Rapid Communications

Modeling ^{19}B as a ^{17}B-n-n three-body system in the unitary limit

Emiko Hiyama,[1,2] Rimantas Lazauskas,[3] F. Miguel Marqués,[4] and Jaume Carbonell[5]

[1]*Department of Physics, Kyushu University, Fukuoka, 819-0395, Japan*

[2]*RIKEN Nishina Center, Wako 351-0198 Japan*

[3]*IPHC, IN2P3-CNRS/Université Louis Pasteur BP 28, F-67037 Strasbourg Cedex 2, France*

[4]*LPC Caen, Normandie Université, ENSICAEN, Université de Caen, CNRS/IN2P3, F-14050, Caen, France*

[5]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, 91406 Orsay Cedex, France*

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We present a model description of the bound ^{19}B isotope in terms of a ^{17}B-n-n three-body system where the two-body subsystems ^{17}B-n and n-n are unbound (virtual) states close to the unitary limit. The ^{19}B ground state is well described in terms of two-body potentials only, and two low-lying resonances are predicted. Their eventual link with the Efimov physics is discussed. This model can be naturally used to describe the recently discovered resonant states in ^{20,21}B.

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The interaction of a low-energy neutron (n) with a nucleus of mass number A (with spin-parity J_A^π) is a balance between the attractive folded neutron-nucleus potential V_{nA} and the effective Pauli repulsion between the incident and target neutrons. For $A = 1$, all the n-A spin ($S = J_A \pm 1/2$) and isospin (T) channels are attractive: This attraction manifests through the large and negative values of the singlet scattering lengths ($a_0^{nn} = -18.5$ fm and $a_0^{np} = -23.5$ fm) and by the existence of the ^{2}H bound state in the only triplet n-p ($^3S_1$) channel [1].

The first consequence of Pauli repulsion already manifests for $A = 2$, in the neutron scattering on ^{2}H in the $S = 3/2^+$ channel. The scattering length in this channel is positive, $a_{3/2} = 6.35$ fm, and reflects a strong repulsion, while the $S = 1/2^+$ channel, despite having a positive scattering length $a_{1/2} = 0.65$ fm, remains attractive accommodating the ^{3}H bound state. For $A = 3$ and $T = 1$ both spin channels are repulsive with $a_0 = 5.2$ fm and $a_1 = 4.8$ fm. The same happens in the case of n-^{3}He (isospin mixture) with $a_0 = 5.9$ fm (its real part) and $a_1 = 3.1$ fm. The only existing reaction with $A = 4$, n-^{4}He, is also repulsive with $a_{1/2} = 2.61$ fm.

When increasing the neutron number in the target, the valence neutrons start filling p (and higher angular momentum) orbitals, the Pauli principle becomes less constraining and the net balance is again attractive with the corresponding negative values of a_S, starting from the ^{7}Li ground state $J^\pi = 3/2^-$ with $a_2 = -3.6$ fm. This is also true for ^{8}He with $a_S \approx -3$ fm [2,3] and ^{9}Li with $a_S \approx -14$ fm [2], although in these cases the n-A total spin S was not determined.

For even higher numbers of neutrons, the virtual state of the n-A system starts approaching the threshold. A spectacular consequence of this trend manifests with the ^{17}B isotope, where the n-^{17}B virtual state is located at the extreme vicinity of the ^{18}B ground-state threshold. This effect is reflected by the largest value of the neutron-nucleus scattering length observed so far, $a_S \approx -100$ fm [4]. However, the limited resolution and acceptance of the experiment did not allow a lower bound to be fixed for the scattering length [4], and only an upper bound $a_S < -50$ fm was determined.

Despite this experimental uncertainty, the potentially huge n-^{17}B scattering length confers a unique and intriguing character to ^{19}B, a two-neutron Borromean halo nucleus [5] exhibiting a weakly bound core-n-n structure in which all the two-body subsystems are unbound [6]. Moreover, due to its extremely weak binding ($2n$ separation energy of $S_{2n} = 0.14 \pm 0.39$ MeV [7]), ^{19}B has no bound excited states. As such, the ground state can be seen as an extended three-body system resulting from the very large scattering lengths of its two two-body components (of the order of -20 and -100 fm respectively).

