# Light vector mesons ( $\omega$ , $\rho$ , and $\phi$ ) in strong magnetic fields: A QCD sum rule approach

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The mass modifications of the light vector mesons  $(\omega, \rho, \text{and } \phi)$  are investigated in asymmetric nuclear matter in the presence of strong magnetic fields, using a quantum chromodynamics (QCD) sum rule approach. These are computed from the medium modifications of the nonstrange and strange light quark condensates as well as scalar gluon condensate. The quark and gluon condensates are calculated from the medium changes of the scalar fields (nonstrange and strange) and a scalar dilaton field in the magnetized nuclear matter, within a chiral SU(3) model. The scalar dilaton field within the model breaks the scale invariance of QCD and simulates the gluon condensate. The anomalous magnetic moments for the nucleons are also taken into account in the present study.

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## I. INTRODUCTION

The study of properties of hadrons under extreme conditions, e.g., high densities and/or temperatures, is an important topic of contemporary research in strong interaction physics. The subject is of relevance to ultrarelativistic heavy ion collision experiments, where the experimental observables of these high energy nuclear collisions are affected by the medium modifications of the hadrons. Furthermore, the heavy colliding nuclei have large isospin asymmetry as the number of the neutrons is much larger than the number of protons of these nuclei. It is thus important to study the effects of isospin asymmetry on the hadron properties. The estimation of huge magnetic fields being created in noncentral ultrarelativistic heavy ion collision experiments necessitates the study of magnetic field effects on the properties of the hadrons. The medium modifications of the hadrons have been studied extensively in the literature. The different formalisms for these studies are the effective hadronic models, e.g., quantum hadrodynamics (QHD) model [1], the QCD sum rule (QCDSR) approach [2,3], the quark meson coupling (QMC) model [4], the chiral effective models, as well as using the coupled channel approach. The models like the Nambu Jona Lasinio model, which simulate the spontaneous chiral symmetry breaking of QCD (through four fermion interactions), have been extensively used in the literature [5-9] to study the strongly interacting matter. The AdS/CFT correspondence and the conjecture of gravity/gauge duality [10] have also been used to study the hadrons [11].

The in-medium masses of the light vector mesons ( $\rho$ ,  $\omega$ , and  $\phi$ ) are studied in the present work using a QCD sum rule approach [12–23], in asymmetric nuclear matter in the

presence of strong magnetic fields. The medium modifications are because of the changes of the light quark condensates and gluon condensate in the magnetized isospin asymmetric hadronic matter, which are calculated within mean field approximation, from the changes in the expectation values of nonstrange ( $\sigma$ ) and strange ( $\zeta$ ) scalar-isoscalar fields, the third component of a scalar isovector field ( $\delta$ ), and a dilaton field  $(\chi)$  from their vacuum values, within a chiral SU(3) model [24,25]. The quark condensates are obtained from the explicit symmetry breaking term within the chiral SU(3)model, in terms of the scalar fields,  $\sigma$ ,  $\zeta$ , and  $\delta$ , and the gluon condensate is related to the dilaton field  $\chi$ , which mimics the scale symmetry of QCD, through a logarithmic potential. The model has been used to describe nuclear matter [24], finite nuclei [25], and the bulk properties of (proto) neutron stars [26]. Using the chiral SU(3) model, the vector mesons have also been studied [27], accounting for the Dirac polarization effects [28]. The model has been used to study the kaons and antikaons in isospin asymmetric nuclear (hyperonic) matter [29-32]. The model has been generalized to the charm and bottom sectors and the in-medium masses of open charm [33-37], open bottom mesons [38,39], charmonium [36,40], and bottomonium states [41]. Using the mass modifications of the charmonium states and the open charm mesons, the partial decay widths of the charmonium states to the  $D\bar{D}$ pair in the hadronic medium have been studied using the  ${}^{3}P_{0}$  model [36,42,43]. The in-medium partial decay widths of the charmonium (bottomonium) to  $D\overline{D}$  ( $B\overline{B}$ ) [44,45] have also been studied using a field theoretic model for composite hadrons, from the mass modifications of these heavy flavor mesons calculated within the chiral effective model. The effects of magnetic fields on these heavy flavor mesons (D, B,charmonium, and bottomonium states) in asymmetric nuclear matter have also been studied [46-50], by including the coupling terms with the electromagnetic field to the baryons in the Lagrangian density of the chiral effective model. The masses of these heavy flavor mesons have been studied accounting for the effects of the anomalous magnetic moments of the

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nucleons. The light vector mesons ( $\omega$ ,  $\rho$ , and  $\phi$ ) in strange hadronic matter have been studied using a QCD sum rule approach [51], using the medium-dependent quark and gluon condensates, calculated within the chiral SU(3) model. In the present investigation, we compute the medium modifications of these vector mesons in asymmetric nuclear matter in the presence of strong magnetic fields using the QCD sum rule approach, with the quark and gluon condensates obtained from the scalar fields and dilaton field within the chiral model.

