Time-reversal invariance violation in neutron-nucleus scattering

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Parity (P) and time-reversal (T) violating effects are enhanced a million times in neutron reactions near p-wave nuclear compound resonances. Planning and interpretation of corresponding experiments require values of the matrix elements of the T,P-violating nuclear forces between nuclear compound states. We calculate the root-mean-square values and the ratio of the matrix elements of the T,P-violating and P-violating interactions using statistical theory based on the properties of chaotic compound states. We present the results in terms of the fundamental parameters in five different forms: in terms of the constants of the contact nuclear interaction, meson exchange constants, QCD θ-term, quark chromo-electric dipole moments $\tilde{d}_u$ and $\tilde{d}_d$, and axion interaction constants. Using current limits on these parameters, we obtain upper bounds on the ratio of the matrix elements and on the ratio of T,P-violating and P-violating parts of the neutron reaction cross sections. Our results confirm that the expected sensitivity in neutron-reactions experiments may be sufficient to improve the limits on the T,P-violating interactions.

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I. INTRODUCTION

A very popular way to search for time-reversal (T) and parity (P) violation and to test unification theories is based on the measurements of electric dipole moments (EDMs) of elementary particles and atomic systems. So far this method has produced stringent limits on EDMs which exclude or bound many models (see reviews in Refs. [1–5]). Studies of T,P-violating (also known as T,P-odd) effects via EDM also give limits on the axion and relaxion interactions [6]. An efficient alternative method is measurement of T,P-odd effects in neutron-nucleus scattering. This method is motivated by the millionfold enhancement of parity violation in neutron reactions near p-wave nuclear compound resonances, which was predicted in Refs. [7–10]. The first confirmation was obtained in experiments performed at the Joint Institute for Nuclear Research in Dubna [11,12]; then a very extensive experimental study was done in several laboratories, including the Joint Institute for Nuclear Research (Dubna), Petersburg Institute of Nuclear Physics, KEK (Tsukuba), and especially in Los Alamos (see reviews in Refs. [13,14]). This activity continues now (see, for example, the recent experimental paper [15] and references therein). A similar mechanism of enhancement should work for the T,P-odd effects [16–19]. An unusual statistics of P-violating and T,P-violating effects, namely random-sign observables not vanishing upon averaging, was demonstrated in Refs. [20,21]. Experiments searching for T,P-violating effects are in progress in Japan and the United States [15,22–26].

Without any enhancement, the effects of P violation in low-energy nuclear reactions are extremely small, $\sim 10^{-7}$ (e.g., in the proton scattering on hydrogen and helium, and neutron radiative capture by protons) [27]. The formula for a P-violation effect near a p-wave compound resonance may be presented as [7–10][1]

$$P \sim \frac{W_{sp}}{E_s - E_p}\sqrt{\frac{\Gamma_s^n}{\Gamma_p^n}}\,, \tag{1}$$

where W_{sp} is the matrix element of the parity-violating interaction mixing s and p resonances, $E_s - E_p$ is the energy interval between these resonances, and Γ_s^n, Γ_p^n are the neutron widths of these resonances. We see that there are two reasons for the enhancement of P violation near p-wave compound resonances. First, in a nucleus excited by neutron capture the interval $E_s - E_p$ between the chaotic compound states (resonances) of opposite parity is very small, and this enhances by three orders of magnitude the mixing of these states by the weak P-violating interaction between nucleons. The second reason is that the admixture of opposite-parity states allows neutron capture in the s wave to contribute to the p-wave resonance. At small neutron energies the s-wave amplitude is three orders of magnitude larger than the p-wave amplitude ($\sqrt{\Gamma_s^n/\Gamma_p^n} \sim 10^3$). As a result of these two 10^3 factors, the P-violating parts reach 1–10% of reactions' cross sections and become accessible to experimental scrutiny. T,P-violating effects are also produced by the parity-violating interaction; therefore, Eq. (1) and the enhancement mechanism works for them too [16–19].

For the experiments to produce useful results we need theory for their interpretation. At first glance, it seems

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[1]We omit the numerical coefficient which depends on the specific process induced by the neutron capture.

impossible, since chaotic compound states are very complicated. However, chaos allows us to develop a statistical theory, similar to the Maxwell-Boltzmann theory for macroscopic systems, which actually gives very accurate predictions. We developed such a theory, including a method to calculate matrix elements between chaotic states in finite systems (in excited nuclei, atoms, and molecules) [28–34]. We briefly present the ideas below.

