# **Estimates of hyperon polarization in heavy-ion collisions at collision energies**  $\sqrt{s_{NN}} = 4-40 \text{ GeV}$

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Global polarization of  $\Lambda$  and  $\bar{\Lambda}$  hyperons in Au + Au collisions at collision energies  $\sqrt{s_{NN}} = 4-40$  GeV in the midrapidity region and total polarization, i.e., averaged over all rapidities, are studied within the scope of the thermodynamical approach. The relevant vorticity is simulated within the model of the three-fluid dynamics (3FD). It is found that the performed rough estimate of the global midrapidity polarization quite satisfactorily reproduces the experimental STAR data on the polarization, especially its collision-energy dependence. The total polarization increases with the collision energy rise, which is in contrast to the decrease of the midrapidity polarization. This suggests that at high collision energies the polarization reaches high values in fragmentation regions.

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### **I. INTRODUCTION**

Huge global angular momentum is generated in noncentral heavy-ion collisions at high energies that can be partially transformed into spin alignment of constituents  $[1-3]$ . The latter can be measured by the polarization of hyperons and vector mesons. Global polarization of  $\Lambda$  and  $\bar{\Lambda}$  hyperons was measured [\[4\]](#page-6-0) by the STAR experiment in the energy range of the Beam Energy Scan (BES) program at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven. It was measured in the midrapidity region of colliding nuclei. The measured polarization is generally reproduced within the hydrodynamic  $[5,6]$  and kinetic  $[7-11]$  model calculations based on the thermodynamics in the hadronic phase [\[12–14\]](#page-6-0), as well as within an alternative approach directly based on the axial vortical effect  $[15-17]$  within the quarkgluon string transport model  $[18]$ . The axial vortical effect is associated with axial-vector current induced by vorticity. This current implies that the right (left)-handed fermions move parallel (opposite) to the direction of vorticity. As the momentum of a right (left)-handed massless fermion is parallel (opposite) to its spin, all spins become parallel to the direction of vorticity, i.e., aligned.

In the present paper we estimate the global polarization of  $\Lambda$  and  $\bar{\Lambda}$  hyperons in Au + Au collisions based on the thermodynamical approach  $[12-14]$ . The relevant vorticity is simulated within the model of the three-fluid dynamics (3FD) [\[19\]](#page-6-0). We perform a collision-energy scan in the energy range of the Facility for Antiproton and Ion Research (FAIR) in Darmstadt [\[20\]](#page-6-0), the Nuclotron based Ion Collider fAcility (NICA) in Dubna [\[21\]](#page-6-0), and BES at RHIC.

The 3FD model describes of the major part of bulk observables: the baryon stopping [\[22,23\]](#page-6-0), yields of different hadrons, their rapidity and transverse momentum distributions [\[24,25\]](#page-6-0), and also the elliptic  $[26]$  and directed  $[27]$  flow. It also reproduces [\[28\]](#page-6-0) recent STAR data on bulk observables [\[29\]](#page-6-0).

The question we address in this paper is whether the 3FD model is able to reproduce the observed global midrapidity polarization without any additional adjustment of the model parameters. In other words, are the bulk and flow properties of the produced matter internally interconnected with its polarization? Based on this analysis we make predictions for the global midrapidity polarization in the FAIR-NICA energy range.

We also address the question, why does the observed global polarization of hyperons in the midrapidity region drop with the collision energy rise while the total angular momentum accumulated in the system substantially increases at the same time? To this end, we also estimate total polarization of hyperons, i.e., the mean global polarization over all rapidities.

#### **II. THE 3FD MODEL**

The 3FD model takes into account a finite stopping power resulting in counterstreaming of leading baryon-rich matter at the early stage of nuclear collisions [\[19\]](#page-6-0). This nonequilibrium stage is modeled by means of two counterstreaming baryonrich fluids initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. Later on these fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather than only nucleons. Newly produced particles, dominantly populating the midrapidity region, are associated with a fireball (f) fluid. These fluids are governed by conventional hydrodynamic equations coupled by friction terms in the right-hand sides of the Euler equations. The friction results in energy-momentum loss of the baryon-rich fluids. A part of this loss is transformed into thermal excitation of these fluids, while another part leads to formation of the

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<span id="page-1-0"></span>

FIG. 1. The total angular momentum (conserved quantity) and the angular momentum accumulated in the participant region in semicentral ( $b = 8$  fm) Au + Au collision as functions of  $\sqrt{s_{NN}}$ . Calculations are done with the first-order transition and crossover EoS's.

fireball fluid. Thus, the 3FD approximation is a minimal way to implement the early-stage nonequilibrium of the produced strongly interacting matter at high collision energies.