The outline of the paper is as follows : In Sec. II, we describe briefly the chiral SU(3) model used to calculate the quark and gluon condensates in the nuclear medium in the presence of strong magnetic fields. The in-medium values of these condensates are calculated from the medium changes of the scalar fields of the explicit symmetry breaking term and of the dilaton field, which mimics the gluon condensate of QCD in the chiral SU(3) model. In Sec. III, we present the QCD sum rule approach using which the in-medium masses of the light vector mesons ( $\omega$ ,  $\rho$ ,  $\phi$ ) are studied. Section IV discusses the results of the mass modifications of the light vector mesons ( $\omega$ ,  $\rho$ , and  $\phi$ ) in the magnetized asymmetric nuclear matter. In Sec. V, we summarize the findings of the present investigation.

### II. THE HADRONIC CHIRAL SU(3) × SU(3) MODEL

We use an effective chiral SU(3) model [25] to obtain the in-medium quark and gluon condensates for the study of modifications of the masses of the light vector mesons using a QCD sum rule approach. The model is based on the nonlinear realization of chiral symmetry [52–54] and broken scale invariance [24,25,27]. The concept of broken scale invariance leading to the trace anomaly in QCD,  $\theta^{\mu}_{\mu} = \frac{\beta_{\rm QCD}}{2g}G^a_{\mu\nu}G^{\mu\nu a}$ , where  $G^a_{\mu\nu}$  is the gluon field strength tensor of QCD, is simulated in the effective Lagrangian at tree level through the introduction of the scale breaking terms [55,56],

$$\mathcal{L}_{\text{scalebreaking}} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\left(\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right).$$
(1)

The Lagrangian density corresponding to the dilaton field,  $\chi$  leads to the trace of the energy momentum tensor as [40,51,57]

$$\theta^{\mu}_{\mu} = \chi \frac{\partial \mathcal{L}}{\partial \chi} - 4\mathcal{L} = -(1-d)\chi^4.$$
<sup>(2)</sup>

Equating the trace of the energy momentum tensor arising from the trace anomaly of QCD with that of the present chiral model given by Eq. (2), gives the relation of the dilaton field to the scalar gluon condensate. The trace of the energy momentum tensor in QCD is given as [58]

$$T^{\mu}_{\mu} = \sum_{q_i = u, d, s} m_{q_i} \bar{q}_i q_i + \left\langle \frac{\beta_{\text{QCD}}}{2g} G^a_{\mu\nu} G^{\mu\nu a} \right\rangle \equiv -(1-d)\chi^4.$$
(3)

In the above, the first term of the energy-momentum tensor, within the chiral SU(3) model is the negative of the explicit chiral symmetry breaking term. This relates the light quark condensates to the values of the scalar fields  $\sigma$ ,  $\zeta$ , and  $\delta$  in the

mean field approximation as [51]

$$m_{u}\langle \bar{u}u \rangle = \frac{1}{2}m_{\pi}^{2}f_{\pi}(\sigma + \delta),$$
  

$$m_{d}\langle \bar{d}d \rangle = \frac{1}{2}m_{\pi}^{2}f_{\pi}(\sigma - \delta),$$
  

$$m_{s}\langle \bar{s}s \rangle = \left(\sqrt{2}m_{k}^{2}f_{k} - \frac{1}{\sqrt{2}}m_{\pi}^{2}f_{\pi}\right)\zeta.$$
(4)

Using the QCD  $\beta$  function occurring in the right-hand side of Eq. (3) at one loop order, for  $N_c = 3$  colors and  $N_f = 3$ flavors, one gets the dilaton field related to the scalar gluon condensate as

$$\binom{\alpha_s}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu}$$

$$= \frac{8}{9} \bigg[ (1-d)\chi^4 + \left( m_\pi^2 f_\pi \sigma + \left( \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right) \bigg].$$
(5)

The coupled equations of motion for the nonstrange scalar isoscalar field  $\sigma$ , scalar isovector field  $\delta$ , the strange scalar field  $\zeta$ , and the dilaton field  $\chi$ , derived from the Lagrangian density of the chiral SU(3) model, are solved to obtain the values of these fields in the asymmetric nuclear medium in the presence of magnetic field.