An increase in the excitation energy of a nucleus increases the number of its active particles k and available orbitals p, leads to an exponential increase of the density of energy levels $\sim p!/[(p-k)!k!]$, and brings the system into a state where the residual interaction between particles exceeds the intervals between the energy levels. The eigenstates $|n\rangle = \sum_i C_i^n|i\rangle$ become chaotic superpositions of thousands or even millions of Hartree-Fock basic states $|i\rangle$. All medium and heavy nuclei and atoms with an open f shell have chaotic excited compound states in the discrete spectrum and/or chaotic compound resonances. The idea of Refs. [28,29] is to treat the expansion coefficients C_i^n as Gaussian random variables, with the average values $\overline{C_i^n} = 0$ and variance

$$\overline{(C_i^n)^2} = \frac{1}{\bar{N}}\Delta(\Gamma_{\rm spr}, E^n - E_i), \tag{2}$$

$$\Delta(\Gamma_{\rm spr}, E^n - E_i) = \frac{\Gamma_{\rm spr}^2/4}{(E^n - E_i)^2 + \Gamma_{\rm spr}^2/4}, \tag{3}$$

where $\bar{N} = \pi\Gamma_{\rm spr}/2d$ is the normalization constant found from $\sum_i (C_i^n)^2 = 1$, d is the average energy distance between the compound states (resonances) with the same angular momentum and parity, and $\Gamma_{\rm spr}$ is the spreading width of the component calculated using the Fermi golden rule [35]; $\bar{N}$ is called the number of principal components.[2]

We have tested this distribution of C_i^n by the numerical calculations of chaotic compound states in cerium and protactinium atoms [36–42], in highly charged ions with an open f shell [43–48], in the two-body random interaction model [49–52], and using an analytical approach [33,53].

The function $\overline{(C_i^n)^2} = \Delta(\Gamma_{\rm spr}, E^n - E_i)/\bar{N}$ gives the probability to find the basis component $|i\rangle$ in the compound state $|n\rangle$; i.e., it plays the role of the statistical partition function. The difference from the conventional statistical theory is that the partition function depends on the total energy of the isolated system E^n instead of on the temperature of a system in a thermostat [recall the Boltzmann factor $\exp(-E_i/T)$]. One may compare this with the microcanonical distribution where the equipartition is assumed within the shell of the states with fixed energy E_i.

Expectation values of matrix elements of any operator W in a chaotic compound state are found as $\overline{|\langle n|W|n\rangle|^2} = \sum_i \overline{(C_i^n)^2}|\langle i|W|i\rangle|^2$. For example, this formula with $W = a_k^+ a_k$ (the occupation-number operator) gives the distribution of the orbital occupation numbers in finite chaotic systems which replaces the Fermi-Dirac (or Bose-Einstein) distribution.[3]

Average values of the non-diagonal matrix elements are equal to zero, $\overline{\langle n|W|m\rangle} = 0$, while the average values of the squared matrix elements $W^2 \equiv \overline{|\langle n|W|m\rangle|^2} = \sum_{i,j} \overline{(C_i^n)^2(C_j^m)^2}|\langle i|W|j\rangle|^2$ are reduced to the sum of matrix elements between the Hartree-Fock basis states $|\langle i|W|j\rangle|^2$, where W is any perturbation operator. The distribution of the matrix elements $\langle n|W|m\rangle$ is Gaussian with variance given by the W^2.

For the correlator between two different operators (e.g., P-violating and T,P-violating) we obtain $\overline{\langle n|W_P|m\rangle\langle m|W_{T,P}|n\rangle} = \sum_{i,j}\overline{(C_i^n)^2(C_j^m)^2}\langle i|W_P|j\rangle\langle j|W_{T,P}|i\rangle$ [28–32]. Note that our theory predicts the results averaged over several compound resonances.

