## **Electromagnetic properties of the** *d***∗(2380) hexaquark**

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(Received 24 May 2019; published 23 July 2019)

Experiments with intense photon and electron beams have the potential to provide access to nontrivial properties of the recently discovered *d*∗(2380) hexaquark including its size, structure, magnetic moment, and quadrupole and octupole deformations. In this paper we investigate the sensitivity of ongoing and planned experiments to various properties of the *d*∗(2380), employing models based on both constituent quark and pion cloud frameworks. Our calculations indicate that for photoinduced reactions on the deuteron, the *d*∗(2380) is predominantly produced from the *D*-wave component of the deuteron. We confirm earlier findings that the intrinsic quadrupole deformation of the *d*∗(2380) should be small. We also demonstrate an ability to extract the *d*∗(2380) magnetic moment and put constraints on the *d*∗(2380) *M*3/*E*2 ratio.

DOI: [10.1103/PhysRevC.100.012201](https://doi.org/10.1103/PhysRevC.100.012201)

The properties of the *d*∗(2380) hexaquark have been established quite rigorously in recent years following its observation in proton-neutron scattering and pionic fusion re-actions [\[1–9\]](#page-4-0). It has a mass of  $M_{d^*} = 2380$  MeV, vacuum width  $\Gamma = 70$  MeV, and quantum numbers  $I(J^P) = 0(3^+)$ . It therefore provides a new bosonic, isoscalar configuration in the light-quark sector. The internal structure of the  $d*(2380)$ is quite complicated and to some extent resembles a deuteron in which the nucleons are substituted with  $\Delta$ 's, where  $\Delta$ refers to the lowest lying excited state of the nucleon. The wave function in such a case is given by  $|\Psi_{d*}\rangle = |6q\rangle +$  $|\Delta \Delta_{S\text{-wave}}| + |\Delta \Delta_{D\text{-wave}}|$ . Each of the components in the wave function has its own spatial extension [\[10\]](#page-4-0) with the  $|6q\rangle$  configuration the most compact one:  $R_{d*}$  ( $|6q\rangle$ ) ≈ 0.5 fm  $[R_{d^*}(|\Delta\Delta_{S\text{-wave}})\rangle \approx 0.8$  fm and  $R_{d^*}(|\Delta\Delta_{D\text{-wave}}\rangle) \approx 1.4$  fm]. The  $|6q\rangle$  compact configuration of the  $d*(2380)$  is predicted to be dominant ( $\approx 69\%$ ) while the more extended *D*-wave 'molecular" component is of order 2%.

In a very recent work the possibility of *d*∗(2380) formation within neutron stars was explored in relativistic mean-field calculations where the *d*∗(2380) was introduced as a diffuse noninteracting and noncondensing gas alongside the standard (nucleonic and leptonic) constituents of neutron stars. By solving Tolman-Oppenheimer-Volkoff (TOV) equations using the resulting equation of state, a significant *d*<sup>∗</sup> formation was observed: up to 20% of the matter in the center of heavy stars was predicted to dwell as *d*<sup>∗</sup>. The resulting massradius predictions for neutron stars with this *d*<sup>∗</sup> degree of freedom is currently one of the few that can simultaneously give agreement with both the mass-radius constraint of the recent merger event observed by the Laser Interferometer Gravitational-Wave Observatory (LIGO) [\[11\]](#page-4-0), while giving agreement with the maximum observed (and inferred from gravitational-wave data) neutron star mass of  $\approx 2.17 M_{\odot}$  [\[12\]](#page-4-0).