The physical input of the present 3FD calculations is described in Ref. [\[22\]](#page-6-0). Three different equations of state (EoS's) were used in simulations of Refs. [\[22–28\]](#page-6-0): a purely hadronic EoS [\[30\]](#page-6-0) and two versions of the EoS with the deconfinement transition [\[31\]](#page-6-0), i.e., a first-order phase transition and a crossover one. The friction between the fluids in the hadronic phase was estimated in Ref. [\[32\]](#page-6-0) based on experimental proton-proton cross sections. This friction is implemented in the 3FD simulations of the hadronic phase. There are no estimates of this friction in the quark-gluon phase (QGP). Therefore, the friction in the QGP was fitted for each EoS to reproduce the observed stopping power; see Ref. [\[22\]](#page-6-0) for details. In the present paper only the first-order-phasetransition (1st-order-tr.) and crossover EoS's are used as the most relevant to various observables.

Total angular momentum is conserved with an accuracy of 1% in the 3FD simulations. The angular momentum is defined as

$$
J = \int d^3x \sum_{\alpha = \text{p}, \text{t}, \text{f}} \left( z \ T_{10}^{\alpha} - x \ T_{30}^{\alpha} \right). \tag{1}
$$

where  $T_{\mu\nu}^{\alpha}$  is the energy-momentum tensor of the  $\alpha$  (=p,t,f) fluid and has the conventional hydrodynamical form, *z* is the beam axis, and  $(x, z)$  is the reaction plane of the colliding nuclei. The total angular momentum, *J*<sub>total</sub>, in semicentral (impact parameter  $b = 8$  fm) Au + Au collision as function of collision energy  $\sqrt{s_{NN}}$  is presented in Fig. 1. It is independent of the used EoS. For the  $J_{total}$  calculation the integration in Eq.  $(1)$  runs over the whole system. As seen,  $J_{total}$  rapidly rises with the collision energy. Only a part of the total angular momentum is accumulated in the participant region. Figure 1 also displays the angular momentum accumulated in the participant region, i.e., in the overlap region of the interacting fluids. As seen from Fig. 1, 25–30% of the total

angular momentum is deposited into participant matter in the Au + Au collisions at  $b = 8$  fm.

The participant angular momentum rises with time because the overlap region of the interacting fluids increases in the course of the expansion stage and includes more and more former spectators. Therefore, the participant angular momentum depends, though weakly, on the EoS. Figure 1 presents the participant angular momenta at the "freeze-out" instant of time in the c.m. frame of colliding nuclei, i.e., when average energy density throughout the participant region falls to the freeze-out value of  $\varepsilon_{\text{frz}} = 0.4 \text{ GeV}/\text{fm}^3$ . This is a kind of illustrative freeze-out. In actual calculations of observables a differential, i.e., cell-by-cell, freeze-out is implemented [\[33\]](#page-6-0). The freeze-out occurs when the local energy density drops down to the freeze-out value  $\varepsilon_{\text{frz}}$ .

### **III. VORTICITY IN THE 3FD MODEL**

A so-called thermal vorticity is defined as

$$
\varpi_{\mu\nu} = \frac{1}{2} (\partial_{\nu} \hat{\beta}_{\mu} - \partial_{\mu} \hat{\beta}_{\nu}), \qquad (2)
$$

which is dimensionless. Here  $\hat{\beta}_{\mu} = \hbar \beta_{\mu}, \ \beta_{\mu} = u_{\nu}/T, \ u_{\mu}$  is collective local four-velocity of the matter, and *T* is local temperature. In the thermodynamical approach [\[12–14\]](#page-6-0) in the leading order in the thermal vorticity it is directly related to the mean spin vector of spin 1/2 particles with four-momentum *p*, produced around point *x* on freeze-out hypersurface

$$
S^{\mu}(x, p) = \frac{1}{8m} [1 - n_F(x, p)] p_{\sigma} \epsilon^{\mu\nu\rho\sigma} \varpi_{\rho\nu}(x), \qquad (3)
$$

where  $n_F(x, p)$  is the Fermi-Dirac distribution function and *m* is mass of the considered particle. To calculate the relativistic mean spin vector of a given particle species with given momentum, the above expression should be integrated over the freeze-out hypersurface.

