Mirror states in ^{9}Li and ^{9}C

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I have used a potential model to examine mirror states in ^{9}Li and ^{9}C. The mirror energy difference and observed widths of the supposed $7/2^-$ states are consistent with dominant parentage to the 3^+ excited state of ^{8}Li and ^{8}B. The newly reported s-wave resonance at 4.3 MeV in ^{9}C should have a mirror near $E_x = 5.39(38)$ MeV in ^{9}Li.

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I. INTRODUCTION

In ^{9}Li, all the known states [1] can be understood in a model that includes only $1p$-shell nucleons. These states include the $3/2^-$ ground state (g.s.) and four excited states with (in order of increasing excitation energy) $J^\pi = 1/2^-$, $5/2^-$, $3/2^-$, and $7/2^-$. The properties of these states are listed in Table I. The g.s. magnetic moment [2] disagrees slightly with expectations for a pure $(p_{3/2})$ configuration, but is in good agreement with a full p-shell calculation, implying some $p_{1/2}$ occupancy. The g.s. magnetic moment also agrees with quantum Monte Carlo calculations if two-body meson-exchange currents are included. These are found to provide 20% of this moment in ^{9}Li and 40% in the mirror ^{9}C [3].

A great deal of other theoretical work exists for ^{9}Li and ^{9}C [4–18]. Furumoto *et al.* [11] investigated the g.s. and excited $3/2^-$ state of ^{9}Li in a microscopic structure model. They concluded that the g.s. valence neutrons in ^{9}Li are the same as in ^{10}Be g.s. In a stochastic multiconfiguration mixing method, they obtained the usual five states of ^{9}Li, plus six more negative-parity states with $J = 1/2$ to $7/2$ above about 8 MeV.

Other theoretical approaches include a *ttt* cluster model [12] and a ^{6}He + t cluster model [13]. The latter found that their states began above about 8 MeV and constituted a $K = 1/2^-$ band.

Timofeyuk [14] computed spectroscopic factors for some states of ^{9}Li and the g.s. of ^{9}C in the source term approach (STA) and compared them with S's obtained from direct overlap of shell-model wave functions. Ratios of the latter to the former varied from about 1.3 to 2.4 for various states. Later, this author applied the STA to calculations of widths of neutron and proton resonances for some p-shell nuclei [15], and found that the STA predictions are often smaller than those obtained with the widely used standard practice.

Nollett [16] presented calculated widths for many nuclear states, including ^{9}Li and ^{9}C, using an integral over the interaction region of *ab initio* variational Monte Carlo wave functions. He concluded that failures of the method generally involve broad states and variational wave functions that are not strongly peaked in the interaction region, and that overlap calculations can diagnose cases in which computed widths should not be trusted.

Maris and Vary [17] reviewed g.s. energies and magnetic moments of p-shell nuclei obtained with the *ab initio* no-core shell model approach. They also reviewed excitation energies for some narrow resonances in $A = 6$ to 9 nuclei.

Myo *et al.* [18] studied the Li isotopes systematically in terms of the tensor-optimized shell model (TOSM) by using a bare nucleon-nucleon interaction. Their results for ^{9}Li are summarized in Table II.

In the ^{7}Li(t, p) reaction [19–21], the g.s. is strongly populated with an $L = 0$ angular distribution, and the $5/2^-$ state is strong, with $L = 2$, both as expected from the shell model. The charge exchange reaction ^{9}Be(t, ^{3}He)^{9}Li [22] populates a spin-dipole state at about 6.5 MeV. No hint of positive-parity states has been reported. An interesting unanswered question is the location of the state with dominant structure $^7\mathrm{Li} \times (\mathrm{sd})^2_{0+}$. If its mixing with the g.s. is small, it should be quite strong in the ^{7}Li(t, p) reaction. I return to this point later, below.

Wuosmaa *et al.* [23] investigated ^{9}Li with the reaction ^{8}Li(d, p) (in reverse kinematics). They used distorted-wave analysis to extract spectroscopic factors for the first three states. Results were generally in agreement with a variety of theoretical structure calculations [5–10]. These are discussed further below. For the higher states, they used the widths to estimate the spectroscopic factors as $S = \Gamma_{\mathrm{exp}}/\Gamma_{\mathrm{sp}}$. Their results are also listed in Table I.