The influence of the *d*∗(2380) electromagnetic properties on neutron stars, such as the contribution of the  $d*(2380)$ magnetic moment to formation of the neutron star magnetic field, makes the question of the *d*∗(2380) size and structure important to astrophysics as well as nuclear physics. It was recently shown that the *d*∗(2380) can be produced copiously using photon beams of energy  $E<sub>\gamma</sub> \approx 570$  MeV from a deuteron target [\[13–15\]](#page-4-0). Due to its high spin,  $J_{d^*}^P = 3^+$ , the *d*∗(2380) requires the contribution of higher multipoles (*E*2, *M*3, or *E*4) to be photoproduced from the deuteron  $[I(J<sup>P</sup>) = 0(1<sup>+</sup>)]$ . The ratios of the strengths of different multipoles in EM transitions are sensitive to the shape and magnetic moments, as already evidenced for the  $\Delta^+$ , where it was possible to extract the intrinsic quadrupole deformation of the  $\triangle$  resonance from ratios of the *E*2 and *M*1 multipoles [\[16,17\]](#page-4-0). In this work we will employ such methodologies to investigate the sensitivities that can be achieved for the new  $d*(2380)$ . Our calculations are based on geometrical arguments only, without any interparticle interaction. The aim of the current Rapid Communication is to provide rough estimates of some important properties of *d*<sup>∗</sup> planned to be measured and thus to motivate the development ofmore refined theoretical models.

The paper is structured as follows. We first outline the theoretical framework used to describe the  $N\Delta$  transition in the pion cloud model and then adapt this to the case of the d∗. We also describe the extraction of the *d*<sup>∗</sup> electric quadrupole and magnetic octupole moments as well as transition electromagnetic moments.

*The*  $\Delta$  *in a pion-cloud model*. We discuss the earlier work regarding the  $\Delta$  with a view to extending this to the *d*<sup>∗</sup>. It was shown in Ref. [\[16\]](#page-4-0) that the wave function of  $\Delta$  resonance can be considered as a two-component sum:  $|\Delta\rangle = \alpha'|\Delta'\rangle + \beta'|N\pi\rangle$ , where  $\Delta'$  is a true compact threequark configuration and  $N\pi$  is the component deriving from a nucleon plus pion cloud. The coefficient  $\alpha'^2$  gives the probability to find the  $\Delta$  in a three-quark state, while  $\beta^2$ corresponds to the probability for the  $\Delta$  to be in a pion-cloud mode. The coefficients are normalized to unity  $\alpha'^2 + \beta'^2 = 1$ .

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To agree with existing experimental data the coefficient  $\beta'$  is determined to be  $-0.52$  [\[16\]](#page-4-0). The quadrupole moment of the  $\Delta$  resonance can be calculated in this model as  $\langle \Psi_{\Delta}^* | \hat{Q}_{\pi} | \Psi_{\Delta} \rangle$ where  $\hat{Q}_{\pi}$  is a quadrupole operator of the form

$$
\hat{Q}_{\pi} = e_{\pi} \sqrt{\frac{16\pi}{5}} r_{\pi}^2 Y_0^2(\hat{r}_{\pi}), \tag{1}
$$

in which  $e_{\pi}$  is the pion charge operator divided by the unit charge  $e$ , and  $r<sub>\pi</sub>$  is the distance between the center of the quark core and the pion. The wave function for the  $\Delta^+$  can be written in the form

$$
|\Delta^+ \uparrow\rangle = \alpha' |\Delta^{+'} \uparrow\rangle + \beta' \frac{1}{3} (2|p' \uparrow \pi^0 Y_0^1\rangle + \sqrt{2}|p' \downarrow \pi^0 Y_1^1\rangle
$$
  
+ $\sqrt{2}|n' \uparrow \pi^+ Y_0^1\rangle + |n' \downarrow \pi^+ Y_1^1\rangle).$  (2)

All coefficients in Eq. (2) are the simple spin-isospin Clebsch-Gordan couplings. Since both neutral pions (zero charge) and the spherically symmetric quark core do not contribute to the quadrupole deformation, one can simplify the wave function by omitting the irrelevant terms:

$$
|\Delta^+ \uparrow\rangle = \beta' \frac{1}{3} \big( \sqrt{2} |n' \uparrow \pi^+ Y_0^1 \big) + |n' \downarrow \pi^+ Y_1^1 \big) \tag{3}
$$

