap of 0.99, and $|f_{\frac{7}{2}}DH;\frac{9}{2}\rangle$ is well represented by $|\beta\rangle = |f_{\frac{7}{2}}D(f_{\frac{7}{2}}f_{\frac{7}{2}})H(p_{\frac{3}{2}}f_{\frac{7}{2}});\frac{9}{2}\rangle$ with an overlap of 0.99. The overlap $\langle\tau_1|\tau_1'\rangle$ is then closely equal to $\langle\alpha|\beta\rangle$ which is 1.00. Therefore, the two wave functions $|\tau_1\rangle$ and $|\tau_1'\rangle$ with the odd neutron in different orbits are actually well approximated by very similar configurations both having four nucleons in the $f_{\frac{7}{2}}$ orbit and one nucleon in the $p_{\frac{3}{2}}$ orbit. The situation is similar for the $\frac{3}{2}{}^-_1$ state of ^{47}Ca, where $|p_{\frac{3}{2}}SSS;\frac{3}{2},\frac{3}{2}\rangle$ is well approximated by $|\gamma\rangle = |p_{\frac{3}{2}}S(f_{\frac{7}{2}}f_{\frac{7}{2}})S(f_{\frac{7}{2}}f_{\frac{7}{2}})S(f_{\frac{7}{2}}f_{\frac{7}{2}});\frac{3}{2},\frac{3}{2}\rangle$ with an overlap of 0.98, and $|f_{\frac{7}{2}}SSF;\frac{7}{2},\frac{7}{2}\rangle$ by $|\delta\rangle = |f_{\frac{7}{2}}S(f_{\frac{7}{2}}f_{\frac{7}{2}})S(f_{\frac{7}{2}}f_{\frac{7}{2}})F(p_{\frac{3}{2}}f_{\frac{7}{2}});\frac{7}{2},\frac{7}{2}\rangle$ with an overlap of 0.99; the overlap $\langle\gamma|\delta\rangle$ is equal to 0.99.

In Table II, similar to Table I, we present the one-dimensional pair wave function for yrast states of three nuclei with even mass numbers, ^{44,46,48}Ca. One sees that as excitation energies become higher, more and more non-S pairs are involved.

As shown in Fig. 2, the overlaps between $|\tau_1\rangle$ and corresponding shell-model wave function for the 4_1^+ state of ^{44}Ca and the 9_1^+ state of ^{48}Ca are small. One would ask whether the simple picture that shell-model wave functions are well represented by one-dimensional pair wave functions survives for these two cases. In order to answer this question, we calculate the overlaps between the shell-model wave function and all possible nucleon-pair basis states for these two cases. We find that the nucleon-pair basis states, $|DI\rangle$ for the 4_1^+ state of ^{44}Ca, and $|SGGG;4,6\rangle$ for the 9_1^+ state of ^{48}Ca, present very large overlaps (0.95 and 0.93, respectively) with corresponding shell-model wave functions. Therefore the simple picture of a one-dimensional nucleon-pair wave function survives, although the nucleon-pair basis state with the lowest energy is not the optimal basis state in these two cases.

Now we come to three neutron-rich open-shell nuclei, ^{132}Cd, ^{131}Ag, and ^{130}Pd. In Fig. 3 we plot yrast-state excitation energies of these nuclei, calculated in the full shell-model configuration space and by using our one-dimensional

TABLE I. One-dimensional nucleon-pair wave functions for yrast states of semi-magic odd-mass ^{43,45,47}Ca, in the form of Eq. (3) and denoted as $|\tau_1\rangle = |jr_1 \dots r_N; J_1, \dots, J_{N-1}\rangle$. S, P, D, F, G, H, I represent positive-parity collective pairs with spin-0, 1, …, 6, respectively. Normalization factors are not listed for simplicity. In many cases, we also present another nucleon-pair basis state $|\tau_1'\rangle$ which has a very large value of overlap with $|\tau_1\rangle$ (following $|\tau_1\rangle$ and the symbol "/" in this Table). The values of normalized overlaps $\langle\tau_1|\tau_1'\rangle$ are listed inside parentheses.