The mirror of ^{9}Li is ^{9}C, and mirrors of all these ^{9}Li states have been tentatively identified in ^{9}C, as listed in Table III [1,24–26]. Early work on ^{9}C used the ^{12}C(^{3}He, ^{6}He) reaction [24]. More recently, Rogachev *et al.* [25] measured an excitation function for ^{8}B + p elastic scattering and reported a $5/2^-$ state at $E_x = 3.6(2)$ MeV, with a width of 1.4(5) MeV. Even more recently, Brown *et al.* [26] extracted widths for the first four unbound states of ^{9}C (the g.s. is bound by 1.3 MeV). All these results for ^{9}C are listed in Table III.

A new ^{8}B + p [27] experiment has reported the first evidence for a positive-parity state in ^{9}C. They observed an s-wave resonance at $E_x = 4.3(3)$ MeV, with a width of $4.0^{+2.0}_{-1.4}$ MeV. They suggested a J^π of $5/2^+$.

TABLE I. Energies (MeV), widths (keV) and spectroscopic factors in ^{9}Li.

Compilation [1]			Wuosmaa *et al.* [23]				Present	
J^π	E_x	Γ	J^π	E_x	S from width	$S(d,p)$	Γ_{sp}	$S = \Gamma/\Gamma_{sp}$
$3/2^-$	0	bound	$3/2^-$	0		0.90(13)		
$1/2^-$	2.691(5)	bound	$(1/2^-)$	2.691		0.73(15)		
$(5/2^-)$	4.296(15)	100(30)	$(5/2^-)$	4.31	0.55(30)	0.93(20)	125	0.80(24)
	5.38(6)	600(100)	$3/2^-$	5.38	0.29(6)		1620 to 2^+	>0.23(6)
							220 to 1^+	
	6.43(15)	40(20)	$7/2^-$	6.43	0.0085(40)		2180 to 2^+	<0.018(9)
							37 to 3^+	

II. RESULTS AND DISCUSSION

The aim of the present work is to examine the energies and widths of the states in these two mirror nuclei with a simple potential model. The potential is of Woods-Saxon shape, with geometrical parameters r_0, a, $r_{0c} = 1.26$, 0.60, 1.40 fm. Figure 1 depicts the first three core states in ^{8}Li and ^{8}B, and the relevant energies in the $A = 9$ nuclei. In Fig. 2 is plotted the energy differences E_p–E_n for the mirror pairs. These energies are those relative to ^{8}Li(g.s.) + n and ^{8}B(g.s.) + p. The fact that the point for the $7/2^-$ state lies above the general trend for the other states is consistent with the fact that its parentage is primarily to the excited 3^+ state at 2.255 and 2.32 MeV, respectively, in ^{8}Li and ^{8}B.

For the unbound states of ^{9}Li, I varied the potential well depth to reproduce the separation energy of each state and computed the single-particle (sp) width from the phase shifts. I do not predict excitation energies in ^{9}Li. I use the potential model to obtain the relationship between energy and width. I then use this relationship to obtain widths at experimental energies. This approach is superior to the use of penetrabilities and reduced widths.

Above the (centrifugal) barrier, I used the asymptotic form of the widths: $\Gamma_{sp} \approx (2E)^{1/2}$. Whenever a given state is energetically allowed to decay to an excited state, I also computed that sp width. These sp widths are listed in the penultimate column of Table I. The last column contains the ratios Γ_{exp}/Γ_{sp}, which should be equal to the spectroscopic factors.

My result is in excellent agreement with the (d, p) experiment for the $5/2^-$ state. The next two states can also decay to the excited 1^+ state of ^{8}Li. The total width of the $3/2^-$ state is 600(100) keV, and the sp width for decay to the 1^+ is 220 keV, so even if S is near unity for this decay, most of the width is due to decay to the g.s.—providing an S for that decay of $S > 0.23(6)$. For the $7/2^-$ state, a variety of structure calculations indicate that its parentage is predominantly to the 3^+ core state, for which the sp width is 37 keV. The experimental width of 40(20) keV is obviously in agreement. Given the sp width for g.s. decay, the g.s. spectroscopic factor is $S < 0.018(9)$.

TABLE II. Dominant configurations in ^{9}Li from a tensor optimized shell model [18].