The aim of this Rapid Communication is to present a dynamical model describing simultaneously both structures, n-^{17}B scattering and the ground state of ^{19}B, which could be easily applicable to the description of the heavier boron isotopes ^{20,21}B recently observed in Ref. [8]. The model is based on a two-body n-^{17}B local potential, built to this aim, supplemented with a realistic n-n interaction.

The same dynamical approach was used in Ref. [9] to describe ^{19}B. However, it overbound the ground state and predicted bound excited states due to the purely attractive n-^{17}B nonlocal interaction. Similar models, based on an effective field theory (EFT) renormalized zero-range n-core interaction plus a three-body force, were applied to another Borromean nucleus, ^{22}C [10,11]. Although the large scattering length values involved in the ^{19}B subsystems make natural its description in terms of EFT zero-range plus three-body forces, we have preferred in view of further applications to rely only on two-body finite-range interactions. A more consistent attempt to link the n-A low-energy parameters (LEPs) with the reaction cross sections with the target nucleus was proposed in Refs. [12,13].

Due to the experimental uncertainties in the relevant observables,that is, the n-^{17}B scattering length and ^{19}B binding

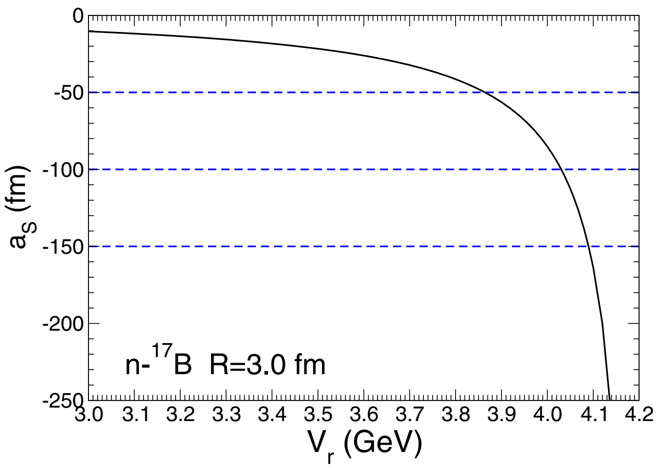

FIG. 1. Dependence of the n-^{17}B scattering length a_S on V_r for $R = 3$ fm. The dashed lines correspond to some selected values of a_S used in other figures.

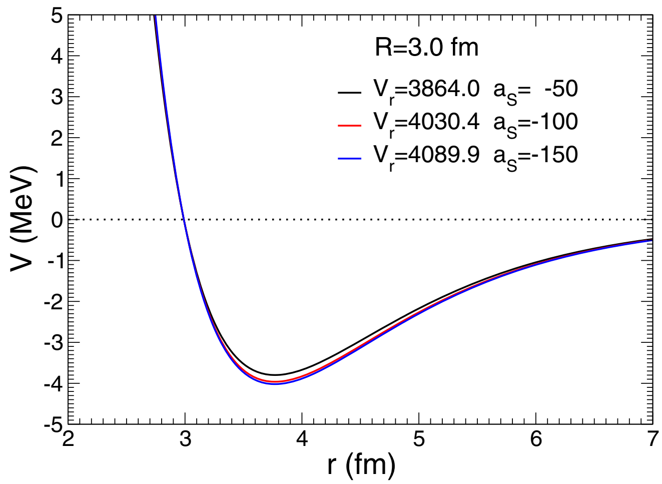

FIG. 2. $V_{n^{17}\mathrm{B}}$ potential for three different values of V_r (in MeV) and the corresponding scattering lengths (in fm).

energy, we will explore the possible range of those parameters up to the unitary limit. Finally, our model predicts the first excited states of ^{19}B to be of resonant character, in agreement with experimental observations [14].