Unlike the conventional hydrodynamics, the system is characterized by three hydrodynamical velocities,  $u_a^{\mu}$  ( $a = p$ , t, and f), in the 3FD model. The counterstreaming of the p and t fluids takes place only at the initial stage of the nuclear collision that lasts from  $\approx$ 5 fm/*c* at  $\sqrt{s_{NN}}$  = 5 GeV [\[34\]](#page-6-0) to  $\approx$ 1 fm/*c* at a collision energy of 39 GeV [\[35\]](#page-6-0). At later stages the baryon-rich (p and t) fluids have already either partially passed though each other or partially stopped and unified in the central region. At lower collision energies, like those of NICA and FAIR, the contribution the f fluid into various quantities, in particular into the vorticity [\[34\]](#page-6-0), is small compared with that of the baryon-rich (p and t) fluids. At higher BES RHIC energies the f-fluid contributions become comparable with those of the baryon-rich (p and t) fluids. The f fluid also is entrained by the the unified baryon-rich fluid but is not that well unified with the latter, thus keeping its identity even after the initial thermalization/unification of the baryonrich fluids. The local baryon-fireball relative velocity is small but not negligible even at the freeze-out stage [\[36\]](#page-6-0). In particular, the friction between the baryon-rich and net-baryon-free fluids is the only source of dissipation at the expansion stage.

Therefore, after the initial thermalization stage the system is characterized by two hydrodynamical velocities,  $u_{\text{B}}^{\mu}$  and  $u_{\text{f}}^{\mu}$ , and two temperatures,  $T_\text{B}$  and  $T_\text{f}$ , corresponding to the unified baryon-rich (B) and fireball (f) fluids.

As a result the system is characterized by two sets of the vorticity related to these baryon-rich and baryon-free fluids,  $\varpi_{\mu\nu}^{B}$  and  $\varpi_{\mu\nu}^{f}$ , respectively, which are defined in terms of their velocities and temperatures. We consider a proper-energy-density-weighted vorticity which allows us to suppress contributions of regions of low-density matter. It is appropriate because production of (anti)hyperons under consideration dominantly takes place in highly excited regions of the system. We also determine the sum of vorticities of the baryon-rich and baryon-free fluids with the weights of their energy densities, and thus define a single quantity responsible for the particle polarization:

$$
\widetilde{\varpi}_{\mu\nu}(\mathbf{x},t) = \frac{\varpi^{\mathrm{B}}_{\mu\nu}(\mathbf{x},t)\varepsilon_{\mathrm{B}}(\mathbf{x},t) + \varpi^{\mathrm{f}}_{\mu\nu}(\mathbf{x},t)\varepsilon_{\mathrm{f}}(\mathbf{x},t)}{\varepsilon(\mathbf{x},t)}
$$
(4)

where  $\varepsilon_B$  and  $\varepsilon_f$  are the proper energy densities of the the baryon-rich and baryon-free fluids, respectively. The proper energy density of all three fluids in their combined local rest frame,  $\varepsilon$ , is

$$
\varepsilon = u_{\mu} T^{\mu \nu} u_{\nu}.
$$
 (5)

where  $T^{\mu\nu} \equiv T^{\mu\nu}_p + T^{\mu\nu}_t + T^{\mu\nu}_f$  is the total energy– momentum tensor, being the sum of conventional hydrodynamical energy-momentum tensors of separate fluids, and the total collective four-velocity of the matter is

$$
u^{\mu} = u_{\nu} T^{\mu \nu} / (u_{\lambda} T^{\lambda \nu} u_{\nu}). \tag{6}
$$