# **III. QCD SUM RULE APPROACH**

In the present section, we briefly describe the QCD sum rule approach used to study the properties of the light vector mesons  $(\omega, \rho, \phi)$  in the nuclear medium in the presence of a magnetic field. The study uses the values of quark and gluon condensates in the magnetized nuclear medium obtained in a chiral SU(3) model as described in the previous section. The current-current correlation function for the vector meson  $V (= \omega, \rho, \phi)$  is written as

$$\Pi^{V}_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0|T j^{V}_{\mu}(x) j^{V}_{\nu}(0)|0\rangle, \qquad (6)$$

where *T* is the time-ordered product and  $J^V_{\mu}$  is the current for the vector meson  $V = \rho$ ,  $\omega$ ,  $\phi$ . Current conservation gives the structure of the correlation function as

$$\Pi^{V}_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)\Pi^{V}(q^2).$$
 (7)

In the large spacelike region,  $Q^2 = -q^2 \gg 1 \text{ GeV}^2$ , the scalar correlation function  $\Pi^V(q^2)$  for the light vector mesons ( $\omega$ ,  $\rho$ , and  $\phi$ ) can be written in terms of the operator product expansion (OPE) as [17,19]

$$12\pi^{2}\tilde{\Pi}^{V}(q^{2} = -Q^{2})$$
  
=  $d_{V}\left[-c_{0}^{V}\ln\left(\frac{Q^{2}}{\mu^{2}}\right) + \frac{c_{1}^{V}}{Q^{2}} + \frac{c_{2}^{V}}{Q^{4}} + \frac{c_{3}^{V}}{Q^{6}} + \cdots\right],$  (8)

where  $\tilde{\Pi}^{V}(q^{2} = -Q^{2}) = \frac{\Pi^{V}(q^{2} = -Q^{2})}{Q^{2}}$ ,  $d_{V} = 3/2$ , 1/6, and 1/3 for  $\rho$ ,  $\omega$ , and  $\phi$  mesons, respectively, and  $\mu$  is a scale chosen to be 1 GeV in the present investigation [51]. The leading term in the OPE, given by the first term, is calculated in the perturbative QCD. The coefficients  $c_{i}^{V}(i = 1, 2, 3)$  in the OPE

contain the information of the nonperturbative effects of QCD in terms of the quark and gluon condensates, as well as the Wilson coefficients [14,16]. The Wilson coefficients are taken as medium independent, with all the medium effects incorporated into the quark and gluon condensates [14,16,17,19,20]. After Borel transformation, the correlator for the vector meson can be written as

$$12\pi^{2}\tilde{\Pi}^{V}(M^{2}) = d_{V} \left[ c_{0}^{V}M^{2} + c_{1}^{V} + \frac{c_{2}^{V}}{M^{2}} + \frac{c_{3}^{V}}{2M^{4}} \right].$$
(9)

On the phenomenological side the correlator function  $\tilde{\Pi}^V(Q^2)$  can be written as

$$12\pi^2 \tilde{\Pi}_{\rm phen}^V(Q^2) = \int_0^\infty ds \frac{R_{\rm phen}^V(s)}{s+Q^2},$$
 (10)

where  $R_{\text{phen}}^V(s)$  is the spectral density proportional to the imaginary part of the correlator,

$$R_{\text{phen}}^V(s) = 12\pi \,\text{Im}\,\Pi_{\text{phen}}^V(s). \tag{11}$$

On Borel transformation, Eq. (10) reduces to

$$12\pi^{2}\tilde{\Pi}^{V}(M^{2}) = \int_{0}^{\infty} ds e^{-s/M^{2}} R_{\text{phen}}^{V}(s).$$
(12)

Equating the correlation functions from the phenomenological side given by Eq. (12) to that from the operator product expansion given by Eq. (9), we obtain

$$\int_{0}^{\infty} ds e^{-s/M^{2}} R_{\text{phen}}^{V}(s) + 12\pi^{2} \Pi^{V}(0)$$
$$= d_{V} \left[ c_{0}^{V} M^{2} + c_{1}^{V} + \frac{c_{2}^{V}}{M^{2}} + \frac{c_{3}^{V}}{2M^{4}} \right], \qquad (13)$$