In building an effective neutron-nucleus potential, it is assumed that a low-energy neutron approaching ^{17}B feels a short-range repulsion due to the Pauli principle with respect to the 12 neutrons in the ^{17}B core, plus a loose attraction due to the folded n-A interaction. A simple form accounting for these facts can be

$$V_{n^{17}\mathrm{B}}(r) \;=\; V_r\,(e^{-\mu r} - e^{-\mu R})\,\frac{e^{-\mu r}}{r}, \tag{1}$$

where R is a hard-core radius, hindering the penetration of the incoming neutron in the nucleus, and μ is a range parameter for the folded n-^{17}B potential. An educated value can be $R = 3$ fm, which corresponds to the rms matter radius of ^{17}B [5], and we take $\mu = 0.7\ \mathrm{fm}^{-1}$ corresponding to the pion mass.

Once the range μ and the size R are fixed, the potential depends on a single strength parameter V_r, which will be adjusted to reproduce the scattering length a_S. The numerical values along this work correspond to $m_n = 939.5654$ MeV, $m_{^{17}\mathrm{B}} = 15879.1$ MeV, i.e., a n-^{17}B reduced mass $m_R = 887.0771$ MeV, and $\hbar^2/2m_R = 21.9473$ MeV fm^2.

The dependence of the n-^{17}B scattering length a_S on the strength parameter V_r within this model is displayed in Fig. 1. The n-^{17}B potentials of Eq. (1) corresponding to $a_S = -50, -100, -150$ fm (dashed lines in Fig. 1) are displayed in Fig. 2. As one can see, despite the large variation of a_S the potentials quickly saturate when approaching the unitary limit. For example, when a_S varies from -50 to -100 fm the potential changes by only a few tens of keV, and at this scale it looks almost independent beyond those values.

Until now we have ignored any spin-spin effect in the n-^{17}B interaction (spin symmetric approximation). In fact ^{17}B is a $J^\pi = 3/2^-$ state and can couple to a neutron in two different spin states, $S = 1^-, 2^-$. If the resonant scattering length is due to the $S = 2^-$ state, as it is assumed in [4], there is no reason for the $S = 1^-$ to be resonant as well. To account for this eventual asymmetry in further calculations, we have introduced a spin-dependent interaction by assuming for each spin S channel a potential $V^{(S)}_{n^{17}\mathrm{B}}$ having the same form as Eq. (1) and driven by the corresponding strength parameter $V_r^{(S)}$. This can be cast in a single expression using the standard operator form:

$$V_{n^{17}\mathrm{B}}(r) \;=\; V_c(r) + (\vec{S}_n\cdot\vec{S}_{^{17}\mathrm{B}})\,V_{ss}(r), \tag{2}$$

in terms of the spin-spin operator

$$2(\vec{S}_n\cdot\vec{S}_{^{17}\mathrm{B}}) = S(S+1) - \tfrac{9}{2}, \tag{3}$$

and the central V_c and spin-spin V_{ss} components which are expressed in terms of $V^{(S)}_{n^{17}\mathrm{B}}$ by inverting the linear system obtained from Eq. (2) for $S = 1$ and $S = 2$:

$$4V^{(S=1)}_{n^{17}\mathrm{B}} = 4V_c - 5V_{ss}, \quad 4V^{(S=2)}_{n^{17}\mathrm{B}} = 4V_c + 3V_{ss}. \tag{4}$$

It follows from that, that the central (V_c) and spin-spin (V_{ss}) components have the same form as Eq. (1):

$$V_i(r) \;=\; V_r^{(i)}\,(e^{-\mu r} - e^{-\mu R})\,\frac{e^{-\mu r}}{r}, \quad i = c, ss,$$

with the strength coefficients given by

$$V_r^{(c)} = \tfrac{1}{8}\big(3V_r^{(1)} + 5V_r^{(2)}\big)V_r^{(ss)} = \tfrac{1}{2}\big(V_r^{(2)} - V_r^{(1)}\big).$$

Using this n-^{17}B potential, the interest of our model lies in the description of the more complex systems composed by ^{17}B and several neutrons in terms of two-body interactions. The first step in this direction is ^{19}B, which will be the subject of the next paragraphs.

