Backbending, seniority, and Pauli blocking of pairing correlations at high rotational frequencies in rapidly rotating nuclei

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Garrett *et al.* systematically investigated band-crossing frequencies resulting from the rotational alignment of the first pair of $i_{13/2}$ neutrons (AB) in rare-earth nuclei. In that study, evidence was found for an odd-even neutron number dependence attributed to changes in the strength of neutron pairing correlations. The present paper carries out a similar investigation at higher rotational frequencies for the second pair of aligning $i_{13/2}$ neutrons (BC). Again, a systematic difference in band-crossing frequencies is observed between odd-N and even-N Er, Yb, Hf, and W nuclei, but in the BC case, it is opposite to the AB neutron-number dependence. These results are discussed in terms of a reduction of neutron pairing correlations at high rotational frequencies and of the effects of Pauli blocking on the pairing field by higher-seniority configurations. Also playing a significant role are the changes in deformation with proton and neutron numbers, the changes in location of single-particle orbitals as a function of quadrupole deformation, and the position of the Fermi surface with regard to the various Ω components of the neutron $i_{13/2}$ shell.

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I. INTRODUCTION

In the realm of high-spin nuclear physics, the deformed rare-earth region has been a focus for experimental studies since nuclei occupying it can accommodate the highest values of angular momentum, see, for example, the following textbooks on nuclear-structure physics [1–9]. Increasing the angular momentum of the nucleus enables its superfluid properties [10] to be investigated. In fact, in 1986 Garrett, Hagemann, and Herskind [11] stated that "*the transition from the correlated to uncorrelated phase has been the 'Holy Grail' of high-spin spectroscopy*" and that "*armed with the improved gamma-ray detection systems it seemed proper to renew this Arthurian quest*." For a summary of this particular topic and other aspects related to pairing correlations in nuclei, including backbending and quasiparticle alignments, the reader is referred to Ref. [12] "*Fifty Years of Nuclear BCS: Pairing in Finite Systems*". The present paper details new information related to the reduction of superfluid correlations at high rotational frequency and seniority in a range of rare-earth nuclei.

As an introduction to the evolution of collectivity as observed in the rare-earth region, the ratio of the excitation energies of the lowest 4^+ and 2^+ levels over a wide range of

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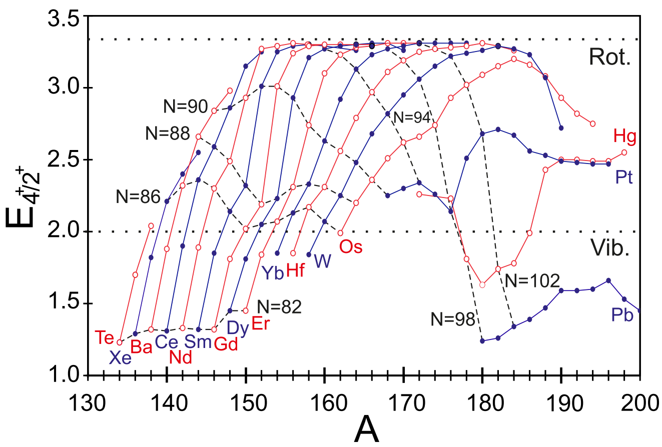

FIG. 1. Ratio of the excitation energies of the lowest 4^+ to 2^+ level in even-even nuclei as a function of mass A, which covers the evolution of collectivity in the rare-earth region. This paper discusses trends at high spin in Er, Yb, Hf, and W nuclei with $88 \leqslant N \leqslant 103$.

even-even nuclei is plotted in Fig. 1 as a function of mass. This well-known "*Casten*" plot [8] illustrates the changing landscape of nuclear deformation with respect to neutron and proton numbers. As can be seen in Fig. 1, the $N = 88$–102 Er, Yb, Hf, and W nuclei, which are the subject of the present paper, evolve from weakly deformed to well-deformed behavior.