However, because of almost perfect unification of the baryonrich fluids and small local baryon-fireball relative velocities [\[35\]](#page-6-0), at the later stages of the collision a very good approximation for  $\varepsilon$  is just

$$
\varepsilon \simeq \varepsilon_B + \varepsilon_f. \tag{7}
$$

A quantitative comparison of the thermal vorticity in semicentral ( $b = 8$  fm) Au + Au collisions at different collision energies  $\sqrt{s_{NN}}$  is performed in terms of average thermal vorticity of the composed matter [Eq. (4)] also averaged over coordinate (*x*) space with the weight of the proper energy density:

$$
\langle \varpi_{\mu\nu}(t) \rangle = \int d^3x \left[ \varpi_{\mu\nu}^{\text{B}}(\mathbf{x}, t) \, \varepsilon_{\text{B}}(\mathbf{x}, t) \right. \\ + \varpi_{\mu\nu}^{\text{f}}(\mathbf{x}, t) \, \varepsilon_{\text{f}}(\mathbf{x}, t) \Big] / \langle \varepsilon(t) \rangle, \tag{8}
$$

where average energy density is

$$
\langle \varepsilon(t) \rangle = \int d^3x \; \varepsilon(\mathbf{x}, t) \Bigg/ \int \theta[\varepsilon(\mathbf{x}, t)] d^3x \tag{9}
$$

with  $\theta(x)$  being equal to 1 for  $x > 0$ , and 0 otherwise. This averaging is performed over two different space regions:

(a) Over the central slab,  $|x| < R - b/2$ ,  $|y| < R - b/2$ and  $|z| < R/\gamma_{cm}$ , where *R* is the radius of the Au nucleus, *b* is the impact parameter, and  $\gamma_{cm}$  is the Lorentz factor associated with the initial nuclear motion along the beam (*z*) axis in the c.m. frame. This central central layer includes the whole participant region in the transverse direction. The data from this central slab are used to imitate the midrapidity global polarization.

(b) Over the whole participant region system, which is restricted by the condition  $T > 100$  MeV. This condition first of all is related to the baryon-rich fluid because the temperature of the produced f fluid is always high. The temperature gradients and hence the thermal vorticity reach very high values at the spectator-participant border, where the temperature itself is not that high. At the same time, the  $\Lambda$  hyperons are efficiently produced only from the hottest regions of the system. Therefore, keeping in mind application to the  $\Lambda$  polarization, we apply this temperature constraint. The temperature is always high in the above discussed central slab, which makes this constraint unnecessary.

In order to keep all the matter in consideration, conventional local 3FD freeze-out was turned off because it removes the frozen-out matter from the hydrodynamical evolution [\[33\]](#page-6-0). Nevertheless, we do apply a simplified freeze-out, that was already mentioned in the end of the previous section. This is an isochronous freeze-out similar to that used in Refs. [\[5,6\]](#page-6-0). The system is frozen out at the time instant *t*frz when

- (a) the average energy density in the central slab,  $\langle \varepsilon(t) \rangle$ <sub>slab</sub>, decreases to its freeze-out value  $\varepsilon_{\text{frz}} =$  $0.4$  GeV/fm<sup>3</sup>, or
- (b) the average energy density in the whole participant region,  $\langle \varepsilon(t) \rangle_{\text{total}}$ , decreases to its freeze-out value  $\varepsilon_{\text{frz}}$ .

The freeze-out in the central slab of the system of colliding nuclei is used to imitate the midrapidity global polarization, while that in the whole participant region is used to estimate the total<sup>1</sup> polarization that also includes averaging over all rapidities.

Time evolution of the average energy density in the central slab and in the whole participant region is displayed in panels (a) of Figs. [2](#page-3-0) and [3,](#page-3-0) respectively. The bold cyan line indicates the freeze-out value  $\varepsilon_{\text{frz}}$ . The simulations were performed with the crossover EoS. We do not present results for the first-order-transition EoS because they are quite similar. As seen from Figs. [2](#page-3-0) and [3,](#page-3-0) the time span prior this global freeze-out is quite short. It should be compared to time of completion of the conventional local 3FD freezeout at the same collision energies: 8 fm/*c* for both total and central-slab freeze-out at 7.7 GeV, 6 fm/*c* for centralslab freeze-out, and 20 fm/*c* for total freeze-out at 39 GeV. This happens because the  $\langle \varepsilon(t) \rangle$  value is calculated over all regions of the system, i.e., including those which would be already locally frozen out to the considered time instant.