I^{P}	^{43}Ca	^{45}Ca	^{47}Ca
$1/2^-$	$\lvert p_{\frac{3}{2}}D\rangle/\lvert f_{\frac{7}{2}}F\rangle$ (1.00)	$\lvert p_{\frac{3}{2}}SD;\frac{3}{2}\rangle$	$\lvert f_{\frac{7}{2}}DFG;\frac{11}{2},\frac{7}{2}\rangle/\lvert f_{\frac{7}{2}}SDD;\frac{7}{2},\frac{3}{2}\rangle$ (0.96)
$3/2^-$	$\lvert f_{\frac{7}{2}}D\rangle$	$\lvert f_{\frac{7}{2}}DG;\frac{9}{2}\rangle/\lvert f_{\frac{7}{2}}SG;\frac{7}{2}\rangle$ (0.96)	$\lvert p_{\frac{3}{2}}SSS;\frac{3}{2},\frac{3}{2}\rangle/\lvert f_{\frac{7}{2}}SSF;\frac{7}{2},\frac{7}{2}\rangle$ (0.98)
$5/2^-$	$\lvert f_{\frac{7}{2}}D\rangle$	$\lvert f_{\frac{7}{2}}DI;\frac{11}{2}\rangle/\lvert f_{\frac{7}{2}}SD;\frac{7}{2}\rangle$ (0.96)	$\lvert f_{\frac{7}{2}}DFI;\frac{11}{2},\frac{15}{2}\rangle$
$7/2^-$	$\lvert f_{\frac{7}{2}}S\rangle$	$\lvert f_{\frac{7}{2}}SG;\frac{7}{2}\rangle/\lvert f_{\frac{7}{2}}SS;\frac{7}{2}\rangle$ (0.99)	$\lvert f_{\frac{7}{2}}SSD;\frac{7}{2},\frac{7}{2}\rangle/\lvert f_{\frac{7}{2}}SSS;\frac{7}{2},\frac{7}{2}\rangle$ (0.99)
$9/2^-$	$\lvert f_{\frac{7}{2}}G\rangle$	$\lvert f_{\frac{7}{2}}GG;\frac{7}{2}\rangle/\lvert f_{\frac{7}{2}}SG;\frac{7}{2}\rangle$ (0.96)	$\lvert p_{\frac{3}{2}}SGG;\frac{3}{2},\frac{7}{2}\rangle$
$11/2^-$	$\lvert f_{\frac{7}{2}}D\rangle$	$\lvert f_{\frac{7}{2}}GG;\frac{3}{2}\rangle/\lvert f_{\frac{7}{2}}SD;\frac{7}{2}\rangle$ (0.95)	$\lvert f_{\frac{7}{2}}DDI;\frac{9}{2},\frac{13}{2}\rangle/\lvert f_{\frac{7}{2}}SDF;\frac{7}{2},\frac{5}{2}\rangle$ (0.95)
$13/2^-$	$\lvert f_{\frac{7}{2}}F\rangle$	$\lvert f_{\frac{7}{2}}SF;\frac{7}{2}\rangle$	$\lvert f_{\frac{7}{2}}SFI;\frac{7}{2},\frac{7}{2}\rangle$
$15/2^-$	$\lvert f_{\frac{7}{2}}G\rangle$	$\lvert f_{\frac{7}{2}}GG;\frac{9}{2}\rangle/\lvert f_{\frac{7}{2}}SG;\frac{7}{2}\rangle$ (0.96)	$\lvert f_{\frac{7}{2}}DDH;\frac{7}{2},\frac{7}{2}\rangle/\lvert f_{\frac{7}{2}}SDH;\frac{7}{2},\frac{5}{2}\rangle$ (0.98)
$17/2^-$	$\lvert f_{\frac{7}{2}}H\rangle$	$\lvert f_{\frac{7}{2}}DG;\frac{11}{2}\rangle$	$\lvert f_{\frac{5}{2}}SGG;\frac{5}{2},\frac{13}{2}\rangle/\lvert f_{\frac{7}{2}}SSI;\frac{7}{2},\frac{7}{2}\rangle$ (0.99)
$19/2^-$		$\lvert p_{\frac{3}{2}}GG;\frac{11}{2}\rangle/\lvert f_{\frac{7}{2}}DH;\frac{9}{2}\rangle$ (0.99)	$\lvert p_{\frac{3}{2}}GII;\frac{11}{2},\frac{17}{2}\rangle$
$21/2^-$		$\lvert f_{\frac{5}{2}}GG;\frac{13}{2}\rangle/\lvert f_{\frac{7}{2}}DH;\frac{11}{2}\rangle$ (1.00)	$\lvert f_{\frac{7}{2}}GGI;\frac{11}{2},\frac{17}{2}\rangle/\lvert f_{\frac{7}{2}}SHH;\frac{7}{2},\frac{13}{2}\rangle$ (0.92)
$23/2^-$		$\lvert f_{\frac{5}{2}}GH;\frac{13}{2}\rangle/\lvert f_{\frac{7}{2}}FH;\frac{13}{2}\rangle$ (1.00)	$\lvert f_{\frac{5}{2}}GGH;\frac{9}{2},\frac{15}{2}\rangle/\lvert f_{\frac{7}{2}}SFI;\frac{7}{2},\frac{11}{2}\rangle$ (0.98)
$25/2^-$			$\lvert f_{\frac{7}{2}}FGH;\frac{9}{2},\frac{15}{2}\rangle$
$27/2^-$			$\lvert f_{\frac{5}{2}}DGI;\frac{9}{2},\frac{15}{2}\rangle/\lvert f_{\frac{7}{2}}GGG;\frac{11}{2},\frac{19}{2}\rangle$ (1.00)