At low spins, these nuclei display well-established superfluid properties with nucleons pairing up in time-reversed orbits forming nucleonic "*Cooper pairs*." This superfluid phase is supported by the observation that the moment of inertia associated with a rotational band is about half the rigid-body value. With increasing angular momentum, these nuclei undergo a range of phenomena. Of particular significance is that the Coriolis force generated by the collective rotation of the nucleus acts to break apart the paired fermions in order to align their spins with the rotational axis: This is referred to as the Coriolis antipairing (CAP) effect [13]. In fact, it was initially hypothesized that, at a critical angular momentum, a transition out of the superfluid paired phase would occur, in a manner analogous to the quenching of superconductivity by a sufficiently high magnetic field. The signature of this phenomenon was predicted to be an abrupt change in the moment of inertia which would then approach the rigid-body value.

Such a signature was first observed in Stockholm in 1971 by Johnson, Ryde, and Sztarkier [14] while investigating the high angular momentum properties of rare-earth nuclei. Due to the abrupt change in the moment of inertia, this observation was referred to as the "*backbending*" phenomenon. However, the change in the moment of inertia did not approach the rigid-body value as predicted by CAP and, thus, pointed to the need for another explanation. Stephens and Simon [15] noted that the strength of the Coriolis force acting on each nucleonic pair was dependent on the single-particle angular momentum of the paired particles and suggested that the observed *backbending* phenomenon resulted from the decoupling of a single pair of high-j particles and their subsequent alignment along the rotational axis. This is now the accepted interpretation of the backbending phenomenon. These concepts were refined and incorporated into highly successful cranking models [16–19]. Hundreds of examples of backbending have now been reported throughout the chart of the nuclides. What was once a surprise and a mystery has become a powerful spectroscopic tool revealing a wealth of nuclear-structure information. A short pedagogical video of the backbending phenomenon may be found in Refs. [20,21].

II. DATA ANALYSIS AND RESULTS

This article builds upon the classic work of Garrett *et al.* [22] who systematically investigated band-crossing frequencies $\hbar\omega_c$, associated with the alignment of the first pair of $i_{13/2}$ neutrons; (AB) in the nomenclature of Riedinger *et al.* [23]. The band-crossing frequency $\hbar\omega_c$ corresponds to the point where the Routhians (energy in the rotating frame) of two different quasiparticle configurations cross one another, changing the energetically preferred configuration [18]. In particular, the AB and BC crossing frequencies represent the frequencies at which the initial configuration X is crossed by the configuration XAB or XBC, respectively. In Ref. [23], a listing of the quasiparticle assignments to specific neutron orbitals close to the Fermi surface for rare-earth nuclei is given. Garrett *et al.* [22] found evidence was for an odd-even neutron-number dependence, $\hbar\omega_c$ (odd N) < $\hbar\omega_c$ (even N), which was attributed to changes in the strength of neutron pairing correlations. The present paper carries out a similar comprehensive investigation for the second pair of aligning $i_{13/2}$ neutrons (BC) occurring at higher rotational frequencies. Again, a systematic difference in band-crossing frequencies is observed among odd-N and even-N Er, Yb, Hf, and W nuclei with $88 \leqslant N \leqslant 103$, but it is opposite to the AB dependence $\hbar\omega_c$ (even N) < $\hbar\omega_c$ (odd N). These results are discussed in terms of the reduction of neutron pairing correlations at higher rotational frequencies, the position of the Fermi surface, and the effects of Pauli blocking on the pairing field by higher-seniority configurations.

It should also be pointed out that the systematic trends of the BC crossing frequency for the $N = 91$ and 92 isotones for Er, Yb, Hf, and W were investigated by several of the present authors [24], see also Ref. [25] for an analysis for the $N = 90$–99 Yb isotopes. The findings discussed hereafter are consistent with these works but are based on the present more global paper. In addition, the present paper is complementary to, and consistent with, the seminal findings of Dracoulis, Kondev, and Walker [26,27], who investigated the changes in the moment of inertia of multiquasiparticle states due to a discrete reduction in pairing with seniority.