From this, the spectroscopic quadrupole moment of the  $\Delta$  can be derived:

$$
Q_{\Delta^{+}} = -\frac{2}{15} \beta'^2 r_{\pi}^2.
$$
 (4)

The proton to  $\Delta$  transition quadrupole moment can be calculated in a similar manner  $\langle \Psi_p^* | \hat{Q}_\pi | \Psi_\Delta \rangle$ , giving a similar result:

$$
Q_{\Delta^+} = Q_{p \to \Delta^+}.\tag{5}
$$

Here the probability of the proton to be in the pion-cloud mode is assumed to be  $\beta = 0.26$  and  $r_\pi = 1.77$  fm, determined from the experimental data  $[16]$ .

*The d*∗(2380) *in a pion cloud model.* One can apply the same formalism to calculate the *d*<sup>∗</sup>(2380) quadrupole deformation under a pure geometrical approach. For the case of the  $d^*$  we assume a "Deltaron" structure consisting of two  $\Delta s$ with wave function as outlined earlier. In this noninteracting case the wave function of the *d*∗(2380) can be written as

$$
|d^*\rangle = |\Delta\Delta\rangle = (\alpha'|\Delta'\rangle + \beta'|N'\pi\rangle)(\alpha'|\Delta'\rangle + \beta'|N'\pi\rangle), \quad (6)
$$

creating three major structures in the wave function:

$$
|d^*\rangle = A + B + C,\tag{7}
$$

where

$$
A = \alpha'^{2} |\Delta' \Delta' \rangle, \ B = \alpha' \beta' |\Delta' N' \pi \rangle, \ C = \beta'^{2} |N' N' \pi \pi \rangle. \tag{8}
$$

For the subsequent analysis we will only consider configurations with two pions in isopin  $I = 0$  state and two nucleons in  $I = 0$  state, the dominant configurations which encapsulate 80% of the available parameter space  $[9]$ .<sup>1</sup> It is interesting to

note that this primitive model gives rise to all the major *d*<sup>∗</sup> structures proposed to date. The first term, *A*, can be related to a six-quark configuration of the *d*∗. However, it should be noted that this configuration is only one out of five possible 6*q* configurations proposed [\[18,19\]](#page-4-0). The second term resembles a "pion-assisted" dibaryon configuration as proposed by Gal *et al.* [\[20\]](#page-4-0). The third term can be further decomposed into two cases: *C*1 two pions in a relative *S*-wave, pion pair in a relative *D* wave to the  $S = 1$  nuclear core, and  $C2$  two pions in a relative *D*-wave, pion pair in a relative *S* wave to the  $S = 1$ nuclear core. The  $C1$  case resembles the  $\sigma$ -cloud model of the *d*<sup>∗</sup> by Kukulin *et al.* [\[21\]](#page-4-0). The *C*2 term is analogous to the *D*-wave  $\Delta\Delta$  configuration [\[22\]](#page-4-0).

To calculate the quadrupole deformation of the *d*<sup>∗</sup> we will use the same ansatz as for the  $\Delta$ :  $\langle \Psi_{d^*}^* | \hat{Q}_\pi | \Psi_{d^*} \rangle$ . Considering the contributions to the  $d^*$  wave function [Eqs. (7) and (8)], the *A* term is spherically symmetric, hence does not give a contribution to the quadrupole moment. The *B* term represents the quadrupole moment of a single  $\Delta^+$  multiplied by the  $\alpha^2$ factor:

$$
\langle B|\hat{Q}_{\pi}|B\rangle = \alpha'^{2}Q_{\Delta^{+}} = (1 - \beta'^{2})Q_{\Delta^{+}} \approx 0.73Q_{\Delta^{+}}.
$$
 (9)

It can be seen that the  $N\Delta\pi$  term is responsible for the nonsphericity of the  $d^*$ , producing a small oblate shape similar in character to that of a single  $\Delta$ . Recent experiments have set limits on a possible  $d^* \to NN\pi$  decay branch and indicate the  $N\Delta\pi$  term in the  $d^*$  wave function is likely to be very small, if it exists at all [\[23\]](#page-4-0). The resulting suppression of the *B* term would further reduce the *d*<sup>∗</sup> quadrupole deformation.