The ^{19}B nucleus will be described as a ^{17}B-n-n three-body system. The n-^{17}B potential of Eq. (1), supplemented by a n-n interaction, will constitute the three-body Hamiltonian. This description is physically justified by the large values of the scattering length in each two-body subsystem.

For the n-n potential, we have chosen two different models: The Bonn A potential [15] and a charge-dependent (CD) version of the semirealistic MT13 interaction [16]. The Bonn A potential, being charge independent, provides the n-n

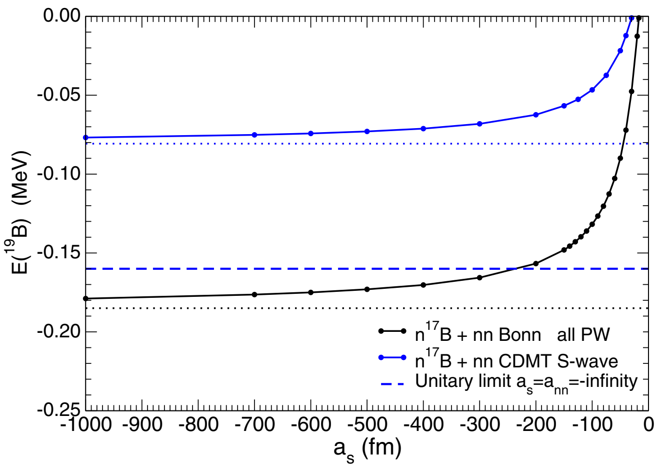

FIG. 3. ^{19}B ground-state energy with respect to the first particle threshold as a function of a_S, for $R = 3$ fm. Blue (gray) and black lines correspond, respectively, to model versions (i) and (ii).

low-energy parameters $a_{nn} = -23.75$ fm and $r_{nn} = 2.77$ fm and acts in all partial waves.

We have built a CD version of MT13 starting from the original parameters of the NN singlet state (that is $V_R = 1438.720$ MeV fm, $\mu_R = 3.11$ fm, $\mu_A = 1.55$ fm):

$$V_{nn} = V_R \frac{e^{-\mu_R r}}{r} - V_A \frac{e^{-\mu_A r}}{r}, \quad (5)$$

and adjusting the strength of the attractive term to $V_A = 509.40$ MeV fm in order to reproduce the n-n LEP $a_{nn} = -18.59$ fm and $r_{nn} = 2.93$ fm, in close agreement with the experimental values. In order to cross-check the results, the three-body problem was solved independently by using two different formalisms: Faddeev equations in configuration space [17,18] and the Gaussian expansion method [19].

We have first computed the ^{19}B ground-state energy $E(^{19}\mathrm{B})$ as a function of the scattering length a_S in the spin-symmetric case, that is, with $V_r^{(1)} = V_r^{(2)} = V_r$. Results, measured with respect to the ^{17}B-n-n threshold (i.e., $-S_{2n}$), are displayed in Fig. 3. They concern two different versions of the model: (i) a purely S-wave interaction both in $V_{n^{17}\mathrm{B}}$ and in V_{nn} (solid blue or gray line) and (ii) letting the interaction of Eq. (1) act in all partial waves (solid black line) supplied with the Bonn A model for V_{nn}.

In view of these results, the following remarks are in order:

(1) The quantum numbers of the ^{19}B ground state are $L = 0$ and $S = 3/2$, which are separately conserved, and so $J^\pi(^{19}\mathrm{B}) = 3/2^-$. Notice that since the total angular momentum is $L = 0$ the total parity is given by the intrinsic parity of ^{17}B, which is a $3/2^-$ state.

(2) In the range of a_S values compatible with the experiment ($a_S < -50$ fm [4]) and in both versions of our model, ^{19}B is bound. Its binding energy decreases when a_S increases and the binding disappears for a critical value of the scattering length a_S^c which slightly depends on the model version: $a_S^c \approx -30$ fm for (i) and $a_S^c \approx -15$ fm for (ii).