The AB and BC crossings, the subject of this paper, are observed in alignment plots as strong backbends or upbends with a significant gain in alignment [18]. Extraction of a precise value of the crossing frequency from the Routhian (or alignment) plot is often difficult, especially when strong upbends are observed. In cases where a backbend occurs, the usual method [18] of extracting the crossing frequency from the experimental Routhians was adopted and, when an upbend was present, a technique that fits the dynamical moment of inertia as a function of rotational frequency was used. The latter proved to be reliable, and when compared,

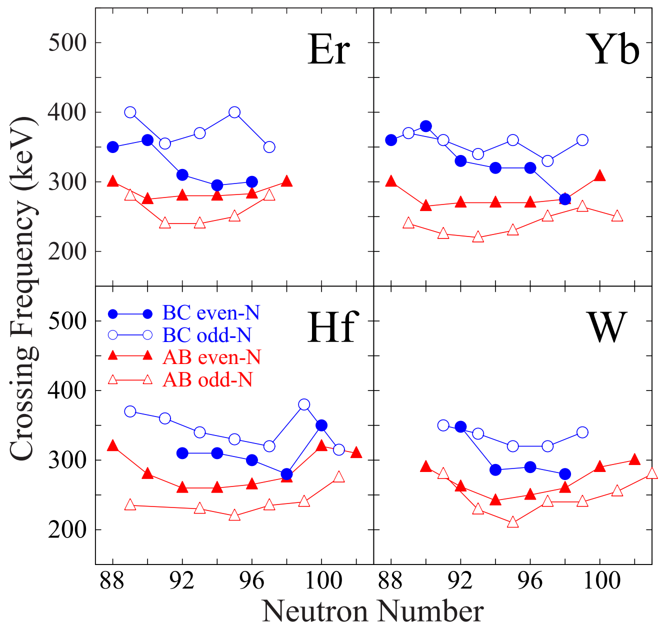

FIG. 2. Systematics of the experimental band-crossing frequencies $\hbar\omega_c$ for the first (AB) and second (BC) $i_{13/2}$ neutron alignments in Er, Yb, Hf, and W rare-earth nuclei with $88 \leqslant N \leqslant 103$ [28]. Typical error bars are $\Delta\hbar\,\omega_c \approx$ 5–10 keV.

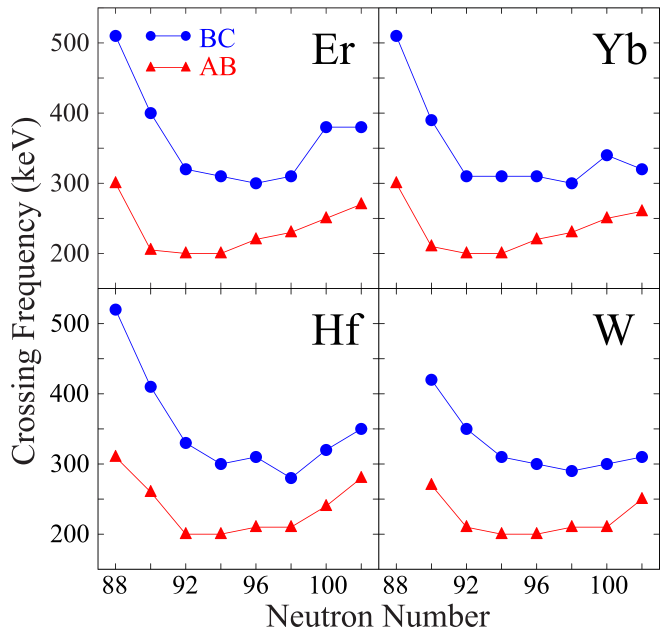

FIG. 3. Theoretical AB and BC crossing frequencies in even-even nuclei, extracted from cranking calculations using a Woods-Saxon potential with pairing. The deformation values were taken from Ref. [37] with the pairing strength corresponding to 80% of the odd-even mass difference.

these two techniques yielded consistent crossing frequencies within ≈5–10 keV. The extracted values were also found to be consistent with the published values to a similar degree of accuracy.