The  $C1$  term (*D*-wave  $\sigma$  cloud) would not contribute to the quadrupole moment due to the zero net charge of the pion pair. To calculate the*C*2 term we adopt a relative coordinate system in which  $\hat{r}_{\pi}$  is substituted by  $\hat{r}_{\pi-\pi}$ , the separation between the two pions. The  $r_{\pi}$  is related to  $r_{\pi-\pi}$  by  $r_{\pi-\pi} = \sqrt{2}r_{\pi}$ , the relative distance between the pions instead of the pion-core distance. Similarly  $e_{\pi}$  would be transformed to 2:  $(e_{\pi^+} - e_{\pi^-})$ . To simplify the integration we employ the spherical harmonics addition theorem, decomposing  $Y_0^2(\hat{r}_{\pi-\pi})$  into the product  $Y_m^2(\hat{r}_{\pi_1}) Y_{-m}^2(-\hat{r}_{\pi_2}).$ 

$$
Y_0^2(\hat{r}_{\pi-\pi}) = \sqrt{\frac{5}{16\pi}} \sum_{m=-2}^2 (-1)^m Y_m^2(\hat{r}_{\pi_1}) Y_{-m}^2(-\hat{r}_{\pi_2}). \tag{10}
$$

The few nonzero elements in these calculations are shown together with their resulting weights below:

(a) 
$$
\langle Y_1^1 Y_{-1}^1 | Y_0^2 | Y_1^1 Y_{-1}^1 \rangle : \frac{-1}{16\sqrt{5\pi^3}},
$$
  
\n(b)  $\langle Y_0^1 Y_0^1 | Y_0^2 | Y_0^1 Y_0^1 \rangle : \frac{-1}{4\sqrt{5\pi^3}},$   
\n(c)  $\langle Y_1^1 Y_0^1 | Y_0^2 | Y_1^1 Y_0^1 \rangle : \frac{1}{8\sqrt{5\pi^3}},$   
\n(d)  $\langle Y_1^1 Y_1^1 | Y_0^2 | Y_1^1 Y_1^1 \rangle : \frac{1}{16\sqrt{5\pi^3}}.$  (11)

Summing these contributions gives

$$
\langle C|\hat{Q}_{\pi}|C\rangle = -0.52Q_{\Delta^{+}}.\tag{12}
$$

<sup>&</sup>lt;sup>1</sup>The case with two pions in the  $I = 1$  state and two nucleons in the *I* = 1, like  $pp\pi^{-}\pi^{0}$  and  $mn\pi^{+}\pi^{0}$ , are very interesting since isospin selection rules also imply  $l_{\pi\pi} = 1$  and  $l_{NN} = 1$ .



FIG. 1.  $d^*(2380)$  quadrupole deformation produced by (b)  $N\Delta\pi$  (*B* term), (c) double-pionic cloud (*C* term), and (a) the total effect.

The double-pion cloud produces a prolate quadrupole deformation, similar to that observed for the case of the nucleon. Note that the *B* and *C* terms have opposite signs. Combining all terms and taking  $Q_{\Delta^+} = 0.043$  *e* fm<sup>2</sup> from Ref. [\[24\]](#page-4-0),<sup>2</sup> one gets  $Q_{d^*} \sim 0.21 Q_{\Delta^+} = 0.009 e \text{ fm}^2$  with a magnitude mainly arising from the *B*-*C* term cancellations. It should be noted that as discussed earlier, experimental decay studies suggest a small contribution from the **B** term  $(N\Delta\pi)$  to the  $d^*$  wave function. With this assumption the quadrupole moment of the  $d^*$  might be smaller or even change the sign. There is a calculation for the *d*<sup>∗</sup> quadrupole deformation from the Beijing group  $[25]$ . In their case the main contribution to the *d*<sup>∗</sup> quadrupole moment comes from the interference between the *S*-wave  $\Delta\Delta$  configuration and the small *D*-wave  $\Delta\Delta$ component of the *d*<sup>∗</sup> wave function. In such an approach the  $d^*$  should have a prolate shape with  $Q_{d^*} = 0.025 e \text{ fm}^2$ , similar in magnitude and sign to the result derived here from the *C* term alone.