We have done many tests comparing the statistical theory results for electromagnetic amplitudes [40], electron recombination rates [43–48,54], and parity-violation effects [28,29] with the experimental data and with numerical simulations. For example, we obtained a thousandfold enhancement of the electron recombination rate with many highly charged tungsten ions due to the very dense spectrum of chaotic compound resonances [44–48,54]. These results agree with all available experimental data and predict recombination rates for ions with a high ionization degree, where experiments are limited by existing techniques. Our results are important for thermonuclear reactors which are made from tungsten. Tungsten ions contaminate plasma and significantly affect the energy output.

Using the theory of chaotic nuclear compound resonances, we calculate in this paper the ratio w/v of the root-mean-square values of the matrix elements of the T,P-odd (w) and P-odd (v) matrix elements. We show the results in terms of the fundamental parameters in five different forms: in terms of the constants of the contact nuclear interaction, meson exchange constants, QCD θ-term, quark chromo-EDMs $\tilde{d}_u$ and $\tilde{d}_d$, and axion interaction constants. Using the latest bounds on θ, $\tilde{d}_u$, and $\tilde{d}_d$ and axion interaction constants we arrive at bounds on the magnitude of possible T violation. In the Conclusion section the results are compared with the expected experimental sensitivity to the T,P-violating effects.

II. P- AND T,P-VIOLATING INTERACTIONS

The ratio of the time-reversal-invariance violating (TRIV) and parity violating (PV) parts of the neutron nuclear cross sections induced by mixing of s- and p-wave nuclear compound resonances, $\Delta\sigma_{PT}/\Delta\sigma_P$, can be expressed as [15,55,56]

$$\frac{\Delta\sigma_{PT}}{\Delta\sigma_P} = \kappa\frac{\langle\psi_p|W_{PT}|\psi_s\rangle}{\langle\psi_p|W_P|\psi_s\rangle}. \tag{4}$$

Here the factor κ includes amplitudes of the partial neutron widths which depend on spin channels $J = I \pm 1/2$, where

[2]Basis states $|i\rangle$ with shell-model energies E_i close to the energy of a compound state E^n (within the spreading width $\Gamma_{\rm spr}$) have the highest weight ($\sim 1/\bar{N}$) and dominate in the normalization sum $\sum_i (C_i^n)^2 = 1$. The number of such states is $\bar{N}$.

[3]However, numerical calculations [36,43,44,49] give occupation numbers which are close to the Fermi-Dirac distribution.

I is the spin of the target nucleus and J is the spin of the compound resonance. For example, for $J = 0$, one obtains $\kappa = 1$, as in this case κ does not depend on neutron partial widths [16–18].

The ratio $\Delta\sigma_{PT}/\Delta\sigma_P$ for the neutron-deuterium scattering was calculated in Ref. [57]. However, experiments are planned for heavier nuclei where we expect a millionfold enhancement of the T,P-odd and P-odd effects.

In the short-range interaction limit, the PV operator W_P and TRIV operator W_{PT} are

$$W_P = \frac{Gg}{2\sqrt{2}m}\{(\sigma\mathbf{p}), \rho\}, \tag{5}$$

$$W_{PT} = \frac{G\eta}{2\sqrt{2}m}(\sigma\nabla)\rho. \tag{6}$$

Here G is the weak-interaction Fermi constant, m is the nucleon mass, $\mathbf{p}$ and σ are nucleon momentum and spin, respectively, and ρ is the nucleon density. Nucleon dimensionless constants $g_{p,n}$ and $\eta_{p,n}$ characterize the strength of the interactions. Note that in the standard definition of angular wavefunctions the matrix element of W_P between bound states is imaginary (since the momentum operator $\mathbf{p} = -i\nabla$) and the matrix element of the TRIV operator W_{PT} is real.

We define v^2 to be the average of the absolute value of the squared PV matrix element, and w^2 to be the average value of the squared TRIV matrix element between the s and p compound resonances, such that

$$v = \sqrt{\overline{\langle\psi_p|W_P|\psi_s\rangle\langle\psi_s|W_P|\psi_p\rangle}}, \tag{7}$$

$$w = \sqrt{\overline{\langle\psi_p|W_{PT}|\psi_s\rangle\langle\psi_s|W_{PT}|\psi_p\rangle}}. \tag{8}$$

Correlations might exist between the matrix elements of PV and TRIV interactions. The quantity parametrizing such correlations, the correlator, is defined as

$$C = \frac{\overline{|\langle\psi_p|W_P|\psi_s\rangle\langle\psi_s|W_{PT}|\psi_p\rangle|}}{vw}. \tag{9}$$

The correlator, which takes values between zero and one, can be useful to deduce the values and signs of TRIV effects, since much is already known about the PV effects. The correlator C was calculated by the same technique as the mean-square matrix element and was found to be [32]

$$|C| \approx 0.1. \tag{10}$$

This result tells us that the correlations between the matrix elements are relatively small so we may neglect them.