nucleon-pair states, with the effective interaction recently developed for this region [44]. In our calculation, we adopted seven lowest-energy proton-hole pairs of positive parity and spin-0,1,2,3,4,6,8 and eleven lowest-energy neutron pairs of positive parity and spin-0,1,2,3,4,5,6,7,8,10,12, to construct pair basis states. In Fig. 3 one sees that the two sets of calculated excitation energies are very consistent with each other. Low-energy levels of these nuclei are close to the limit of current experimental access: only the 2_1^+ state of ^{132}Cd was studied experimentally [41]; and both calculations reproduce this experimental datum. In Fig. 4 we present the overlaps between shell-model wave functions and corresponding one-dimensional pair wave functions. One sees that with the exceptions of states with spin $I = 10$ and 12 for ^{132}Cd and ^{130}Pd, and four excited states of ^{131}Ag, the overlaps are typically larger than or around 0.8. Therefore these states are essentially represented by our one-dimensional nucleon-pair wave functions adopted via a very simple procedure in this paper.

TABLE II. Same as Table I except that wave functions are in the form of Eq. (2) and denoted as $|r_1 \dots r_N; J_2, \dots, J_{N-1}\rangle$ for semi-magic even-even ^{44,46,48}Ca.

I^{P}	^{44}Ca	^{46}Ca	^{48}Ca
0^+	$\lvert SS\rangle$	$\lvert SSS;0\rangle$	$\lvert SSSS;0,0\rangle$
1^+	$\lvert HI\rangle$	$\lvert HII;6\rangle$	$\lvert SSDF;0,2\rangle$
2^+	$\lvert SD\rangle$	$\lvert SDG;2\rangle$	$\lvert SSSD;0,0\rangle$
3^+	$\lvert SF\rangle$	$\lvert SSF;0\rangle$	$\lvert SSSF;0,0\rangle$
4^+	$\lvert SG\rangle$	$\lvert SGI;4\rangle$	$\lvert SSSG;0,0\rangle$
5^+	$\lvert GI\rangle$	$\lvert SSH;0\rangle$	$\lvert SSSH;0,0\rangle$
6^+	$\lvert SI\rangle$	$\lvert SGG;4\rangle$	$\lvert SSDG;0,2\rangle$
7^+	$\lvert FG\rangle$	$\lvert SFG;3\rangle$	$\lvert SSFH;0,3\rangle$
8^+	$\lvert GG\rangle$	$\lvert DFG;5\rangle$	$\lvert SSGG;0,4\rangle$
9^+	$\lvert FI\rangle$	$\lvert FGG;6\rangle$	$\lvert SSGI;0,4\rangle$
10^+	$\lvert II\rangle$	$\lvert SII;6\rangle$	$\lvert SSHI;0,5\rangle$
11^+		$\lvert HII;6\rangle$	$\lvert SFFI;3,6\rangle$
12^+		$\lvert DHH;7\rangle$	$\lvert SGHI;4,8\rangle$
13^+		$\lvert HHI;10\rangle$	$\lvert GHII;9,8\rangle$
14^+			$\lvert GGII;6,12\rangle$