In Fig. 1, the individual shapes of the *B* and *C* terms and the resulting distribution are plotted.

*Calculation of the transition quadrupole moment.* To calculate the  $E2(\gamma d \rightarrow d^*)$  transition probability we need to calculate the  $Q_{d\rightarrow d^*}$  quadrupole transition moment  $Q_{d\rightarrow d^*}$  =  $\langle \Psi_d^* | \hat{Q} | \Psi_{d^*} \rangle$ . Before tackling the complicated  $d \to d^*$  case, we first consider the deuteron itself. The wave function of the deuteron can be written as

$$
|\Psi_d\rangle = \alpha |\Psi_d^S\rangle + \beta |\Psi_d^D\rangle, \quad \alpha^2 + \beta^2 = 1. \tag{13}
$$

Here  $|\Psi_{d}^{S}\rangle$  is the *S*-wave component of *pn* inside the deuteron and  $|\Psi_d^D\rangle$  is the *D* wave. The *D*-wave probability in the deuteron is small, with  $P_D = \beta^2 \approx 4\%$ . To calculate the deuteron quadrupole moment we need to fold the quadrupole operator into the deuteron wave functions  $Q_d = \langle \Psi_d^* | \hat{Q} | \Psi_d \rangle$ . Since  $\hat{Q} \sim Y_0^2$  then it follows that  $\langle (\Psi_d^S)^* | \hat{Q} | \Psi_d^S \rangle = 0$  due to the sphericity of the *S*-wave component. The  $\langle (\Psi_d^D)^* | \hat{Q} | \Psi_d^D \rangle$ term would be suppressed by the low  $P_D$  probability in the wave function, so the main contribution to the deuteron quadrupole moment arises from the  $\langle (\Psi_d^D)^* | \hat{Q} | \Psi_d^S \rangle$  term. It should be noted that the nucleon has a small intrinsic quadrupole deformation [\[16\]](#page-4-0), as the *S*-wave component is not fully spherical. However, this is a small effect compared to the contributions calculated here and is neglected.

The *S*-wave part of the *d*<sup>∗</sup> is assumed to be essentially spherical, as also assumed in calculations of the deuteron. Therefore the dominant term contributing to the transition quadrupole moment, and therefore the *E*2 transition probability, is  $\langle (\Psi_d^D)^* | \hat{Q} | \Psi_{d*} \rangle$ . The  $d^*$  is excited from the *D*-wave part of the deuteron *only*. The deuteron to *d*<sup>∗</sup> transition quadrupole moment can then be written as

$$
Q_{d \to d^*} = Q_d \frac{\langle R_{d^D} | R_{d^*} \rangle}{\langle R_{d^D} | R_{d^S} \rangle},
$$
(14)

where  $R_{d^S}$  and  $R_{d^D}$  are the radial parts of a deuteron *S*-wave and *D*-wave wave function, and  $R_{d^*}$  is a radial part of the *d*<sup>∗</sup> wave function. By measuring the transition quadrupole moment we directly measure the  $d^*$  compactness in terms of the deuteron size. Therefore the strength of *d*<sup>∗</sup> production via an *E*2 transition, which can be extracted in photoproduction experiments using double-polarization measurements in which photon and deuteron spins are aligned, has the potential to provide measurement of the *d*<sup>∗</sup> compactness. Taking the known deuteron wave function and combining it with the  $d^*(\Delta \Delta)$  wave function from Ref. [\[25\]](#page-4-0) one can evaluate

$$
\frac{\langle R_{d^D} | R_{d^*} \rangle}{\langle R_{d^D} | R_{d^S} \rangle} \approx \frac{0.15}{0.22} \approx 0.7; \quad Q_{d \to d^*} \approx 0.20 \text{ e fm}^2. \quad (15)
$$