(3) The ^{19}B binding energy is compatible with the experimental value $E = -0.14 \pm 0.39$ MeV [7] for both versions of the model and in all the range of a_S, starting from $a_S = -50$ fm until the unitary limit in the n-^{17}B channel, i.e., $a_S \to -\infty$. This limit (dotted lines) corresponds to $E_u = -0.081$ MeV in version (i) and $E_u = -0.185$ MeV in version (ii). Note that in both cases the ground-state energy for $a_S = -150$ fm is only ≈20−30 keV distant from the unitary limit.

(4) It is interesting to consider also the unitary limit in the n-n channel, i.e., $a_{nn} \to -\infty$. This is realized, in the purely S-wave model version (i), by setting $V_A \approx 531.0$ MeV in the n-n potential of Eq. (5). The full unitary result, where both $a_S = a_{nn} \to -\infty$, is indicated by the dashed blue (gray) line in this figure. It corresponds to a ^{19}B energy $E_{uu} = -0.160$ MeV, compatible with the experimental result. The *ab initio* nuclear physics in the S-wave unitary limit of the NN interaction was recently considered in Ref. [20]. It was claimed that the gross properties of the nuclear chart were already determined by very simple NN interactions, provided they ensure $a_0 = a_1 \to -\infty$. The ^{19}B ground state constitutes a nice illustration of this remarkable property, though at the level of cluster description and based on other dynamical contents.

(5) The main difference between versions (i) and (ii) in Fig. 3 is essentially due to the contribution of the higher angular-momentum terms in $V_{n^{17}\mathrm{B}}$. In the present state of experimental knowledge, this contribution is totally uncontrolled but we have made an attempt to quantify it by assuming V to be the same in all partial waves. For $a_S = -150$ fm this accounts for about 60 keV of extra binding. The difference due to the n-n interaction is smaller, and mainly given by the fact that the Bonn A model is charge independent and has a larger a_{nn} absolute value. Once this is corrected, the difference in the ^{19}B binding energy at $a_S = -150$ fm reduces to 8 keV, which is mainly accounted for by the higher partial waves in the n-n interaction. As expected from the EFT arguments, the details of the interaction turn out to be negligible for a system close to the unitary limit.

The spatial probability amplitude, i.e., the squared modulus of the total wave function $\Psi(r, R)$ in terms of the Jacobi coordinates, is represented in Fig. 4 (upper panel), for $a_S = -100$ fm and $E = -0.130$ MeV in model version (ii). For the sake of comparison, we have computed the same amplitude and in the same scale of another two-neutron halo nucleus, ^{6}He ($E = -0.97$ MeV), using the α-n-n three-body model from Ref. [21]. It is displayed in the lower panel of the same Fig. 4. Due to the very weak binding energy, the wave function of ^{19}B is much more extended. Unlike the ^{6}He wave function it displays very large asymmetry, being strongly elongated in the ^{17}B-(nn) direction. An exceptional feature of ^{19}B is the presence of the void in the few fm space around the center of mass of the three clusters ^{17}B-n-n. This feature establishes ^{19}B as a veritable two-neutron halo nucleus having a molecule-like structure with three centers.

Besides providing a satisfactory description of the ^{19}B ground state, our model accommodates two broad resonances. Letting the interaction of Eq. (1) act in all partial waves with $a_S = -150$ fm ($V_r = 4089.9$ MeV fm) and adopting the S-wave model for the n-n interaction, two resonant states are found for total angular momentum $L = 1$ and $L = 2$, with energies $E_{1^-} \approx 0.24(2)$–$0.31(4)i$ MeV and $E_{2^+} \approx 1.02(5)$–$1.22(6)i$ MeV. The parameters of these resonances, however, depend on the value of a_S, which remains