The time evolution of the average proper-energy-weighted thermal *zx* vorticity in the central slab and in the whole participant region is displayed in panels (b) of Figs. [2](#page-3-0) and [3,](#page-3-0) respectively. The bold cyan line indicates the global

<sup>&</sup>lt;sup>1</sup>To distinguish it from the global one at midrapidity.

<span id="page-3-0"></span>



FIG. 2. Time evolution of (a) the proper energy density averaged over the central slab in the semicentral  $(b = 8$  fm) Au + Au collision at  $\sqrt{s_{NN}}$  = 3.8, 4.9, 7.7, 11.5, 19.6, 39 GeV; the cyan band is placed at the freeze-out energy density  $\varepsilon = 0.4 \text{ GeV}/\text{fm}^3$ . (b) The properenergy-density-weighted thermal *zx* vorticity averaged over the central slab; the cyan band indicates the freeze-out, corresponding to the  $\varepsilon = 0.4$  GeV/fm<sup>3</sup> band in panel (a). Calculations are done with the crossover EoS.

freeze-out, which correspond to the similar lines at value  $\varepsilon_{\text{frz}}$ in panels (a) of Figs. 2 and 3.

The central-slab thermal vorticity rapidly decreases with time. At the early stages it practically coincides with the total one because this central region includes all the participant region. Later on the central vorticity becomes more than an order of magnitude lower than the total one. The central-slab vorticity at the freeze-out decreases with increasing collision energy because the vortical field is pushed out to the fragmentation of regions [\[36,37\]](#page-6-0). The violation of this trend at energies

FIG. 3. The same as in Fig. 2 but averaging over the whole participant region at collision energies  $\sqrt{s_{NN}}$  = 3.8, 4.9, 7.7, 11.5, 19.6, 27 GeV.

19.6 and 39 GeV is because of somewhat unstable numerics at 39 GeV energy.

At the same time, the average total thermal vorticity at the freeze-out generally rises with the collision energy, as can be expected from the corresponding increase of the total angular momentum accumulated in the participants; see Fig. [1.](#page-1-0) At the lowest considered energy of 3.8 GeV, the central-slab and total values of the vorticity are very similar because the vorticity is more homogeneously distributed over the beam direction [\[9,34,38\]](#page-6-0) than at higher collision energies. The average total thermal vorticity as a function of time changes much slower as compared with the central one: at lower collision energies it moderately decreases while at higher energies it even slightly rises with time.

### **IV. POLARIZATION**

<span id="page-4-0"></span>In terms of the mean spin vector  $(3)$ , the polarization vector of an *S*-spin particle is defined as

$$
P_S^{\mu} = S^{\mu}/S. \tag{10}
$$

In the experiment, the polarization of the  $\Lambda$  hyperon is measured in its rest frame, therefore the  $\Lambda$  polarization is

$$
P_{\Lambda}^{\mu} = 2S_{\Lambda}^{*\mu},\tag{11}
$$

where  $S_{\Lambda}^{*\mu}$  is mean spin vector of the  $\Lambda$  hyperon in its rest frame. In the  $\Lambda$  rest frame the zeroth component  $S^0_\Lambda$ identically vanishes and the spatial component becomes [\[9\]](#page-6-0)

$$
\mathbf{S}_{\Lambda}^{*}(x, p) = \mathbf{S}_{\Lambda} - \frac{\mathbf{p}_{\Lambda} \cdot \mathbf{S}_{\Lambda}}{E_{\Lambda}(E_{\Lambda} + m_{\Lambda})} \mathbf{p}_{\Lambda}.
$$
 (12)

Substitution of the expression for **S** from Eq. [\(3\)](#page-1-0) and averaging this expression over the  $\mathbf{p}_{\Lambda}$  direction (i.e., over  $\mathbf{n}_p$ ) results in the following polarization in the direction orthogonal to the reaction plane  $(xz)$  [\[9\]](#page-6-0) (see also  $[12-14]$ ):

$$
\langle P_{\Lambda} \rangle_{\mathbf{n}_p} = \frac{1}{2m_{\Lambda}} \left( E_{\Lambda} - \frac{1}{3} \frac{\mathbf{p}_{\Lambda}^2}{E_{\Lambda} + m_{\Lambda}} \right) \varpi_{zx},\tag{13}
$$

where  $m_{\Lambda}$  is the  $\Lambda$  mass and  $E_{\Lambda}$  and  $\mathbf{p}_{\Lambda}$  are the energy and momentum of the emitted  $\Lambda$  hyperon, respectively. Here we put  $(1 - n_{\Lambda}) \simeq 1$  because the  $\Lambda$  production takes place only in high-temperature regions, where Boltzmann statistics dominates.