A. Rough estimate of w/v

Naively one would expect from Eqs. (5) and (6) the following relation: $w/v \sim \eta/g$. However, this ratio is actually $A^{1/3}$ times smaller than the ratio of interaction constants [31], where A is the number of nucleons. Indeed, for $\nabla\rho$ in Eq. (6),

$$\nabla\rho \sim \frac{\rho}{R_N} \sim \frac{\rho}{r_0 A^{1/3}}, \tag{11}$$

where r_0 is the internucleon distance, and $R_N = r_0 A^{1/3}$ is the nuclear radius. The momentum in Eq. (5) is approximated as $p \sim p_F \sim \hbar/r_0$. Thus, the ratio of matrix elements is smaller than the ratio of interaction constants in Eqs. (5) and (6) by a factor of $A^{1/3}$:

$$\frac{w}{v} \sim \frac{\eta}{gA^{1/3}}. \tag{12}$$

For elements with the number of nucleons in the range 100–250, $A^{1/3} \approx 5$. A detailed discussion of this suppression factor including many-body effects can be found in Ref. [31].

B. Dependence of matrix elements on nucleon interaction constants

A general expression for the root-mean-square value of the matrix element v of the PV operator (and the matrix element w of the TRIV operator) was derived in Ref. [29]:

$$v = \frac{1}{\sqrt{N}}\left\{\sum_{abcd} \nu_a(1-\nu_b)\nu_c(1-\nu_d)\frac{1}{4}|V_{ab,cd} - V_{ad,cb}|^2 \Delta(\Gamma_{\text{spr}}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d)\right\}^{\frac{1}{2}}. \tag{13}$$

Here ν are the orbital occupation numbers given by the Fermi-Dirac distribution in an excited nucleus, numerical values of the matrix elements of the two-nucleon interaction $V_{ab,cd}$ (see Fig. 1) are presented in Refs. [29,32], and $\Delta(\Gamma_{\text{spr}}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d)$ is the "spread" δ function [Eq. (3)] of the change in energy $\epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d$. Equation (13) has a clear interpretation. The Δ function means in fact an approximate energy conservation with an accuracy up to the spreading width Γ_{spr} (since the single-particle states are not stationary states in this problem). In the case $\Gamma_{\text{spr}} \to 0$ we have $\Delta(\Gamma_{\text{spr}}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d) \to (\pi\Gamma_{\text{spr}}/2)\delta(\epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d)$. To have a transition, initial states must be occupied (this gives ν_a and ν_c) and final states empty (this gives $1-\nu_b$ and $1-\nu_d$).

The dependence of v and w on the nucleon interaction constants g and η [which appear in Eqs. (5) and (6)] can be

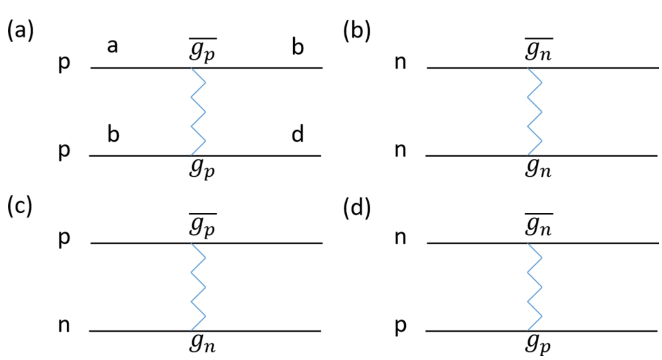

FIG. 1. Possible configurations of weak interactions $V_{ab,cd}$ [29] within the nucleus between protons (p) and neutrons (n). In each diagram, the upper vertex is P-violating. Constants g_p and g_n characterize the strength of the interactions. (a) Interactions between two protons; (b) interactions between two neutrons; (c) and (d) interactions between protons and neutrons, which contribute to the squared PV matrix element v^2 by $(V_{np} + V_{pn})^2 = V_{np}^2 + V_{pn}^2 + 2V_{np}V_{pn}$. When summing matrix elements in (c) and (d), the terms $V_{np}V_{pn}$ have random signs and the result is much smaller than the sums of V_{np}^2 and V_{pn}^2.