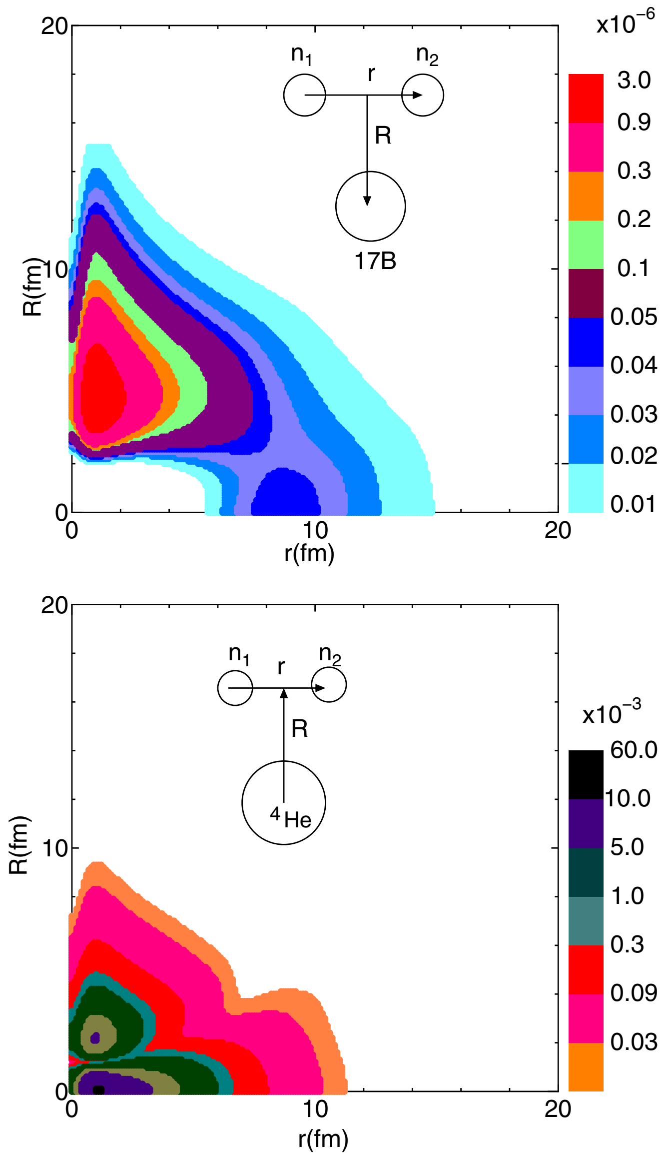

FIG. 4. Ground-state probability amplitude $|\Psi(r, R)|^2$ as a function of the Jacobi coordinates: Upper panel ^{19}B for $a_S = -100$ fm in model version (ii) and lower panel for ^{6}He [21].

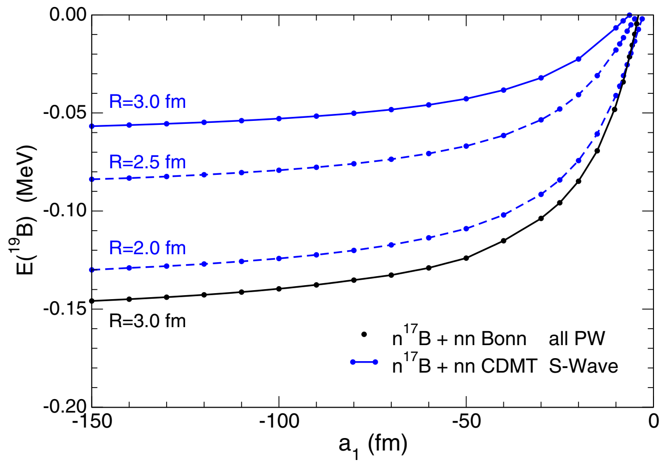

FIG. 5. ^{19}B ground-state energy with respect to the first particle threshold as a function of a_1, for a fixed $a_2 = -150$ fm and several values of R. Blue (or gray) and black lines correspond, respectively, to model versions (i) and (ii).

unknown. Their widths display also a strong dependence on the size parameter (R) of the n-^{17}B interaction, which we fixed once for all to $R = 3$ fm: If R is increased, keeping a_S fixed, the resonance widths decrease. Further experimental data are needed for a fine tuning of these parameters.

In fact, several resonances in the continuum of ^{19}B have been recently observed [14], although their precise energy and quantum numbers have not been determined yet. It is worth noting that despite its simplicity, our model of the n-^{17}B interaction is able to account for the ^{18}B virtual state, the ^{19}B ground state, and two resonances without any need of three-body forces. This follows from the strong resonant character of both n-^{17}B and n-n channels as well as the large spatial extension of the ^{19}B ground state.