Particles are produced across the entire freeze-out hypersurface. Therefore, to calculate the global polarization vector, the above expression should be integrated over the freeze-out hypersurface  $\Sigma$  and particle momenta

$$
\langle P_{\Lambda} \rangle = \frac{\int (d^3 p/p^0) \int_{\Sigma} d\Sigma_{\lambda} p^{\lambda} n_{\Lambda} P_{\Lambda}}{\int (d^3 p/p^0) \int_{\Sigma} d\Sigma_{\lambda} p^{\lambda} n_{\Lambda}}.
$$
 (14)

Because of the isochronous freeze-out,  $(d^3 p/p^0) d\Sigma_{\lambda} p^{\lambda} =$  $d^3 p \, d^3 x$ .

We apply further approximations, after which the present evaluation of the global polarization becomes more an estimation than a calculation. We associate the global midrapidity polarization with the polarization of  $\Lambda$  hyperons emitted from the above discussed central slab. We decouple averaging of  $\overline{\omega}_{zx}$  and the term in parentheses in Eq. (13). Here we neglect the longitudinal motion of the  $\Lambda$  hyperon at the freeze-out stage in the central slab and therefore approximate the average  $\Lambda$  energy by the mean midrapidity transverse mass:  $\langle E_{\Lambda} \rangle = \langle m_T^{\Lambda} \rangle_{\text{midrap}}$ , which was calculated earlier in Ref. [\[25\]](#page-6-0). Applying all the above approximations, we arrive at the estimate of the global midrapidity  $\Lambda$  polarization in the direction orthogonal to the reaction plane (*xz*):

$$
\langle P_{\Lambda} \rangle_{\text{midrap.}} \simeq \frac{\langle \varpi_{zx} \rangle_{\text{cent. slab}}}{2} \left( 1 + \frac{2}{3} \frac{\langle m_T^{\Lambda} \rangle_{\text{midrap.}} - m_{\Lambda}}{m_{\Lambda}} \right). \tag{15}
$$

Results of this estimate are presented in panel (a) of Fig. 4. The corresponding 3FD simulations of  $Au + Au$  collisions were performed at fixed impact parameter  $b = 8$  fm. This value of *b* was chosen in order to roughly comply with the



FIG. 4. (a) Global, i.e., in the central-slab region, and (b) total, i.e. averaged over the whole participant region, polarization of  $\Lambda$ hyperons in Au + Au collisions at  $b = 8$  fm as a function of collision energy  $\sqrt{s_{NN}}$ . The blue bands indicate polarization uncertainty due to a change in the freeze-out criterion from  $\varepsilon_{\text{frz}} = 0.3$  to 0.5 GeV/fm<sup>3</sup> for the crossover EoS. STAR data on global  $\Lambda$  and also  $\bar{\Lambda}$  polarization in the midrapidity region (pseudorapidity cut  $|\eta| < 1$ ) [\[4\]](#page-6-0) are also displayed.

centrality selection 20–50% in the STAR experiment [\[4\]](#page-6-0). The correspondence between experimental centrality and the mean impact parameter was taken from Glauber simulations of Ref. [\[39\]](#page-6-0).

As seen from Fig. 4, such a rough estimate of the global midrapidity polarization quite satisfactorily reproduces the experimental data, especially the collision-energy dependence of the polarization. This energy dependence is related to the decrease of the thermal vorticity in the central region (see Fig. [2\)](#page-3-0) with the collision energy rise. The latter is a consequence of pushing out the vorticity field into the fragmentation region, which was discussed in Ref. [\[36\]](#page-6-0) in detail. This effect of pushing out was found already in Ref. [\[37\]](#page-6-0). The difference between results of the first-order-phase-transition and crossover EoS's is negligible. Apparently this is related to the fact that these two EoS's equally well reproduce the bulk of the available experimental data in this energy range. The performed estimate predicts that the global midrapidity polarization further increases at NICA/FAIR energies, reaching values of 5% at  $\sqrt{s_{NN}}$  = 3.8 GeV. This prediction approximately agrees with that made in Ref. [\[18\]](#page-6-0) based on the axial vortical effect  $[15-17]$ .