Figure 2 displays the AB band-crossing frequencies for the ground-state (lowest positive-parity) band and the BC crossing frequencies for the AE quasiparticle configuration (lowest negative parity) in the even-even isotopes of Er, Yb, Hf, and W nuclei. In addition, the AB crossings observed from the bands based on the *E* quasiparticle and the BC crossings from the *A* quasiparticle in the odd-even neighbors are also shown in Fig. 2. The data used for each nucleus was extracted from the Radware directory of level schemes [28] and from the original papers listed within the latter. The *E* quasineutron (mixed components from the $f_{7/2}$ and $h_{9/2}$ shells) was chosen since it is the favored sequence of the E/F signature pair, and, hence, bands built upon it are more numerous. However, the same trends were observed when analyzing the *F* and AF band-crossing systematics, thus, leading to the same conclusions.

III. DISCUSSION

Examination of Fig. 2 indicates a consistent behavior in all four elements for both the AB and the BC crossings. Over a range of neutron numbers, the AB crossing is such that $\hbar\omega_c$ (odd N) $<$ $\hbar\omega_c$ (even N), but for the BC crossing, the opposite trend is observed with $\hbar\omega_c$ (even N) $<$ $\hbar\omega_c$ (odd N). The data exhibit a variety of other interesting features, such as the near convergence of the AB and BC crossing frequencies systematically in Yb, Hf, and W nuclei at $N = 98$, and the large jump in the BC crossing frequency observed in Yb and Hf at $N = 99$. In addition, the odd-even trends appear to be quenched in the more neutron-deficient isotopes ($N \leqslant 88$) especially for the BC crossing. The latter suggests that other competing elements are at work and take priority in the neutron-deficient isotopes. These are probably associated with the less deformed and "*softer*" prolate shapes of the $N \approx 88$ isotopes, see, for example, Refs. [29,30]. Also, the BC crossing differences in Fig. 2 between even-*N* and odd-*N* Er isotopes are systematically larger than in the Yb, Hf, or W cases. This may be correlated with the fact that the octupole structures in the even-*N* Er nuclei interact strongly with the AE quasiparticle band structures and, thus, influence the AE to AEBC band-crossing frequency [25]. The latter also points out that the BC crossing frequency is more sensitive to the placement of the Fermi surface with respect to the position of the $i_{13/2}$ single-particle levels than the AB alignment. Thus, an alternative interpretation for the Er isotopes is that because of their increased deformation the placement of the Fermi surface impacts their BC crossing frequency behavior more so than the less deformed Yb, Hf, and W isotones.

In order to gain insight into the experimental observations, theoretical Woods-Saxon cranking calculations [19,31,32] have been performed. From these calculations, the AB and BC crossing frequencies for even-even nuclei were extracted, and these are plotted in Fig. 3 as a function of neutron number. Both the AB and the BC crossing frequencies appear roughly constant over the range of neutron numbers, consistent with the experimental observations of Fig. 2. However, for neutron-deficient nuclei with $N < 92$, the crossing frequencies display a sharp rise. This effect is probably associated with the lower deformation values for the nuclei in the "*transitional*" region below $N \approx 90$ as seen in Fig. 1. The fact that, with decreasing