presented in the following form [29,32]:

$$v = \frac{1}{\sqrt{N}}\sqrt{\left(\Sigma_{pp}^{(P)}g_p\right)^2 + \left(\Sigma_{nn}^{(P)}g_n\right)^2 + \left(\Sigma_{pn}^{(P)}g_pg_n\right)}, \quad (14)$$

$$w = \frac{1}{\sqrt{N}}\sqrt{\left(\Sigma_{pp}^{(PT)}\eta_p\right)^2 + \left(\Sigma_{nn}^{(PT)}\eta_n\right)^2 + \left(\Sigma_{pn}^{(PT)}\eta_p\eta_n\right)}, \quad (15)$$

where g_p and g_n are proton and neutron weak constants—they characterize the strength of the P-odd weak potential; η_p, η_n are constants that characterize the strength of the T,P-odd potential, and Σ are sums of the weighted squared matrix elements of the weak interaction between nucleon orbitals defined in Eq. (13). Contributions of the cross terms $\Sigma_{pn}^{(P)}g_pg_n$ and $\Sigma_{pn}^{(PT)}\eta_p\eta_n$ are small compared to the other terms since they contain products of different matrix elements which have random signs, while in the terms containing squared interaction constants all contributions are positive.

Therefore, we can present v and w in the following form:

$$v = K_P\sqrt{g_n^2 + kg_p^2}, \quad (16)$$

$$w = K_{PT}\sqrt{\eta_n^2 + k\eta_p^2}. \quad (17)$$

The coefficient k should be slightly smaller than 1 since in heavy nuclei the number of neutrons $N = 1.5Z$, where Z is the number of protons. To make a simple estimate of the sensitivity of v and w to changes in the interaction constants, we assume in the next step that Σ from Eqs. (14) and (15) are proportional to the number of interaction terms in the nucleus. There are $Z^2/2$ interaction terms between protons, $N^2/2$ such terms between neutrons, and ZN terms between a proton and a neutron (Fig. 1). Thus we can write

$$k = \frac{Z^2 + 2ZN}{N^2 + 2ZN} = 0.76. \quad (18)$$

Numerical calculations of v and w were done in Refs. [29–32] for specific values of the interaction constants g_p, g_n, η_p, and η_n. The values of these constants have been updated since those calculations. Therefore, we would like to find updated values of these constants to insert into Eqs. (16) and (17). The general expressions for g_p and g_n are [28,58–60]

$$g_p = 2\times 10^5\, W_\rho\left[176\frac{W_\pi}{W_\rho}f_\pi - 19.5h_\rho^0 - 4.7h_\rho^1 + 1.3h_\rho^2 - 11.3\left(h_\omega^0 + h_\omega^1\right)\right], \quad (19)$$

$$g_n = 2\times 10^5\, W_\rho\left[-118\frac{W_\pi}{W_\rho}f_\pi - 18.9h_\rho^0 + 8.4h_\rho^1 - 1.3h_\rho^2 - 12.8\left(h_\omega^0 + h_\omega^1\right)\right], \quad (20)$$

where f and h are the weak NN-meson couplings, and W_ρ and W_π are constants which account for the repulsion between nucleons at small distances as well as for the finite range of the interaction potential. We take $W_\rho = 0.4$ and $W_\pi = 0.16$ as in Refs. [58,60].

TABLE I. Values of g_p and g_n based on the meson exchange constants from different publications (left-hand column): Desplanques, Donoghue, and Holstein (DDH) [59,60]; Noguera and Desplanques (ND) [29]; Dubovik and Zenkin (DZ) [66]; Feldman, Crawford, Dubach, and Holstein (FCDH) [67]. In the line of Wasem [63] we use the best DDH values for all the values of h except $f_\pi = h_\pi^1$, which was recently derived by lattice QCD methods [62,63] to be $h_\pi^1 = 1.1\times 10^{-7}$.