The global midrapidity polarization of  $\bar{\Lambda}$  hyperons differs only by replacement of  $\langle m_T^{\hat{A}} \rangle_{\text{midrap.}}$  with  $\langle m_T^{\hat{A}} \rangle_{\text{midrap.}}$  in Eq. (15) from that for  $\Lambda$ 's, and quantitatively does not exceed 5% of that for  $\Lambda$  hyperons. Therefore, we do not display it in Fig. [4.](#page-4-0)

As mentioned above, the freeze-out applied in this calculation differs from that used in previous studies of the bulk and flow observables. We studied sensitivity of the polarization to a change of the freeze-out criterion that indirectly simulates the effect of different freeze-out procedures. The results of the change of the freeze-out energy density from  $\varepsilon_{\text{frz}} = 0.3$ to  $0.5 \text{ GeV}/\text{fm}^3$  for the calculations with the crossover EoS are presented in Fig. [4.](#page-4-0) The lower  $\varepsilon_{\text{frz}}$  corresponds to the lower border of the displayed band. As seen from Fig. [4,](#page-4-0) the resulting variation of the central-slab polarization gradually changes from 30% at the energy of 4.3 GeV to 5% at 39 GeV. Results for the first-order-transition EoS are similar.

To further estimate uncertainties of the present estimation we performed calculations in a considerably smaller central box:  $|x| < 2$  fm,  $|y| < 2$  fm, and  $|z| < 2$  fm/ $\gamma_{cm}$ , i.e., with the box used in Ref. [\[40\]](#page-6-0) to estimate densities achieved in the center of colliding nuclei. The difference from results in the central slab used in the above case depends on the collision energy but generally does not exceed 20%. Another source of uncertainty is feed-down contribution due to decays of higher-mass hyperons, which are not included in the present estimate. According to Refs. [\[5,7,14\]](#page-6-0), including  $\Lambda$ 's from resonance decays reduces the  $\Lambda$  polarization by 15% to 20%. However, the resonance decays increase the  $\Lambda$  polarization by approximately 20% according to Ref. [\[9\]](#page-6-0).

In the case of total  $\Lambda$  polarization, the integration in Eq. [\(13\)](#page-4-0) runs over the whole participant range confined by the condition  $T > T_0$  with  $T_0 = 100$  MeV. In such averaging the above applied decoupling of averaging of  $\varpi_{zx}$  and the term in parentheses in Eq. [\(13\)](#page-4-0) is even less justified than in the central slab. Therefore, we do an even more rough estimate of the mean total polarization of emitted  $\Lambda$  hyperons,

$$
\langle P_{\Lambda} \rangle_{\text{total}} \approx \frac{\langle \varpi_{zx} \rangle_{T>T_0}}{2},\tag{16}
$$

by neglecting the term in parentheses in Eq. [\(13\)](#page-4-0). Note that this term is a correction, though not a negligible one. Sometimes it results in 30% correction for the central-slab polarization. Thus, this is another point to add to the abovediscussed list of uncertainties.

Results of this estimate of the total  $\Lambda$  polarization are presented in panel (b) of Fig. [4.](#page-4-0) The total  $\Lambda$  polarization increases with collision energy rise. This is in contrast to the energy dependence of the midrapidity polarization. This increase is quite moderate as compared with the rapid rise of the angular momentum accumulated in the participant region; see Fig. [1.](#page-1-0)