In Table III we list our one-dimensional nucleon-pair wave function $|\tau_1\rangle$ for yrast states of ^{132}Cd and ^{130}Pd. For these two even-even nuclei, the proton parts of $|\tau_1\rangle$ are S_π-pair condensate states when $I \leqslant 6$, and have one S_π broken into K_π pair for $8 \leqslant I \leqslant 14$. For the two valence neutrons, the pair spin is 0, 2, 4, 6 for $I = 0, 2, 4, 6$, and repeats the same values for $I = 8, 10, 12, 14$, successively. Clearly, the structures of these two nuclei are very simple from the perspective of collective-nucleon-pair basis states.

The results of one-dimensional pair wave functions for yrast states of ^{131}Ag are listed in Table IV. One sees that configurations of valence protons are simple, all yrast states of spin between 1/2 to 21/2 are dominated by one proton-hole S pair coupled to the odd proton hole in the $g_{\frac{9}{2}}$ orbit, and this S_π pair is excited to D_π and other pairs for yrast states with higher spins. For the two valence neutrons, the ground state (with $I = \frac{9}{2}$) is constructed by the S_ν pair. Going to higher states, the neutron pair is excited to the D_ν pair, G_ν pair, and I_ν pair, successively, according to Fig. 3 and Table IV. Therefore the dominant configurations for yrast states of the ^{131}Ag nucleus are similar to those of its even-even neighboring nuclei, and low-lying states of these three open-shell nuclei are well described in terms of the generalized-seniority picture.

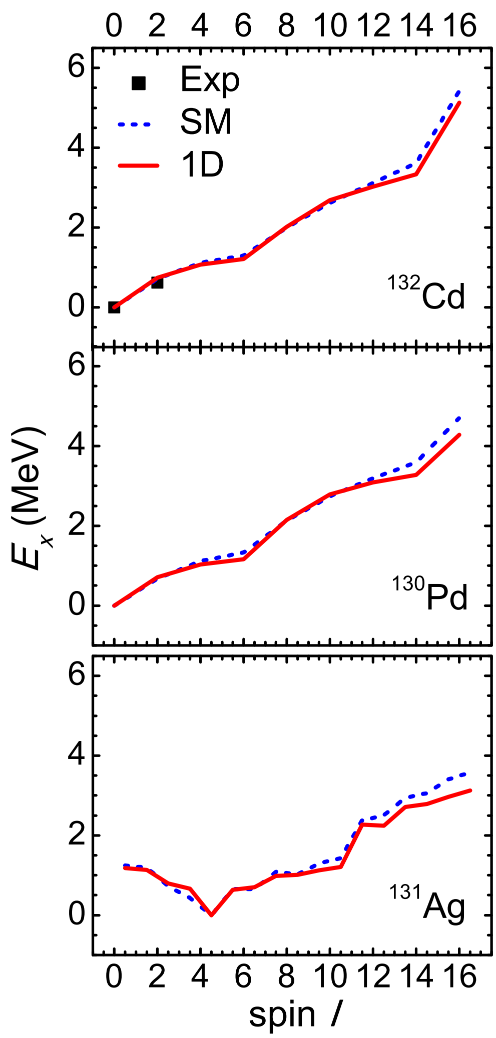

FIG. 3. Excitation energies (denoted as E_x, in units of MeV) vs the spin of the state, for positive-parity yrast states of ^{132}Cd, ^{130}Pd, and ^{131}Ag. The experimental data are taken from Ref. [41] and denoted by "Exp", and the results obtained in the shell-model space and those given by the one-dimensional pair structures are denoted by "SM" and "1D", respectively. The two-body effective interaction and the single-particle energies for this region are taken from Ref. [44].