where the second term in the left-hand side of the above equation is the contribution from scattering of the vector meson with the baryons in the hadronic medium. In the nuclear medium as is the case of the present work of the study of in-medium masses of vector meson,  $\Pi^V(0) = \frac{\rho_B}{4M_N}$  for  $V = \omega$ ,  $\rho$ . Further,  $\Pi^V(0)$  vanishes for the  $\phi$  meson since the  $\phi$ -meson nucleon coupling is zero in the parametrization for the vector meson-baryon interactions within the chiral SU(3) model [25]. This is a reasonable assumption because of the fact that the  $\phi$ -meson nucleon scattering amplitude is negligibly small as compared to the  $\omega$  nucleon as well as  $\rho$ -nucleon scattering amplitudes [13,17,59,60]. The spectral density is assumed to be of the form of a resonance part  $R_{\text{phen}}^{V(\text{res})}(s)$  and a perturbative continuum as [17,51]

$$R_{\rm phen}^{V}(s) = R_{\rm phen}^{V({\rm res})}(s)\theta(s_{0}^{V}-s) + d_{V}c_{0}^{V}\theta(s-s_{0}^{V}).$$
(14)

For  $M > \sqrt{s_0^V}$ , the exponential function in the integral of the left-hand side of Eq. (13) is expanded in powers of  $s/M^2$  for  $s < s_0^V$  and one obtains the finite energy sum rules (FESR) [17] by equating the powers in  $1/M^2$  of both sides of Eq. (13).

In the hadronic medium, the FESRs are obtained as [51]

$$F_V^* = d_V \left( c_0^V s_0^{*V} + c_1^V \right) - 12\pi^2 \Pi^V(0), \tag{15}$$

$$F_V^* m_V^{*\,2} = d_V \left( \frac{\left(s^{*_0^V}\right)^2 c_0^V}{2} - c_2^{*V} \right),\tag{16}$$

$$F_V^* m_V^{*\,4} = d_V \left( \frac{\left( s_0^{*V} \right)^3}{3} c_0^V + c_3^{*V} \right). \tag{17}$$

The coefficient  $c_2^{*V}$  contains the quark and gluon condensates in the medium, and  $c_3^{*V}$  corresponds to the four quark condensate, which is calculated using a factorization method [3] along with a parameter  $\kappa_i$ , i = u, d, s which measures the deviation from exact factorization ( $\kappa_i = 1$ ). For the nonstrange vector mesons,  $\rho$  and  $\omega$ , these coefficients are given as

$$c_0^{(\rho,\omega)} = 1 + \frac{\alpha_s(Q^2)}{\pi}, \quad c_1^{(\rho,\omega)} = -3(m_u^2 + m_d^2), \quad (18)$$

$$c_{2}^{*(\rho,\omega)} = \frac{\pi^{2}}{3} \left\langle \frac{\alpha_{s}}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle + 4\pi^{2} \langle m_{u} \bar{u}u + m_{d} \bar{d}d \rangle, \quad (19)$$

$$c^*{}_3{}^{(\rho,\omega)} = -\alpha_s \pi^3 \times \frac{448}{81} \kappa_q (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2), \tag{20}$$

where we take  $\kappa_u \simeq \kappa_d = \kappa_q$ . For the  $\phi$  meson, these coefficients are given as [2,17]

$$c_0^{\phi} = 1 + \frac{\alpha_s(Q^2)}{\pi}, \quad c_1^{\phi} = -6m_s^2,$$
 (21)

$$c_{2}^{*\phi} = \frac{\pi^{2}}{3} \left\langle \frac{\alpha_{s}}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle + 8\pi^{2} \langle m_{s} \bar{s} s \rangle, \qquad (22)$$

$$c_{3}^{*}{}^{\phi} = -8\pi^{3} \times \frac{224}{81} \alpha_{s} \kappa_{s} \langle \bar{s}s \rangle^{2}.$$
 (23)

Solving the FESR for the vector meson  $V(\rho, \omega, \phi)$  in vacuum, assuming the vacuum mass of the vector meson, determines the value of the coefficient  $\kappa_q$  ( $\kappa_s$ ) of the four-quark condensate, along with the parameters  $F_V$  and  $s_0^V$  in vacuum [51]. Equations (15), (16), and (17) are then solved to obtain the medium dependent mass  $m_V^*$ , the scale  $s_0^{*V}$ , and  $F_V^*$  for the vector meson V, using the value of  $\kappa_i$  as determined from the FESR in vacuum.