A peculiar feature is seen in Fig.  $4(b)$ . The lower and upper borders of the band, corresponding to lower and higher freeze-out energy densities,  $\varepsilon_{\text{frz}} = 0.3$  and 0.5 GeV/fm<sup>3</sup>, respectively, at low collision energies  $\sqrt{s_{NN}} \le 11.5 \text{ GeV}$ , cross and then change their places at high collision energies. Thus, the total polarization rises with decrease of the freeze-out energy density at high collision energies. This can be expected from the evolution of the thermal vorticity displayed in Fig. [3.](#page-3-0) This observation indirectly indicates that the  $\Lambda$  polarization in the fragmentation regions reaches high values at high collision

energies. Indeed, the fragmentation regions become dominant at later time instants because of their longer evolution (as compared to the central region) due to relativistic time dilation caused by their high-speed motion with respect to the central region. Therefore, at late freeze-out, i.e., at lower  $\varepsilon_{\text{frz}}$ , we see a larger relative contribution from the fragmentation regions in the total polarization than that at the early freeze-out. The increase of the total polarization with simultaneous decrease of the midrapidity one additionally confirms the conjecture on high values of the  $\Lambda$  polarization reached in the fragmentation regions at high collision energies.

In view of high degree of the polarization and therefore large values of  $\varpi_{zx}$ , the expansion of the exponential function in terms of  $\varpi$  is definitely inapplicable (see Eqs. (34) and (35) in Ref. [\[12\]](#page-6-0)). Note that this expansion was used in deriving the formula for the polarization in Ref.  $[12]$ . This is another source of uncertainty of the present estimate of the total polarization at high energies.

At lower collision energies, values of the total and midrapidity polarization are very close to each other, which reflects a more homogeneous distribution of the vortical field over the bulk of the produced matter. This spread into the bulk is an effect of dissipation (or the shear viscosity in terms of the conventional hydrodynamics). In the 3FD dynamics it is a result of the 3FD dissipation which increases with collision-energy decrease [\[41\]](#page-6-0).

#### **V. SUMMARY**

We estimated the global polarization of  $\Lambda$  and  $\bar{\Lambda}$  hyperons in  $Au + Au$  collisions in the midrapidity region and the total polarization, i.e., averaged over all rapidities. This estimate was based on the thermodynamical approach  $[12-14]$ . The relevant vorticity was simulated within the 3FD model [\[19\]](#page-6-0). A collision-energy scan in the energy range of FAIR, NICA, and BES-RHIC was performed. The midrapidity results were compared with STAR data [\[4\]](#page-6-0).

It is found that without any adjustment of the model parameters the performed rough estimate of the global midrapidity polarization quite satisfactorily reproduces the experimental STAR data on the  $\Lambda$  polarization, especially the collisionenergy dependence of the polarization. This energy dependence is a consequence of the decrease of the thermal vorticity in the central region with the collision energy rise, which in its turn results from pushing out the vorticity field into the fragmentation regions  $[36,37]$ . The difference between results of the first-order-phase-transition and crossover EoS's is negligible. Apparently this is related to the fact that these two EoS's equally well reproduce the bulk of the available experimental data in this energy range. The performed estimate predicts that the global midrapidity polarization further increases at NICA/FAIR energies, reaching values of 5% at  $\sqrt{s_{NN}}$  = 3.8 GeV. This prediction approximately agrees with that made in Ref. [\[18\]](#page-6-0) based on the axial vortical effect  $[15–17]$ .

The global midrapidity polarizations of  $\bar{\Lambda}$ 's and  $\Lambda$ 's practically do not differ from each other within the present estimate. This is also true for all other hydrodynamic [\[5,6\]](#page-6-0) and kinetic [\[7–11\]](#page-6-0) calculations based on the thermodynamical approach.

<span id="page-6-0"></span>It is not quite clear whether this contradicts the STAR data at the energy of 7.7 GeV because of large error bars of the measured  $\bar{\Lambda}$  polarization. However, there are approaches which naturally explain this difference. One of them is directly based on the axial vortical effect  $[15-17]$ . Application of this approach within the quark-gluon string transport model [18] well reproduces both the  $\bar{\Lambda}$  and  $\Lambda$  polarizations and splitting between them. Another recently suggested approach [42] based on a Walecka-like model can also explain the difference in the  $\bar{A}$ - $\Lambda$  polarizations. However, the ability of this Walecka-like approach to describe absolute values of these polarizations still remains to be seen.

According to our estimate, the total  $\Lambda$  polarization increases with collision energy rise, which is in contrast to the energy dependence of the midrapidity polarization. This increase is quite moderate compared to the rapid rise of the angular momentum accumulated in the participant region. The

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increase of the total polarization with simultaneous decrease of the midrapidity one suggests that at high collision energies the fragmentation-region polarization reaches high values.

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