State	Configuration	Intensity
g.s.	$(p_{3/2})^5$	0.46
	$(p_{3/2})^3{}_{3/2\,1/2}(p_{1/2})^2{}_{01}$	0.19
$1/2^-$	$(p_{3/2})^4{}_{02}(p_{1/2})$	0.67
$5/2^-$	$(p_{3/2})^4{}_{21}(p_{1/2})$	0.57
$3/2_2^-$	$(p_{3/2})^3{}_{3/2\,3/2}(p_{1/2})^2{}_{01}$	0.38
	$(p_{3/2})^4{}_{11}(p_{1/2})$	0.27
$7/2^-$	$(p_{3/2})^4{}_{31}(p_{1/2})$	0.80

I turn now to ^{9}C. Single-particle widths for decays to the g.s. and (whenever appropriate) to excited states are listed in Table III. Brown *et al.* [26] observed decays of the excited $(3/2^-)$ to the 1^+ excited state of ^{8}B, and the $(7/2^-)$ state to ^{8}B(3^+). Because the sp width for $3/2^- \rightarrow 1^+$ decay is about 1.85 MeV, the total width of 2.75(11) MeV provides a limit on the g.s. spectroscopic factor of $S > 0.35(4)$. If the spectroscopic factors in mirror nuclei are equal, the ratios Γ_{exp}/Γ_{sp} should be the same in ^{9}Li and ^{9}C. This means that the ratios $\Gamma_{exp}(^9\text{Li})/\Gamma_{exp}(^9\text{C})$ should be equal to $\Gamma_{sp}(^9\text{Li})/\Gamma_{sp}(^9\text{C})$, independently of the relevant spectroscopic factors. These ratios are plotted in Fig. 3. The fact that all the experimental ratios are larger than the sp ones might indicate that the ^{9}Li widths are systematically too large, or that the ones in ^{9}C are systematically too low. A preference for the latter might be suggested by the data for the $5/2^-$ state of ^{9}C, for which Rogachev *et al.* reported a width of 1.4(5) MeV and Brown *et al.* a width of 0.673(50) MeV. The discrepancy here is only a 1.4σ effect. In comparison with the mirror state in ^{9}Li, the spectroscopic factor strongly prefers the larger width. Brown's width agrees with the compilation for the broad supposed second $3/2^-$ state, and even for the $1/2^-$ state, the difference is only 2.1σ. Still, the comparison with ^{9}Li makes Brown's widths seem too small.

As noted by Wuosmaa *et al.*, a variety of nuclear structure calculations [5–10] predict the g.s. spectroscopic factor of ^{9}Li to be in the range 0.90–1.11, to be compared with the experimental value of 0.90(13). However, Li *et al.* [28] found this S to be 0.68(14). Results for the $1/2^-$ state are more varied, with theoretical values ranging from 0.20 to 0.52 and an experimental result of 0.73(15). For the $5/2^-$ state, the variation is less—from 0.75 to 0.84, with an experimental value of 0.93(20). Millener's new calculations [29] with the Par4 interaction provide $S = 0.93$, 0.38, and 0.79, respectively, for these three states. These new results for all five states and the first three states of ^{8}Li are listed in Table IV.

TABLE III. Energies and widths (both in MeV) in ^{9}C.

J^π	Compilation [1]		Brown *et al.* [26]		Present	
	E_x	Γ	E_x	Γ	Γ_{sp}	Γ/Γ_{sp}
$3/2^-$	0		bound	–		
$1/2^-$	2.218(11)	0.10(2)	2.218(11)	0.052(11)	0.064	0.81(17)
$(5/2^-)$[a]	3.30(05)	?	3.549(20)	0.673(50)	1.8	0.37(3)
$(3/2^-)$	≈4.3	≈2.6	4.40(4)	2.75(11)	2.56 to 2^+	>0.35(4)
					1.85 to 1^+	
$(7/2^-)$			5.75(4)	0.601(50)	2.13 to 2^+	<0.28(3)
					1.51 to 3^+	<0.40(3)

[a]Rogachev *et al.* [25] reported a $5/2^-$ state at $E_x = 3.6(2)$ MeV, with a width of 1.4(5) MeV.

For the second $3/2^-$ state, $S_{\rm th}$ varies from 0.08 to 0.42. The spectroscopic factor of this state was not measured in the ^{8}Li(d,p) experiment, but those authors computed S from Γ/Γ_{sp} to obtain 0.29(6). Note however, that my analysis gives a limit of >0.23(6) in ^{9}Li and >0.35(4) in ^{9}C. In ^{9}Li, the total width is 600(100) keV and the sp width for decay to the 1^+ excited state of ^{8}Li is 220 keV (see Table I), so that $S < 1$ for 1^+ decay gives the aforementioned limit, using the sp width of 1.62 MeV for g.s. decay. Similar remarks hold for ^{9}C. Millener's new S for 1^+ decay is 0.68, so this limit becomes S(g.s.) $> 0.25(6)$ in ^{9}Li and >0.58(4) in ^{9}C, which is larger than any of the theoretical estimates.