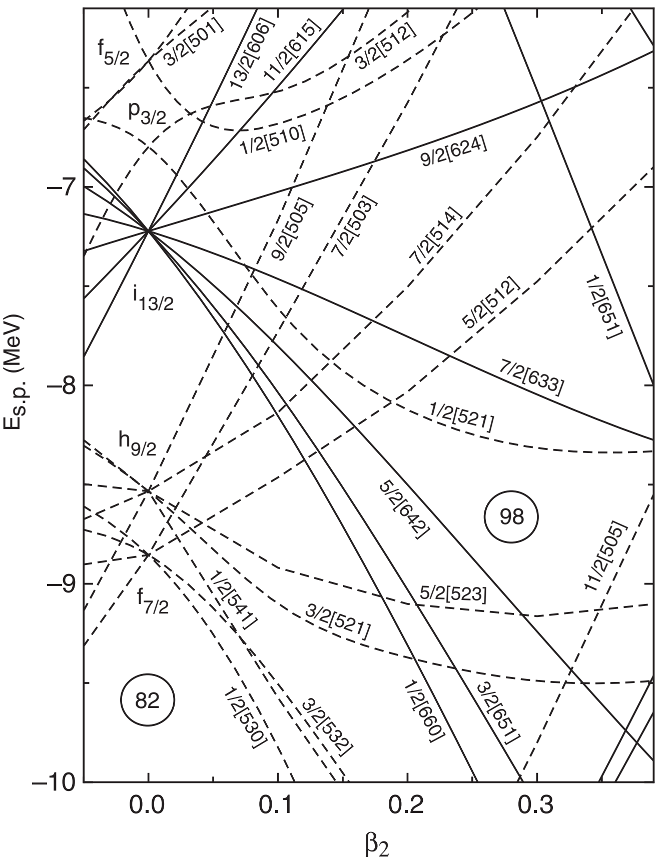

FIG. 4. Single-particle energies as a function of quadrupole deformation (β_2) from the Woods-Saxon model. Note the large prolate deformed gap for $N = 98$ and for the nuclei in the present paper, the deformation varies between $0.2 \leqslant \beta_2 \leqslant 0.3$ [37].

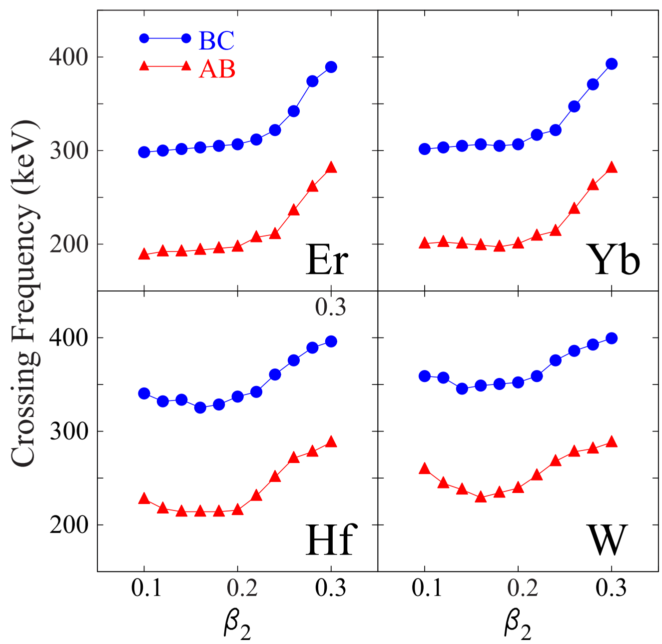

FIG. 5. Calculated AB and BC crossing frequencies plotted as a function of quadrupole deformation (β_2) for $N = 94$ Er, Yb, Hf, and W isotones. A pairing strength of 80% of the odd-even mass difference was used.

deformation, the Fermi surface for these neutron numbers moves away from the low-Ω components of the $i_{13/2}$ shell, see Fig. 4, is consistent with this interpretation. The latter trend has been observed experimentally and was discussed for the AB crossing in Ref. [33]. For these lower deformations and smaller neutron numbers, other possible alignments also begin to play a role [29,30]. At the higher neutron-numbers $N \geqslant 98$, the increase seen in the AB and BC crossing frequencies can be attributed to both the increasing deformation of these isotopes and the Fermi surface being close to a large deformed shell gap at $N = 98$ [34], see Fig. 4. Figure 5 illustrates the latter by examining the effects of increasing deformation values for specific isotopes. For higher deformation values, both the AB and the BC crossing frequencies increase as the Fermi surface moves away from the alignable low-Ω components of the $i_{13/2}$ shell. Interestingly, due to decreasing and significantly smaller values of the neutron odd-even mass differences for these heavier nuclei, it has been proposed that the static pair field may be absent and that the "*normal*" band-crossing patterns are not expected to be observed [35,36].