Note that neither the *S*-wave or *D*-wave part of deuteron can be excited directly into the  $|6q\rangle$  part of the  $d^*$  (one cannot transfer two color bags into one with a colorless photon), so only the  $\Delta\Delta$  part will be relevant to the *d*<sup>\*</sup> production. If the  $\Delta\Delta$  part is indeed 1/3 of the  $d^*$  wave function, as predicted in Ref. [\[10\]](#page-4-0), then the transition quadrupole moment would be further suppressed by this factor.

*Octupole magnetic moment.* As shown in Ref. [\[26\]](#page-4-0) the octupole magnetic moment can be evaluated within a pion cloud model using the octupole moment operator

$$
\hat{\Omega} = e_{\pi} \sqrt{\frac{16\pi}{5}} r_{\pi}^2 Y_0^2 (\hat{r}_{\pi}) \hat{\mu} \tau_z^N \sigma_z^N = \hat{Q} \hat{\mu}.
$$
 (16)

<sup>&</sup>lt;sup>2</sup>We used  $Q_{\Delta^+} = -0.043 e \text{ fm}^2$  from Ref. [\[24\]](#page-4-0) because it is closer to the experimentally determined value, instead of  $Q_{\Delta^+}$  = <sup>−</sup>0.<sup>113</sup> *<sup>e</sup>* fm2 from Ref. [\[17\]](#page-4-0), calculated within the pion cloud formalism.

moment has been calculated in the previous discussion, so it is possible to calculate the magnetic moment of the *d*<sup>∗</sup> in a simple quark model: The *d*<sup>∗</sup> wave function on a quark level can be written as

$$
|d^*\rangle = |u \uparrow u \uparrow u \uparrow d \uparrow d \uparrow d \uparrow.
$$
 (17)

Therefore the magnetic moment of the *d*<sup>∗</sup> can be calculated as

$$
\mu_{d^*} = 3\mu_u + 3\mu_d, \ \mu_u = \frac{2}{3} \frac{e\hbar}{2M_q c}, \ \mu_d = -\frac{1}{3} \frac{e\hbar}{2M_q c},
$$
  
with  $M_q = M_N/3$ ,  $\mu_u = 2\mu_N$ ,  $\mu_d = -2\mu_N$ ,  
 $\mu_{d^*} = 3\mu_N \sim \mu_p$ . (18)

To calculate the *d*<sup>∗</sup> octupole moment we also need to evaluate the magnetic moment of a deuteron core and a  $\Delta N$  core (the *C* and *B* terms in the quadrupole moment calculations, respectively). The "deuteron core" has only a deuteron *S*-wave component, so its magnetic moment would be  $\mu_{C\text{-term}} = \mu_p + \mu_p$  $\mu_n = \mu_N$ . For the *B* term we have  $\mu_{B\text{-term}} = \mu_{\Delta^0} + \mu_n =$  $\mu_n = -2\mu_N$ . The double-pion cloud and the  $N\Delta\pi$  terms have opposite signs. In quadrupole moment they cancel each other, but in octupole moment they add up. In ultimate situation we can have zero quadrupole moment and large octupole moment. Also the octupole moment of the *d*<sup>∗</sup> is positive.

$$
\Omega_{d^*} = (Q_{d^*}^{\text{C-term}} - 2Q_{d^*}^{\text{B-term}})\mu_N
$$
  
~1.98Q<sub>Δ</sub>+ $\mu_N$  = 0.0089 e fm<sup>3</sup>. (19)