All theoretical calculations predict a tiny S(g.s.) for the $7/2^-$ state—ranging from 0.0001 to 0.009. Millener's new value is 0.76×10^{-4}. If all the total width of 0.601(50) MeV in ^{9}C is for decay to the 3^+ of ^{8}B, then S for decay to 3^+ is 0.40(3), to be compared to Millener's new result of $S(3^+) = 0.69$.

As noted above, Hooker *et al.* [27] observed an s-wave resonance at $E_x = 4.3(3)$ MeV in ^{9}C, with a width of $4.0^{+2.0}_{-1.4}$ MeV. I have used the potential model to compute the sp width for a $2s_{1/2}$ resonance at the experimental energy. The result is $\Gamma_{sp} = 2.45$ MeV, indicating that the observed resonance is close to single particle. Estimating the energy of the mirror state in ^{9}Li is tricky because it would be unbound, and an s-wave neutron resonance does not appear in a potential model. However, I have computed the mirror energy difference for several bound s states in ^{9}Li and extrapolated into the unbound region. The resulting expectation for the first positive-parity state in ^{9}Li is a neutron energy of 1.33(38) MeV, i.e., $E_x = 5.39(38)$ MeV. The sp width should be about 1.63(22) MeV. It would be interesting to examine the previous ^{8}Li(d,p) data for such a resonance.

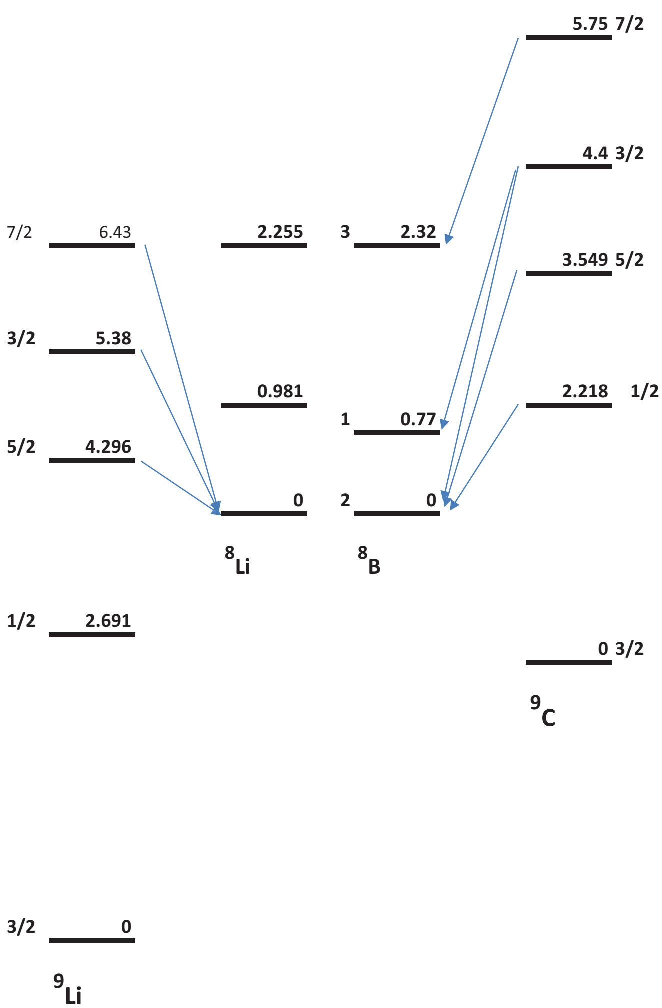

FIG. 1. Energies of first three states in ^{8}Li and ^{8}B [1] together with first five states of ^{9}Li and ^{9}C.

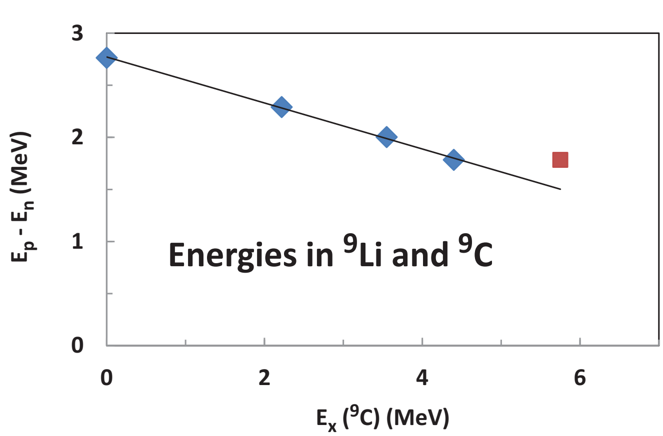

FIG. 2. Plot of E_p in ^{9}C minus E_n in ^{9}Li (both relative to ^{8}B and ^{8}Li ground states, respectively, vs excitation energy in ^{9}C.