In agreement with the work of Garrett *et al.* [22] when discussing the AB crossing systematics, the present paper suggests an explanation involving the Pauli blocking effect of neutron pairing correlations. Here, seniority is significantly responsible for the observed differences of the BC crossing frequencies between odd-N and even-N isotopes. Ground-state bands in even-even nuclei have seniority $s = 0$, whereas the A and E bands in the odd-A nuclei have seniority $s = 1$, and the AE bands in the even-even nuclei have seniority $s = 2$. Because of the Pauli blocking in a higher-seniority configuration, the pairing is reduced and, thus, the band-crossing frequency is lower. This was first observed in the case of the AB band crossing [22] where the ground-state bands ($s = 0$) have a systematically higher crossing frequency than the E bands ($s = 1$) in the odd-A neighbors.[1] In a similar manner for the BC crossing frequency, the AE bands ($s = 2$) from the even-even nuclei exhibit lower crossing frequencies than the A bands ($s = 1$) from the odd-A isotopes. Figure 6 illustrates this effect for the BC crossing with the odd-N systems having a lower average *seniority* for the $A \rightarrow$ ABC alignment when compared with the AE $\rightarrow$ AEBC configuration change in the even-N systems.

The AB and BC crossing frequencies in Er, Yb, Hf, and W $N = 94$ isotones for various pairing strengths are illustrated in Fig. 7. The deformation values were taken from Ref. [37] with 100% of the pairing strength corresponding to the odd-even mass difference. From this figure, it is clear that decreasing the

[1]Further work [38,39] showed that the Pauli blocking effect was configuration dependent when the valence quasiparticle involved an oblate orbital, such as 11/2[505] neutrons, for example. However, in the present paper on BC crossing behavior, such oblate orbits were not included because of the lack of available systematic data.

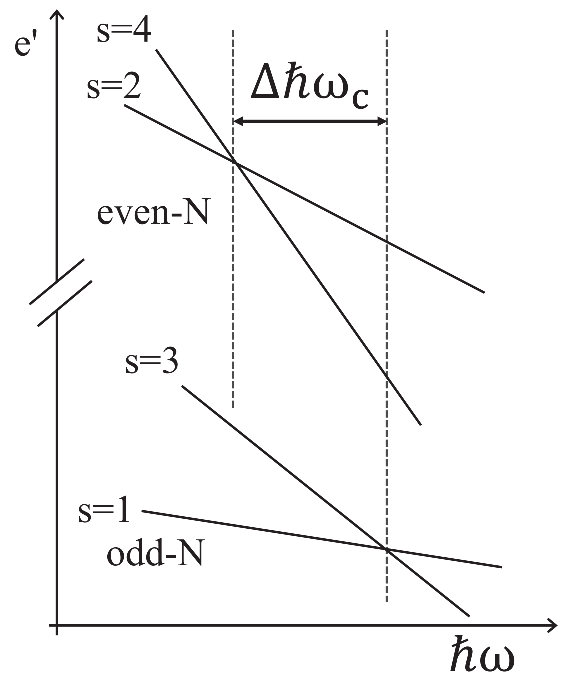

FIG. 6. Schematic illustrating the change in crossing frequency $\Delta\hbar\omega_c$ of the Routhians in arbitrary units for the BC neutron alignment in odd-N ($s = 1 \rightarrow s = 3$ or $A \rightarrow$ ABC) and even-N ($s = 2 \rightarrow s = 4$ or AE $\rightarrow$ AEBC) nuclei due to the Pauli blocking of neutron pairing correlations with increasing seniority s.