We finally comment on a few cases in which the overlaps between our one-dimensional nucleon-pair wave functions and corresponding shell-model wave functions are not large. These "exceptions" arise in Fig. 4 for yrast states with $I = 10$ and 12 of ^{132}Cd and ^{130}Pd, and $I = 7/2$, $17/2$, $23/2$, and $27/2$ of ^{131}Ag. For these cases, we have examined the overlaps between the shell-model wave function and all possible nucleon-pair basis states, and found that the nucleon-pair basis state $|\tau_1\rangle$ with the lowest energy presents actually the largest overlap, $\approx$0.6$-$0.7, which is large but not large enough. This means that the simple picture that shell-model wave functions are represented by one-dimensional nucleon-pair wave functions is not very applicable to these states, in which nucleon-pair basis states with the lowest energies are considerably mixed with other basis states which have similar energies.

TABLE III. One-dimensional nucleon-pair wave functions for yrast states of ^{132}Cd and ^{130}Pd. The wave functions are denoted by $|r_1\rangle_\pi \otimes |r_1'\rangle_\nu$ for ^{132}Cd, and $|r_1 r_2; J_2\rangle_\pi \otimes |r_1'\rangle_\nu$ for ^{130}Pd, with subscripts π and ν for proton and neutron configurations, respectively. S, D, G, I, K represent the collective pairs with positive parity and spin-0, 2, 4, 6, 8, respectively.

	I^P	Configuration
^{132}Cd	0^+	$\lvert S\rangle_\pi \otimes \lvert S\rangle_\nu$
	2^+	$\lvert S\rangle_\pi \otimes \lvert D\rangle_\nu$
	4^+	$\lvert S\rangle_\pi \otimes \lvert G\rangle_\nu$
	6^+	$\lvert S\rangle_\pi \otimes \lvert I\rangle_\nu$
	8^+	$\lvert K\rangle_\pi \otimes \lvert S\rangle_\nu$
	10^+	$\lvert K\rangle_\pi \otimes \lvert D\rangle_\nu$
	12^+	$\lvert K\rangle_\pi \otimes \lvert G\rangle_\nu$
	14^+	$\lvert K\rangle_\pi \otimes \lvert I\rangle_\nu$
	16^+	$\lvert K\rangle_\pi \otimes \lvert K\rangle_\nu$
^{130}Pd	0^+	$\lvert SS;0\rangle_\pi \otimes \lvert S\rangle_\nu$
	2^+	$\lvert SS;0\rangle_\pi \otimes \lvert D\rangle_\nu$
	4^+	$\lvert SS;0\rangle_\pi \otimes \lvert G\rangle_\nu$
	6^+	$\lvert SS;0\rangle_\pi \otimes \lvert I\rangle_\nu$
	8^+	$\lvert SK;8\rangle_\pi \otimes \lvert S\rangle_\nu$
	10^+	$\lvert SK;8\rangle_\pi \otimes \lvert D\rangle_\nu$
	12^+	$\lvert SK;8\rangle_\pi \otimes \lvert G\rangle_\nu$
	14^+	$\lvert SK;8\rangle_\pi \otimes \lvert I\rangle_\nu$
	16^+	$\lvert DK;10\rangle_\pi \otimes \lvert I\rangle_\nu$