#### IV. RESULTS AND DISCUSSIONS

In this section, we study the in-medium masses of light vector mesons ( $\omega$ ,  $\rho$ , and  $\phi$ ) in strongly magnetized nuclear matter, using the QCD sum rule approach from the light quark condensates and the scalar gluon condensate calculated within a chiral SU(3) model. The broken scale invariance of QCD is incorporated in the effective hadronic model through a scale breaking logarithmic potential in terms of a scalar dilaton field  $\chi$  and the gluon condensate in the magnetized nuclear matter is calculated from the medium modification of this scalar dilaton field. In the chiral effective model, the calculations are done in the mean field approximation. In mean field approximation, the meson fields are treated as classical fields. The nonstrange light quark condensates ( $\langle \bar{u}u \rangle$ ,  $\langle dd \rangle$ ) and strange condensate ( $\langle \bar{s}s \rangle$ ) in the asymmetric nuclear matter in the presence of magnetic field are calculated within the model from the values of the nonstrange ( $\sigma$ ,  $\delta$ ) and strange ( $\zeta$ ) scalar fields within the chiral SU(3) model. For given values of baryon density  $\rho_B$ , the magnetic field and the isospin asymmetry parameter  $\eta = (\rho_n - \rho_p)/(2\rho_B) (\rho_p \text{ and } \rho_n \text{ are the proton and neutron number densities, respectively), the mean values of the scalar fields <math>\sigma$ ,  $\zeta$ ,  $\delta$ , and  $\chi$  are obtained by solving the coupled equations of motion of these fields. In the absence of a magnetic field, the in-medium masses of the light vector mesons in asymmetric hadronic matter were studied using the QCD sum rule approach and using the gluon and quark condensates calculated within the chiral SU(3) model [51]. In the present work, the medium modifications of the vector meson masses are studied in the presence of a strong magnetic field.

In the present investigation, the values of the current quark masses are taken as  $m_u = 4$  MeV,  $m_d = 7$  MeV, and  $m_s = 150$  MeV. The vacuum masses of the  $\rho$ ,  $\omega$ , and  $\phi$  mesons are taken to be 770, 783, and 1020 MeV. The coefficients  $\kappa_i$  (i = q, s) are solved from the FESRs in vacuum [51] and are obtained as 7.788, 7.236, and -1.21 for the  $\omega$ ,  $\rho$ , and  $\phi$  mesons. The difference in the values of  $\kappa_q$  obtained from solving the FESRs of the  $\omega$  and  $\rho$  mesons are due to the difference in their vacuum masses.

In the presence of a magnetic field, the proton has contributions from the Landau energy levels. The anomalous magnetic moments of the nucleons are also taken into consideration in studying the in-medium masses of light vector mesons in nuclear matter in the presence of a magnetic field, using the QCDSR approach. The light quark condensates and the scalar gluon condensate are calculated from Eqs. (4) and (5), which are then used to calculate the values of the coefficients  $c_2^{*V}$  and  $c_3^{*V}$  for the vector mesons,  $\rho$ ,  $\omega$ , and  $\phi$  as given by Eqs. (19), (20), (22), and (23). Using the values of these coefficients, the in-medium masses of the light vector mesons are calculated by solving the coupled equations (15), (16), and (17) involving the in-medium values  $F_V^*$ ,  $s_0^{*V}$ , and  $m_V^*$ .

In Figs. 1 and 2, the in-medium masses of  $\omega$  mesons are plotted as functions of baryon density  $\rho_B$  (in units of nuclear matter saturation density  $\rho_0$ ) for values of magnetic fields  $eB = 4m_{\pi}^2$  and  $eB = 12m_{\pi}^2$ , respectively, with values of isospin asymmetry parameter  $\eta$  taken to be 0, 0.3, and 0.5. These masses are plotted including the anomalous magnetic moments (AMM) of the nucleons and are compared to the case when the AMMs are not taken into consideration. The masses of the  $\omega$  meson are also given in Table I for the isospin symmetric ( $\eta = 0$ ) as well as for the extreme isospin asymmetric case of  $\eta = 0.5$  for magnetic fields eB = $4m_{\pi}^2$ ,  $8m_{\pi}^2$ ,  $12m_{\pi}^2$ , (a) without, and (b) with the anomalous magnetic moments (AMM) of nucleons taken into account. The  $\omega$  meson mass is observed to have an initial drop with increase in density for subnuclear densities. This is due to the fact that the contribution of the scattering term, which is proportional to baryon density  $\rho_B$ , is small at low densities, and the medium modification of the  $\omega$  meson mass is dominated by modifications of the light quark condensates in the medium, which leads to a drop in the mass of the  $\omega$  meson. As the density is further increased, the effects of the scattering of the vector meson  $\omega$  from the nucleons become appreciable, leading to a rise in the  $\omega$  meson mass. This observed behavior can be understood from the first two FESRs. The effective