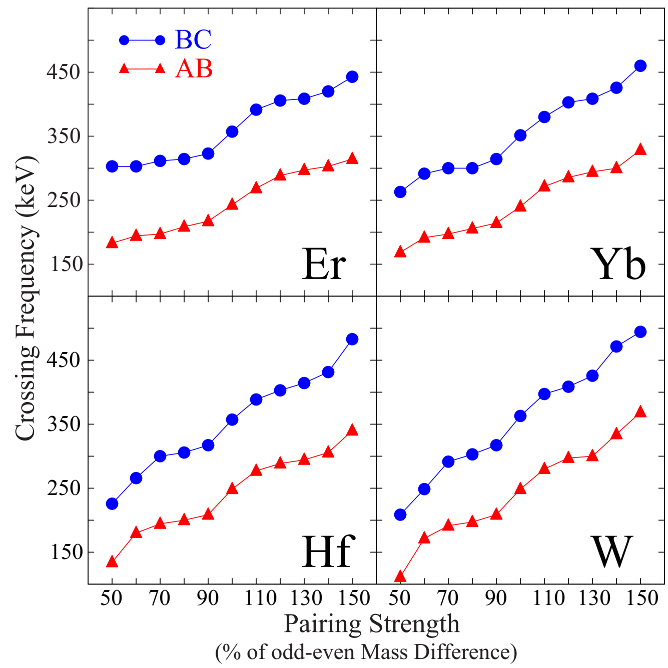

FIG. 7. Calculated AB and BC crossing frequencies plotted as a function of the percentage of the odd-even mass difference for Er, Yb, Hf, and W $N = 94$ isotones. Deformation values are taken from Ref. [37]. Note that the AB and BC trajectories are roughly parallel to one another and that the slopes do not change significantly with proton number.

pairing strength causes a corresponding decrease in both the AB and the BC band crossing frequencies. Garrett *et al.* [40] calculated that, on average, a change in the pairing correlation energy of ≈130 keV per excited quasineutron was required to account for the average 40-keV shift in the AB band crossing frequencies between odd-N and even-N isotopes, see Fig. 2.

If an average of the BC crossing frequency differences in Fig. 2 is computed for the Er, Yb, Hf, and W $N = 94$ isotones and their odd mass neighbors, a value similar to that observed for the AB alignment of $\Delta\hbar\,\omega_c \approx 50$ keV is obtained. Using the same methodology as used by Garrett *et al.* [40], this crossing frequency difference would correspond, using Fig. 7 and odd-even mass differences, to a change in the pairing energy of ≈140 keV per excited quasineutron. This observation is consistent with values calculated for the AB crossing.

The discrete reduction in pairing with seniority was investigated by Dracoulis, Kondev, and Walker [26,27], who looked at changes in the moment of inertia in rare-earth nuclei of multiquasiparticle states with increasing seniority. They calculated, using the Lipkin-Nogami prescription, a discrete reduction in pairing with a geometric dependence on seniority such that no abrupt transition from a superfluid to a normal phase is expected even for high seniority. In ^{178}W, for example, a reduction of the neutron pairing strength of ≈100 keV per unit of seniority was derived (for $s \leqslant 6$), a value consistent with the estimate made for the AB band crossing [22,40], and the findings with respect to the BC band crossing discussed here.

IV. SUMMARY

To summarize, backbending in nuclei, which was once a surprise and a mystery, is now a powerful diagnostic phenomenon. It is sensitive to changes in pairing, deformation, the single-particle spectrum of states, and how the important intruder orbitals respond to rotational frequency. In the present paper, a systematic analysis of band-crossing frequencies of both the first (AB) and the second (BC) $i_{13/2}$ neutron alignments for $A \approx 160$–170 nuclei has been performed. This extends the work of Garrett *et al.* [22] to the higher-seniority BC crossing frequencies and is consistent with the general conclusions reached by their AB crossing analysis. A systematic difference in BC band-crossing frequencies between odd-N and even-N Er, Yb, Hf, and W nuclei was found, but the N dependence is opposite to that of the AB alignment reported earlier. A consistent explanation of these results was discussed in terms of the reduction of neutron pairing correlations at high rotational frequencies and the effects of Pauli blocking on the pairing field by higher-seniority configurations. Changes in deformation with proton and neutron numbers and the location of the single-particle orbitals as a function of quadrupole deformation together with the position of the Fermi surface with regard to the various Ω components of the neutron $i_{13/2}$ shell also play a role. The present paper is complementary to, and consistent with, the findings of Dracoulis, Kondev, and Walker [26,27], who interpreted changes in the moment of inertia of multiquasiparticle rotational sequences in rare-earth nuclei in terms of a reduction in pairing with seniority. It will be interesting to investigate this phenomenon in more extreme cases, such as in more neutron-rich nuclei and in systems closer to the proton drip line where changes in static pairing correlations are expected to occur. In addition, more detailed self-consistent calculations are needed to fully quantify these effects.

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