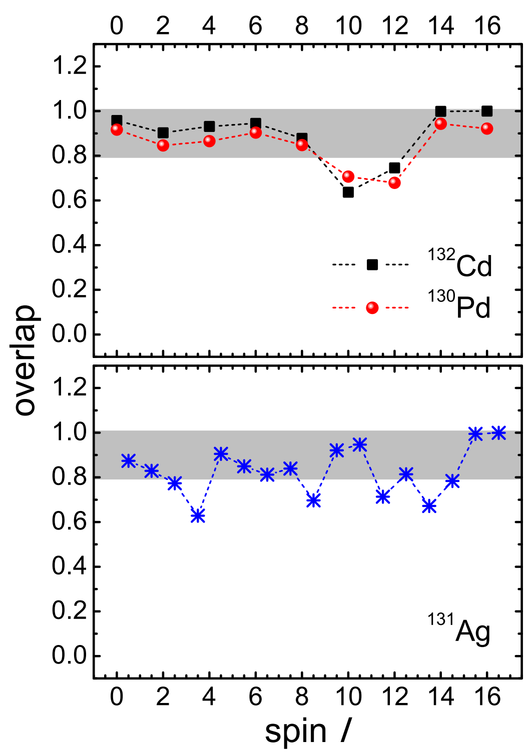

FIG. 4. Overlaps between one-dimensional pair structures and corresponding shell-model wave functions. We consider seven lowest-energy proton-hole pairs of positive parity and spin-0,1,2,3,4,6,8 and eleven lowest-energy neutron pairs of positive parity and spin-0,1,2,3,4,5,6,7,8,10,12. We take the pair basis states with the lowest expectation energies to be the one-dimensional pair wave functions for these yrast states.

TABLE IV. Same as Table III except that the wave functions are denoted as $|jr_1; J_1\rangle_\pi \otimes |r_1'\rangle_\nu$, for the ^{131}Ag nucleus.

	I^{P}	Configuration
^{131}Ag	$1/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|G\rangle_\nu$
	$3/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|G\rangle_\nu$
	$5/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|D\rangle_\nu$
	$7/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|D\rangle_\nu$
	$9/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|S\rangle_\nu$
	$11/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|D\rangle_\nu$
	$13/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|D\rangle_\nu$
	$15/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|G\rangle_\nu$
	$17/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|G\rangle_\nu$
	$19/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$21/2^+$	$\|g_{\frac{9}{2}}S; \frac{9}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$23/2^+$	$\|g_{\frac{9}{2}}D; \frac{13}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$25/2^+$	$\|g_{\frac{9}{2}}D; \frac{13}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$27/2^+$	$\|g_{\frac{9}{2}}I; \frac{15}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$29/2^+$	$\|g_{\frac{9}{2}}G; \frac{17}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$31/2^+$	$\|g_{\frac{9}{2}}I; \frac{21}{2}\rangle_\pi \otimes \|I\rangle_\nu$
	$33/2^+$	$\|g_{\frac{9}{2}}I; \frac{21}{2}\rangle_\pi \otimes \|I\rangle_\nu$

IV. SUMMARY

To summarize, we have studied in this paper the dominant configurations of Ca isotopes with mass number 43–48 and three open-shell nuclei, ^{132}Cd, ^{131}Ag, and ^{130}Pd, within the framework of the nucleon-pair approximation of the shell model.

We have shown that yrast states of these nuclei are well represented by one-dimensional nucleon-pair wave functions. This scenario, discussed in our earlier studies [42,43], is remarkably applicable to odd-mass semi-magic nuclei, ^{43,45,47}Ca. We also investigate the simple picture of nucleon-pair states for low-lying states of three open-shell nuclei, ^{132}Cd, ^{131}Ag, and ^{130}Pd. We find that most low-lying states of these three open-shell nuclei are also reasonably approximated by one-dimensional nucleon-pair wave functions, the building blocks of which show a simple hierarchy as *SDGIK* (with spin-0,2,4,6,8).

Finally, we note that the one-dimensional nucleon-pair wave functions for yrast states of both semi-magic nuclei and open-shell nuclei discussed in this work are readily constructed without diagonalization of the shell-model Hamiltonian; they are assumed to be the nucleon-pair basis states with the lowest energies $\langle\tau|H|\tau\rangle/\langle\tau|\tau\rangle$ among all possible nucleon-pair basis states. This provides us with a very simple approach to study yrast states of semi-magic nuclei as well as open-shell nuclei with only a few valence nucleons.

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