FIG. 1. The mass of  $\omega$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 4m_{\pi}^2$ .

mass squared of the vector meson V is obtained as dividing Eq. (16) by (15) as

$$m_V^{*2} = \frac{\left(\frac{(s^*_V)^2 c_V^V}{2} - c_2^{*V}\right)}{\left(c_0^V s^{*V}_0 + c_1^V\right) - (1/d_V) 12\pi^2 \Pi^V(0)},$$
 (24)

where, as was already mentioned,  $\Pi^V(0) = \rho_B/(4M_N)$ , for  $V = \omega$ ,  $\rho$ , and  $\Pi^V(0)$  vanishes for the  $\phi$  meson as  $\phi$ -nucleon



FIG. 2. The mass of  $\omega$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 12m_{\pi}^2$ .

TABLE I. In-medium masses for the  $\omega$  meson in magnetized nuclear matter for densities of  $\rho_0$  and  $2\rho_0$ , asymmetric parameter  $\eta = 0$  and 0.5, and for magnetic fields  $eB/m_{\pi}^2$  as 4, 8, and 12, (a) without and (b) with the anomalous magnetic moments of the nucleons taken into account. These masses are compared with the in-medium masses of  $\omega$  meson for zero magnetic field.

$eB/m_{\pi}^2$		$\eta = 0$		$\eta = 0.5$	
		$\rho_B = \rho_0$	$\rho_B = 2\rho_0$	$\rho_B = \rho_0$	$\rho_B = 2\rho_0$
	0	777	953.7	788.8	965.6
4	(a)	773.4	950	787.24	964.2
	(b)	775.7	953.4	792.5	967.56
8	(a)	772.8	947.8	787.24	964.2
	(b)	775.6	952.4	793.74	972
12	(a)	772.34	947.3	787.24	964.2
	(b)	775.87	953.34	795.3	974.1

coupling is assumed to be zero in the present work. The scattering term for the nonstrange vector mesons, being proportional to the density, becomes appreciable at higher densities. This leads to a smaller value for the denominator and hence to an increase in the mass of  $\omega$  meson at high densities. This behavior of the  $\omega$  mass with density was also observed for the case of zero magnetic field in Ref. [51]. The effects of the magnetic field, which are through the light quark and gluon condensates, are observed to be much smaller as compared to the density effects. The values of the  $\omega$  meson mass (in MeV) at densities  $\rho_0(2\rho_0)$ , for the values of the isospin asymmetric parameter  $\eta = 0, 0.3, and 0.5$  are observed to be 773.4 (950), 781.55 (956.86), and 787.24 (964.2) at the value of the magnetic field as  $eB = 4m_{\pi}^2$ , when the anomalous magnetic moments (AMM) of nucleons are not considered, and, 775.7 (953.4), 785.2 (960), and 792.5 (967.56), when AMMs are taken into consideration. The isospin asymmetry effects are observed to be larger for the higher value of the magnetic field,  $eB = 12m_{\pi}^2$ , plotted in Fig. 2. The contribution of the scattering term which is the dominant contribution to the mass of the  $\omega$  meson at high densities, being proportional to  $\rho_B$ , is independent of the isospin asymmetry. This leads to lessening of the isospin asymmetry effects at higher densities, as is evident from Figs. 1 and 2. The isospin asymmetry effects on the  $\omega$  meson mass are observed only at densities, of around  $0.5\rho_0$  to about 2  $\rho_0$ , above which these effects are seen to be diminished as the scattering effects start becoming important. The isospin asymmetry effects are observed to be more appreciable for the higher value of the magnetic field,  $eB = 12m_{\pi}^2$ , as can be seen from Fig. 2.

The density isospin asymmetry effects on the masses of the  $\rho$  meson are illustrated in Figs. 3 and 4 for the values of eB as  $4m_{\pi}^2$  and  $12m_{\pi}^2$ , respectively. The in-medium  $\rho$  meson masses are given in Table II, for typical values of the magnetic field, density, and isospin asymmetry parameters, which are calculated (a) without and (b) with the anomalous magnetic moments (AMM) of the nucleons. The  $\rho$  meson mass in the presence of a magnetic field is observed to drop with baryon density, similar to the case of zero magnetic field studied previously [51]. The contribution of the scattering of



FIG. 3. The mass of  $\rho$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 4m_{\pi}^2$ .

the  $\rho$  meson from the nucleons in the nuclear matter is small because of the factor  $(1/d_V)$  in this term, which makes the contribution of the Landau scattering term 9 times smaller than that of the  $\omega$  meson, as  $(1/d_{\rho})/(1/d_{\omega}) = 1/9$ . The effects of the isospin asymmetry are observed to be large at high densities, as was seen for the case of zero magnetic field [51]. The effects of magnetic fields as well as of anomalous



FIG. 4. The mass of  $\rho$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 12m_{\pi}^2$ .