Reference	g_p	g_n
DDH (1980) [59,60]	4.5	0.2
ND (1986) [29,61]	4	1
DZ (1986) [66]	2.4	1.1
FCDH (1991) [67]	2.7	−0.1
Wasem (2012) [63]	2.6	1.5

For the choice of constants $g_p = 4$, $g_n = 1$ [61], numerical calculations give [29]

$$v = K_P\sqrt{1+16k} = 2.08\,\text{meV}. \quad (21)$$

We calculate updated values for g_p and g_n, using the best values of the constants h from Desplanques, Donoghue, and Holstein (DDH) [59] with an updated $f_\pi \equiv h_\pi^1$, which was recently derived by lattice QCD methods [62,63]. Such calculations give $g_p = 2.6$, $g_n = 1.5$ (Table I). Using these values with Eq. (21), we have

$$v_{\text{updated}} = 2.08\,\text{meV}\frac{\sqrt{1.5^2 + 2.6^2k}}{\sqrt{1+16k}} = 1.56\,\text{meV}, \quad (22)$$

where in the last step we used $k = 0.76$.[4] This theoretical estimate is in excellent agreement with the experimental value $1.39^{+0.55}_{-0.38}$ meV [64,65].

Numerical calculations were done for $\eta_p = \eta_n$ and gave $w = 0.2|\eta_n|$ meV [32]. Using this result and Eq. (17) we obtain

$$w_{\text{updated}} = 0.15\,\text{meV}\sqrt{\eta_n^2 + 0.76\eta_p^2}. \quad (23)$$

C. The ratio w/v expressed via meson exchange constants, QCD θ-term, quark chromo-EDMs $\tilde{d}_u$ and $\tilde{d}_d$, and axion exchange constants

Now we can express the ratio w/v in five different ways: as a function of η_p, η_n, by π_0 meson-exchange coupling constants with the nuclei, by QCD CP-violation parameter θ, by quark chromo-EDMs, and finally by axion exchange constants.

First, to express the ratio w/v as a function of η, we use Eqs. (22) and (23) to obtain

$$\frac{w}{v} = 0.10\sqrt{\eta_n^2 + 0.76\eta_p^2}. \quad (24)$$

[4]For $k = 1$ we would get 1.51 meV. Thus we see that our result is not very sensitive to the value of k.

If, following Refs. [32,68], we take $|\eta_p| = |\eta_n|$, the ratio in Eq. (24) becomes

$$\frac{w}{v} = 0.13|\eta_n|. \tag{25}$$

Second, the T,P-odd nuclear forces are dominated by π_0 meson exchange. Such an exchange is described by the interaction [69–71]

$$\mathcal{W}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -\frac{\bar{g}}{8\pi m_N}\left[\nabla_1\left(\frac{e^{-m_\pi r_{12}}}{r_{12}}\right)\right]\cdot\{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times [\bar{g}_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \bar{g}_2(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - 3\tau_{1z}\tau_{2z})] + \bar{g}_1(\tau_{1z}\boldsymbol{\sigma}_1 - \tau_{2z}\boldsymbol{\sigma}_2)\}, \tag{26}$$

where $\bar{g} = 13.6$ is the strong-force T,P-conserving πNN coupling constant, $\bar{g}_0$, $\bar{g}_1$, and $\bar{g}_2$ are the strengths of the isoscalar, isovector, and isotensor T,P-violating couplings, respectively, m_N is the nucleon mass, m_π is the pion mass, $\boldsymbol{\sigma}$ is the nucleon spin, $\boldsymbol{\tau}$ is the nucleon Pauli isospin matrix in vector form, and r_{12} is the separation between nucleons. The coupling constants η can be expressed in terms of $\bar{g}$ [68]:

$$-\eta_p = \eta_n = 5 \times 10^6 \bar{g}(\bar{g}_1 + 0.4\bar{g}_2 - 0.2\bar{g}_0). \tag{27}$$

Then we have

$$\frac{w}{v} = 0.13|\eta_n| = |6.5 \times 10^5 \bar{g}(\bar{g}_1 + 0.4\bar{g}_2 - 0.2\bar{g}_0)|. \tag{28}$$