In the work of Ref. [\[25\]](#page-4-0) the octupole moment is calculated to be  $\Omega_{d^*} = -0.00567 e \text{ fm}^3$ , the opposite sign and smaller magnitude. However, it should be noted that their magnetic moment is about twice as large  $\mu_{d^*} = 7.6 \mu_N$ , so having large magnetic moment and a *C* term only, one can reproduce the result of Ref. [\[25\]](#page-4-0). The estimation of magnetic octupole moment magnitude is important for the feasibility studies of upcoming experiments at photon beam facilities. While experimental determination of the  $\Omega_{d^*}$  might be challenging with rather poor accuracy of the magnitude determination. The  $\Omega_{d^*}$  sign evaluation should be straightforward.

*Transition octupole magnetic moment.* The transition octupole moment can be calculated the same way as the transition quadrupole moment. The argument about the exclusive deuteron *D*-wave contribution holds also for this case. The *D*-wave part of the deuteron wave function contributes to the magnetic moment in two ways: from the spin of the constituents and from the orbital motion of the charged proton  $\mu_d^{D\text{-wave}} = -\frac{3}{2}(\mu_p + \mu_n - 1/2)$ . The orbital part is irrelevant to our calculations<sup>3</sup> and the spin part has opposite sign due to the anti-alignment of nucleon and deuteron spins in the case of the *D*-wave component. Since we consider the deuteron to *d*<sup>∗</sup> transition for the following spin state  $|S_d = 1, S_d^z = +1 \rangle \rightarrow$ 

 $|S_{d^*} = 3, S_{d^*}^z = +1 \rangle$  we need to reevaluate the magnetic moment for another *d*<sup>∗</sup> spin state

$$
|d^*\rangle(S=3, S_z=1) = 1/3(|u \downarrow u \downarrow u \uparrow d \uparrow d \uparrow d \uparrow)
$$
  
 
$$
+|u \uparrow u \uparrow u \downarrow d \downarrow d \uparrow d \uparrow)
$$
  
 
$$
+|u \uparrow u \uparrow u \uparrow d \uparrow d \downarrow d \downarrow)
$$
 (20)

$$
\mu_{d^*}(S_z = 1) = 1/3(-\mu_u + 3\mu_d + \mu_u + \mu_d + 3\mu_u - \mu_d)
$$
  
=  $\mu_u + \mu_d = \mu_N.$  (21)

The transition moment can be roughly evaluated as

$$
\Omega_{d \to d^*} = -Q_{d \to d^*} \sqrt{(\mu_p + \mu_n) \mu_d^* (S_z = 1)} \sim -Q_{d \to d^*} \mu_N
$$
  
= -0.021 e fm<sup>3</sup>. (22)

Double-polarized photoproduction measurements with a tensor polarized deuteron target should be an ideal tool to access the *d*<sup>∗</sup> magnetic octupole transition moment and the extraction of the *d*<sup>∗</sup> magnetic moment. Such a target is planned to be used at the CEBAF large acceptance spectrometer (CLAS) detector at the Thomas Jefferson National Accelerator Facility in the near future. The experiments at CLAS are planned to be done with an electron beam, allowing the potential extraction of the magnetic octupole form factor and not just  $G_{M3}(0)$ .

*The M3/E2 ratio*. It was demonstrated with  $\Delta$  photoexcitation that the ratio of multipole transitions can be calculated and measured more precisely than the transitions themselves. For the case of the  $p \to \Delta$  transition, the *E*2/*M*1 ratio can be accessed:

$$
\frac{E2}{M1}(p \to \Delta) = \frac{M_N \omega_\gamma Q_{p \to \Delta}}{6\mu_{p \to \Delta}} \approx 3\%.
$$
 (23)

It is interesting to note that one of the first attempts to calculate the  $p \to \Delta E2/M1$  ratio was done in the 1960's using Weisskopf widths from nuclear physics and it gave remarkable agreement [\[27\]](#page-4-0).