TABLE II. In-medium masses for  $\rho$  meson in magnetized nuclear matter for densities of  $\rho_0$  and  $4\rho_0$ , asymmetric parameter  $\eta = 0$  and 0.5, and for magnetic fields  $eB/m_{\pi}^2$  as 4, 8, and 12, (a) without and (b) with the anomalous magnetic moments of the nucleons taken into account. These masses are compared with the in-medium masses of  $\rho$  meson for zero magnetic field.

$eB/m_{\pi}^2$		$\eta = 0$		$\eta = 0.5$	
		$\rho_B = \rho_0$	$\rho_B = 4\rho_0$	$\rho_B = \rho_0$	$\rho_B = 4\rho_0$
0		622.2	391	636.3	473.3
4	(a)	618	374.9	634.5	468.5
	(b)	620.7	398	640.85	477.5
8	(a)	617.26	335.8	634.5	468.5
	(b)	620.65	411.7	642.3	502.8
12	(a)	616.68	315.8	634.5	468.5
	(b)	620.9	433.7	644.2	527.6

magnetic moments are observed to be appreciable at high densities, as can be seen in Table II. Accounting for the anomalous magnetic moments (AMM) of the nucleons, the values of the  $\rho$  meson mass for symmetric nuclear matter  $(\eta = 0)$  at densities  $\rho_0(3\rho_0)$  are 620.7 (445.4) and 620.9 (455.9) for  $eB = 4m_{\pi}^2$  and  $eB = 12m_{\pi}^2$ , respectively. These values are modified to 632.1 (468.9) and 634.85 (500) at  $\eta = 0.3$  and 640.85 (508.7) and 644.2 (545.36) at  $\eta = 0.5$ , for the same magnetic fields. The drop in the  $\rho$  meson mass is observed to be smaller when the AMMs of the nucleons are taken into consideration. The isospin asymmetry effect is observed to be much larger for the larger magnetic field,  $eB = 12m_{\pi}^2$ , especially at higher densities, as can be seen in Fig. 4. In Table II, the values of the  $\rho$  meson mass in the presence of magnetic field for the symmetric nuclear matter as well as for asymmetric nuclear matter with  $\eta = 0.5$ , at densities  $\rho_0$  and  $4\rho_0$ , are compared with the values obtained for zero magnetic field in Ref. [51]. As can be seen from Table II, in the absence of a magnetic field, the in-medium  $\rho$ mass calculated at the nuclear matter saturation density  $\rho_0$  in symmetric nuclear matter is 622.2 MeV [51], which is similar to the value obtained in Ref. [12], using the linear density approximation. The mass of  $\rho$  meson at  $\rho_0$  at zero magnetic field, may be compared with the value of 670 MeV obtained using the QCD sum rule approach in Ref. [19] and, of around 530 MeV, in an improved QCD sum rule calculation [15]. In the present work, the effects of magnetic field on the  $\rho$ meson mass in isospin asymmetric nuclear matter have been investigated.

The  $\phi$  meson masses in the magnetized nuclear matter are presented in the Table III, for given values of density, isospin asymmetry parameter and the magnetic field, (a) without and (b) with the anomalous magnetic moments of the nucleons being taken into consideration. The in-medium masses of the  $\phi$  meson, plotted in Figs. 5 and 6 for the values of *eB* as  $4m_{\pi}^2$  and  $12m_{\pi}^2$ , respectively, are observed to decrease sharply with density up to a density of around  $\rho_0$ , above which there is observed to be very less modification in the mass of the  $\phi$  meson. There is no contribution to the  $\phi$  meson mass from the scattering term. This is because of the fact that

TABLE III. In-medium masses for  $\phi$  meson in magnetized nuclear matter for densities of  $\rho_0$  and  $4\rho_0$ , asymmetric parameter  $\eta = 0$  and 0.5, and for magnetic fields  $eB/m_{\pi}^2$  as 4, 8, and 12, (a) without and (b) with the anomalous magnetic moments of the nucleons taken into account. These masses are compared with the in-medium masses of  $\phi$  meson for zero magnetic field.