Third, using the previous results $\bar{g}\bar{g}_0 = -0.37\theta$ [72], where θ is the QCD CP-violation parameter, and $\bar{g}\bar{g}_1 = \bar{g}\bar{g}_2 = 0$, we can write the ratio w/v as a function of θ:

$$\frac{w}{v} = 4.8 \times 10^4|\theta|. \tag{29}$$

Using updated results [4,73]

$$\bar{g}\bar{g}_0 = -0.2108\theta, \tag{30}$$

$$\bar{g}\bar{g}_1 = 46.24 \times 10^{-3}\theta, \tag{31}$$

we can write, still with $\bar{g}\bar{g}_2 = 0$,

$$\frac{w}{v} = 5.7 \times 10^4|\theta|.$$

Using the current limit on θ, obtained from constraints on neutron EDMs, $|\theta| < 10^{-10}$ [4], we obtain

$$w/v < 10^{-5}. \tag{32}$$

Fourth, we can connect our result to the quark chromo-EDM $\tilde{d}$ [2]:

$$\bar{g}\bar{g}_1 = 4 \times 10^{15}(\tilde{d}_u - \tilde{d}_d)/\text{cm}, \tag{33}$$

$$\bar{g}\bar{g}_0 = 0.8 \times 10^{15}(\tilde{d}_u + \tilde{d}_d)/\text{cm}. \tag{34}$$

Then

$$\frac{w}{v} = |6.5 \times 10^{20}(4(\tilde{d}_u - \tilde{d}_d) - 0.16(\tilde{d}_u + \tilde{d}_d))|/\text{cm}. \tag{35}$$

Using the current limits (Table IV in Ref. [74]; see also Ref. [75])

$$|\tilde{d}_u - \tilde{d}_d| < 6 \times 10^{-27}\text{cm}, \tag{36}$$

$$|\tfrac{1}{2}\tilde{d}_u + \tilde{d}_d| < 3 \times 10^{-26}\text{cm}, \tag{37}$$

we obtain

$$w/v < 2 \times 10^{-5}. \tag{38}$$

Finally, a T,P-violating interaction, similar to the pion-exchange-induced Eq. (26), may be due to exchange by any scalar particle which has both scalar (with the interaction constant g^s) and pseudoscalar (with the interaction constant g^p) couplings to nucleons. The most popular examples are the dark-matter candidates axion [76,77] and relaxion [78–80], which have very small masses.[5] A numerical estimate shows that due to the long range of the interaction the matrix elements in the small-mass case are ~1.5 times larger than the pion exchange matrix elements; i.e., we have instead of Eq. (28) the following estimate:

$$\frac{w}{v} \sim |1 \times 10^6 g^s g^p|. \tag{39}$$

The limit on $g^s g^p$ may be obtained from the proton EDM calculation,[6]

$$d_p = \frac{g^s g^p e}{8\pi^2 m_p}, \tag{40}$$

and measurement [75], $|d_p| < 2 \times 10^{-25}e$ cm, $|g^s g^p| < 1 \times 10^{-9}$. Using limits from the proton EDM and the ^{199}Hg nuclear-Schiff-moment measurements in Ref. [75], the authors of Ref. [81] concluded that the limit on $|g^s g^p|$ is between 10^{-9} and 10^{-11}. This gives a rather weak limit on w/v induced by axion exchange:

$$w/v < 10^{-3}\text{–}10^{-5}. \tag{41}$$

III. CONCLUSION

Using the bound in Eq. (38), and assuming $\kappa \approx 1$ in Eq. (4) (which is reasonable [55] and matches experimental results [15,26]), we arrive at

$$\frac{\Delta\sigma_{PT}}{\Delta\sigma_P} \lesssim 2 \times 10^{-5}. \tag{42}$$

The limit based on the axion exchange in Eq. (41) is weaker. The current expected experimental sensitivity is [26,82]

$$\left.\frac{\Delta\sigma_{PT}}{\Delta\sigma_P}\right|_{\text{exp. sensitivity}} < 10^{-6}. \tag{43}$$

Thus we confirm that the expected experimental sensitivity may be sufficient to improve the limits on the TRIV interactions, or possibly to detect them.

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[5]The limits on the T,P-violating electron-nucleon interactions mediated by the axion or relaxion exchange from EDM measurements were obtained in Ref. [6], where more references may be found.

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