$$
\frac{E2}{M1}(p \to \Delta) = \frac{2.4 \times 10^{-8} R^4 k^5}{2.1 \times 10^{-2} k^3} = 1.14 \times 10^{-6} R^4 k^2 \approx 3\%.
$$
\n(24)

Here  $R = 0.8$  fm is the radius of the proton and  $k$  is the photon momentum. For the *M*3/*E*2 transition the Weisskopf width dependence would be [\[28\]](#page-4-0)

$$
\frac{M3}{E2}(d \to d^*) \sim \frac{R^4 k^7}{R^4 k^5} = \left(\frac{\omega_\gamma}{M_N}\right)^2 \left(\mu_p - \frac{1}{4}\right)^2 \frac{8}{392} \approx 5\%,\tag{25}
$$

in which the size term cancels. However, from nuclear physics moments measurements we know that large deformation can lead to sizable deviation from Weisskopf coefficients. In such a case one needs to substitute them by a transition moment:

$$
\frac{B_w(M3)}{B_w(E2)} = \frac{250\mu_N^2}{144e^2} \approx 1.9 \times 10^{-2} \rightarrow \frac{\Omega_{d \to d^*}}{Q_{d \to d^*}} \approx 0.11,
$$
\n(26)

$$
\frac{M3}{E2}(d \to d^*) \sim \frac{\omega_\gamma^2 \Omega_{d \to d^*}}{Q_{d \to d^*}} \sim \frac{\omega_\gamma^2 Q_{d \to d^*} \mu_{d \to d^*}}{Q_{d \to d^*}}
$$

$$
\sim \omega_\gamma^2 \mu_{d \to d^*} \approx 30\%.
$$
 (27)

<sup>&</sup>lt;sup>3</sup>It gives rise to the structure  $\langle Y_0^2 | Y_0^1 | Y_0^0 \rangle = 0$ .

<span id="page-4-0"></span>In the *M*3/*E*2 ratio the size term from the quadrupole moment cancels leaving a proportionality to the transition magnetic moment. In the case of  $d \rightarrow d^*$  one can assess one more type of excitation, *E*4, which may give sensitivity to the *D*wave configuration inside the *d*∗. This could be accessed with double-polarized photoproduction measurements, in which the deuteron and photon spins are anti-aligned. The production cross section is expected to be small, but such information would be very valuable. One word of caution needs to be offered here. In our calculations of transition quadrupole and octupole moments as well as the *M*3/*E*2 ratio the main dependence came from deuteron. However, if we compare the

strength of transition moments with the  $d^*$  moments we see a

$$
\left|\frac{\Omega_{d^*}}{\Omega_{d\to d^*}}\right| \approx 0.43, \quad \left|\frac{Q_{d^*}}{Q_{d\to d^*}}\right| \approx 0.045. \tag{28}
$$

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sizable difference:

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An order of magnitude difference in these ratios can point us to a possible further suppression of *E*2 transition in favor of *M*3. If the  $d^*$  magnetic moment is indeed  $\mu_{d^*} = 7.6\mu_N$  as anticipated in Ref. [25], the *M*3/*E*2 ratio gets as high as 80%.

*Summary.* We have calculated the *d*<sup>∗</sup> electromagnetic properties exploiting simple theoretical models. The quadrupole and octupole moments were calculated in a pion cloud model and reasonable agreement was obtained with the Resonating Group Method of Ref. [25]. The electromagnetic transition moments from the deuteron to the *d*<sup>∗</sup> were also calculated for the first time. These results will help guide future experimental investigations of *d*<sup>∗</sup> with electromagnetic beams, where there is the potential to reveal important new information on the *d*<sup>∗</sup> structure. Particularly, sensitivities may be obtained from measurements of photoproduction from a tensor polarized deuteron target. We hope these initial theoretical studies will motivate more detailed theoretical work in the future.

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