$eB/m_{\pi}^2$		$\eta = 0$		$\eta = 0.5$	
		$\rho_B = \rho_0$	$\rho_B = 4\rho_0$	$\rho_B = \rho_0$	$\rho_B = 4\rho_0$
0		1001.49	998.79	1001.79	998.38
4	(a)	1001.11	998.4	1001.6	997.85
	(b)	1001.28	998.08	1002.03	997.74
8	(a)	1000.9	998.9	1001.6	997.85
	(b)	1001.28	997.87	1002.14	997.5
12	(a)	1001.09	999.2	1001.6	997.85
	(b)	1001.22	997.63	1002.28	997.38

the nucleon- $\phi$  meson coupling is zero in the parameter set chosen for the vector meson-baryon interactions within the chiral SU(3) model. It might be noted here that  $\phi \to K\bar{K}$  is OZI allowed, whereas the OZI rule forbids the decay of the  $\phi$ meson to pions. There is, however, observed to be a violation of the OZI rule by about 5% in the  $\phi \to 3\pi$  channel [17,18]. The decay  $\phi \to 3\pi$  and the decay  $\omega \to 3\pi$  (through direct decay as well as the two-step decay  $\omega \to \rho\pi$  followed by  $\rho \to 2\pi$ ) imply that there is mixing between the  $\phi$  meson and the nonstrange ( $\rho$ ,  $\omega$ ) vector mesons. However, the decay of  $\phi$  is dominated by the OZI allowed  $\phi \to K\bar{K}$  and the mass modification of the  $\phi$  meson is predominantly due to the change of the strange condensate in the hadronic medium. The drop in  $\phi$  meson mass is much smaller than the mass



FIG. 5. The mass of  $\phi$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 4m_{\pi}^2$ .



FIG. 6. The mass of  $\phi$  meson plotted as a function of the baryon density in units of nuclear matter saturation density for magnetized nuclear matter (for  $\eta = 0, 0.3, 0.5$ ) with  $eB = 12m_{\pi}^2$ .

drop of the  $\rho$  meson (where the scattering term has negligible contribution) because of the large drop of the nonstrange quark condensate as compared to the drop of the strange quark condensate in the nuclear medium. At higher densities, the strange condensate remains almost constant and hence the  $\phi$  meson mass changes very little with density, whereas the nonstrange quark condensate continues to drop as density is increased, leading to a monotonic drop of the  $\rho$  meson mass with density. These behaviors of the  $\rho$  and  $\phi$  vector meson masses were also observed for the case of zero magnetic field [51]. The effects of anomalous magnetic moments of the nucleons are observed to be larger at higher densities, even though the magnitudes of these still remain small. The strange quark condensate as well as the scalar gluon condensate have very small effects from isospin asymmetry, leading to the modifications of the  $\phi$  meson mass to be very similar in the isospin symmetric and asymmetric nuclear matter.

### V. SUMMARY

In summary, in the present investigation, we have calculated the masses of the light vector mesons ( $\omega$ ,  $\rho$ , and  $\phi$ ) in the nuclear matter in the presence of a strong magnetic field, using the QCD sum rule approach, from the modifications of the light quark and scalar gluon condensates, calculated within a chiral effective model. The medium modifications are from the changes of the light quark condensates and the scalar gluon condensate in the magnetized isospin asymmetric hadronic matter, which are calculated within mean field approximation, from the changes in the expectation values of the scalar fields ( $\sigma$ ,  $\zeta$ , and  $\delta$ ) and a dilaton field ( $\chi$ ) from their vacuum values, respectively, within a chiral SU(3) model. The masses of the  $\rho$  mesons are dominantly governed by the nonstrange quark condensates which lead to decrease in these masses with increase in density. For the  $\omega$  meson, the drop in the nonstrange quark condensates leads to a drop in the mass at subnuclear matter densities, which, however, is overcome by the  $\omega$ -nucleon scattering term, leading to an increase in the  $\omega$  mass as the density is further increased. The scattering term, which dominates for the  $\omega$  meson mass at high densities, is of the form  $(\rho_p + \rho_n)$  and is thus independent of the isospin asymmetry of the nuclear medium. The mass of the  $\phi$  meson is dominated by the behavior of the strange condensate  $\langle \bar{s}s \rangle$ in the medium, leading to an initial drop with density and the mass changes slightly as the density is further increased. The isospin asymmetry effects are observed to be more for the  $\rho$ meson as compared to the  $\omega$  and  $\phi$  mesons. The effects of anomalous magnetic moments of the nucleons are seen to be appreciable at higher densities and higher magnetic fields for the  $\rho$  meson mass. The density effects are observed to be the dominant medium effects on the masses of these light vector mesons.

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