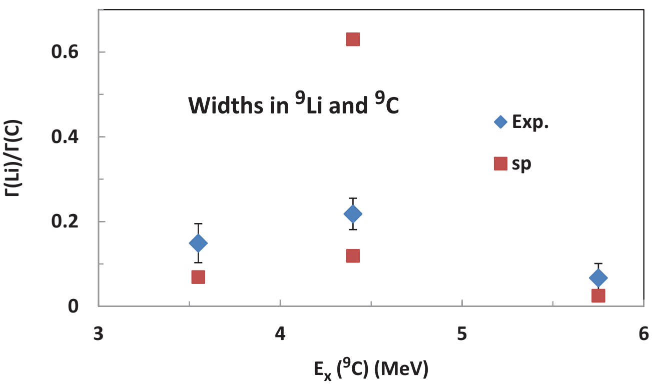

FIG. 3. Widths in ^{9}Li divided by widths in ^{9}C: Diamonds are experimental, squares are single-particle. For the upper state, the square is for 3^+ parentage; For the middle state, lower square is for 1^+ parentage, upper is for g.s. parentage.

Now that I have an estimate of the first s state in ^{9}Li, I can estimate the expected energy of the first $(sd)^2$ state. For $(sd)^2$ zero-coupled pairs, a Hamiltonian [30] that has been successful in other p-shell nuclei [31,32] has $\langle s^2, Vs^2\rangle = -1.54$, $\langle d^2, Vs^2\rangle = -1.72$, and $\langle d^2, Vd^2\rangle = -2.78$, all in MeV. If the first s state is indeed $5/2^+$, with an estimated energy of $E_n = 1.33(38)$ MeV, then the energy of the $5/2^+ - 3/2^+$ centroid should be about $E_n = 1.59$ MeV. I take this to be the $2s_{1/2}$ single-particle energy in ^{8}Li. The Hamiltonian above then produces the first $(sd)^2$ state at about $E_{2n} = 1.08$ MeV, i.e., $E_x \approx 7.2$ MeV. This state will have the structure ^{7}Li(g.s.) $\times$ $(sd)^2_0$, with the majority configuration of the last two neutrons being s^2. This state should be quite strong in the reaction ^{7}Li(t, p). I think it would be very worthwhile to take another look at that reaction. Of the three previous investigations of this reaction, the one at highest energy [21] displayed a spectrum to an excitation energy of about 12 MeV, which showed no narrow peaks above the 6.43-MeV state. Of course, any low-J states at such high excitation might be expected to be quite broad. Parts of the spectrum were obscured by impurity peaks from ^{12}C in the target. In fact, their spectrum has a hint of a broad peak near 7.7 MeV that deserves further attention.

TABLE IV. Spectroscopic factors for levels of ^{9}Li to first three states of ^{8}Li.[a]

J^π_n	2^+	1^-	3^-
$3/2^-_1$	0.9035, 0.0258	0.4520, 0.0535	1.344, - - -
$1/2^-$	0.3781, - - -	0.5075, 0.0026	
$5/2^-$	0.1018, 0.6869	0.00043, - - -	0.2347, 0.2079
$3/2^-_2$	0.0674, 0.0174	0.0665, 0.6160	0.0132, - - -
$7/2^-$	0.76×10^{-4}, - - -		0.0455, 0.6458

[a]Reference [29]. For each core state, first S is for $1p_{3/2}$, second is $1p_{1/2}$.

III. SUMMARY

I have examined the known states of ^{9}Li and ^{9}C in a potential model. For the $7/2^-$ states, the mirror energy difference and decay widths are consistent with a predominant parentage to the 3^+ core states in ^{8}Li and ^{8}B. Using computed sp widths from the potential model, observed widths in ^{9}Li are consistent with spectroscopic factors from the ^{8}Li(d, p) reaction. Widths in ^{9}C reported by Brown *et al.* may be systematically too low. I estimate the first s state in ^{9}Li to have an excitation energy of about 5.4 MeV, and the first $(sd)^2$ state at about 7.2 MeV.

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