If we introduce the spin dependence of the interaction, Eq. (2), and assume the nonresonant scattering length a_1 to be smaller in absolute value than a_2, the weaker potential leads to ^{19}B binding energies smaller than those in Fig. 3. For example, if we fix $a_2 = -150$ fm the ^{19}B binding energy decreases when a_1 varies from the spin-symmetric case $a_1 = a_2 = a_S$ up to some critical value a_1^c, still negative, beyond which ^{19}B is no longer bound. The results are displayed in Fig. 5.

The very existence of this critical value a_1^c, as well as its negative sign, is independent of the model version (blue and black solid lines) and the size parameter R, that was also varied here to check the robustness of this result (dashed blue or gray lines). In all the cases that we have explored, its value lies within $-7 < a_1^c < -3$ fm (Fig. 5). On the other hand, in order to reach the critical value a_1^c an extremely strong spin-spin interaction is required with $V_r^{(1)}/V_r^{(2)} \approx 0.6$, for the case $a_2 = -150$ fm. We may hardly find any physical arguments to support the existence of such a strong spin dependence in the n-^{17}B interaction. Therefore, we conclude that the value of a_1 should be also negative and not small, and that ^{19}B remains bound even when the spin symmetry is broken.

The proximity of the n-^{17}B interaction to the unitary limit strongly suggests that ^{17}B could be a genuine nuclear candidate to exhibit the Efimov physics [22], that is, the existence of a family of bound states whose consecutive energies are scaled by a universal factor f^2. This is what would happen by setting $a_1 = a_2 = a_{nn} = -\infty$ as in the blue (gray) dashed line of Fig. 3. However, the ^{17}B-n-n system, representing a light-light-heavy structure, turns out to be quite an unfavorable case to exhibit the sequence of excited Efimov states due to the requirement of a very large factor f. However, in the real world, as well as in our model, ^{19}B has only one bound state and it is governed by three different scattering lengths, from which only the n-n scattering length is relatively well known and can be fixed to its experimental value. For the case when only the n-^{17}B interaction is tuned the universal factor turns out to be $f \approx 2000$ [22,23]. It follows that the appearance of the first $L = 0^+$ excited state of Efimov nature in ^{19}B would manifest only when the n-^{17}B scattering length reaches several thousands of fm. We conclude that, independent of the particular value of a_S, it is highly unlikely that any Efimov excited state is observed in ^{19}B. On the other hand, the universal features related to Efimov physics [24] are, however, genuinely preserved in this system.

It is worth mentioning that a hypothetical molecular-like ^{17}B-^{17}B-n system would constitute a favorable heavy-heavy-light structure in order to generate Efimov states [22]. However, the long-range part of the effective hyper-radial interaction in this system would be dominated by the repulsive Coulomb interaction between the ^{17}B nuclei, largely exceeding the $1/r^2$ tail of Efimovian attraction.

In summary, and motivated by the recent experimental results on heavy boron isotopes [4,8,14], we have developed a model to describe the main phenomenological facts of these neutron-rich nuclei. It is based on a n-^{17}B effective interaction which, supplemented with a realistic n-n potential, provides a satisfactory description of ^{19}B ground and resonant states in terms of a ^{17}B-n-n three-body system.

The key ingredient of the model is the proper description of the extremely shallow virtual state of ^{18}B, which manifests through a very large and negative value of the n-^{17}B scattering length, $a_S < -50$ fm, in the $S = 2$ channel [4].

It is worth noticing that despite its simplicity, the model is able to account for the n-^{17}B virtual state, the ^{19}B ground state, and two broad resonances without introducing any three-body force. Such a possibility is certainly due to the strong resonant character of the n-^{17}B and n-n channels as well as the small binding energy and large spatial extension of ^{19}B. In the spin-independent approximation, ^{19}B is bound for $a_S < -30$ fm. Its binding energy is compatible with the experimental value for all the range of a_S until the unitary limit $a_S \to -\infty$.

We have considered the stability of our ^{19}B results when a spin-spin interaction term is introduced in $V_{n^{17}\mathrm{B}}$. By fixing a resonant value in the $S = 2$ channel ($a_2 = -150$ fm), we found that the system remains bound for a large domain of the $S = 1$ scattering length $a_1 < a_1^c < 0$. The existence of this negative critical value a_1^c, beyond which the binding disappears, is independent of the model parameters, and its particular value depends only slightly on them. One has typically $a_1^c \approx -5$ fm. Such a large asymmetry of a_S, however, requires unphysically large differences among the potential strengths of the two spin channels.

Work is in progress to extend the application of this model to describe the recently observed resonances in ^{20}B and ^{21}B [8] in terms of ^{17}B + 3n and +4n, respectively, by solving the corresponding four- and five-body equations using the techniques developed in Ref. [25]. On the other hand, the precise measurement of the main experimental observables related to this problem, such as the ^{18}B scattering length, the ^{19}B binding energy, and the energy and quantum numbers of the first ^{19}B resonant states, would provide a more stringent constraint to the parameters of the present model, and could eventually suggest the introduction of n-^{17}B P waves and/or ^{17}B-n-n three-body forces.

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[1] L. Koester, H. Rauch, and E. Seymann, At. Data Nucl. Data Tables **49**, 65 (1991).
[2] H. Al Falou *et al.*, J. Phys.: Conf. Ser. **312**, 092012 (2011).
[3] H. T. Johansson *et al.*, Nucl. Phys. A **842**, 15 (2010).
[4] A. Spyrou *et al.*, Phys. Lett. B **683**, 129 (2010).
[5] T. Suzuki *et al.*, Nucl. Phys. A **658**, 313 (1999).
[6] M. V. Zhukov *et al.*, Phys. Rep. **231**, 151 (1993).
[7] L. Gaudefroy *et al.*, Phys. Rev. Lett. **109**, 202503 (2012).
[8] S. Leblond *et al.*, Phys. Rev. Lett. **121**, 262502 (2018).
[9] I. Mazumdar, V. Arora, and V. S. Bhasin, Phys. Rev. C **61**, 051303(R) (2000).
[10] M. T. Yamashita *et al.*, Phys. Lett. B **697**, 90 (2011).
[11] B. Acharya, C. Ji, and D. R. Phillips, Phys. Lett. B **723**, 196 (2013).
[12] G. Blanchon, A. Bonaccorso, and N. Vinh-Mau, Nucl. Phys. A **739**, 259 (2004)
[13] G. Blanchon, A. Bonaccorso, D. M. Brink, A. García-Camacho, N. Vinh Mau, Nucl. Phys. A **784**, 49 (2007)
[14] J. Gibelin *et al.*, in *Proceedings of the XXII International Conference on Few-Body Problems in Physics, Caen (France) july 2018*, Springer Proceedings in Physics (Springer Nature, Switzerland, 2019).
[15] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[16] R. A. Malfliet and J. A. Tjon, Nucl. Phys. A **127**, 161 (1969).
[17] R. Lazauskas and J. Carbonell, Phys. Rev. C **71**, 044004 (2005).
[18] R. Lazauskas and J. Carbonell, Phys. Rev. C **72**, 034003 (2005).
[19] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).
[20] S. König, H. W. Grießhammer, H.-W. Hammer, and U. van Kolck, Phys. Rev. Lett. **118**, 202501 (2017).
[21] E. Hiyama and M. Kamimura, Nucl. Phys. A **588**, c35 (1995); E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Phys. Rev. C **53**, 2075 (1996).
[22] P. Naidon and S. Endo, Rep. Prog. Phys. **80**, 056001 (2017).
[23] D. Hove, E. Garrido, P. Sarriguren, D. V. Fedorov, H. O. U. Fynbo, A. S. Jensen, and N. T. Zinner, Phys. Rev. Lett. **120**, 052502 (2018).
[24] A. Kievsky and M. Gattobigio, Phys. Rev. A **87**, 052719 (2013).
[25] R. Lazauskas, E. Hiyama, and J. Carbonell, Phys. Lett. B **791**, 335 (2019).