Distortion effects in $\alpha + \alpha$ system*

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α-particle specific distortion effects in the $\alpha + \alpha$ system are examined by the addition of square-integrable distortion functions into the usual resonating-group formulation. The results indicate that such effects are not too important and can be neglected in calculations on other light nuclear systems.

[NUCLEAR REACTIONS ^{4}He(α,α), E=0–20 MeV; calculated phase shifts. Resonating-group method with specific distortion effects.]

I. INTRODUCTION

In a recent publication,[1] hereafter referred to as TT, we have discussed the results of our investigation on specific distortion effects in the $d+\alpha$ system. In that investigation, we have examined these distortion effects by introducing a sum of square-integrable distortion functions with linear variational amplitudes into the usual single-channel resonating-group formulation. The results showed that specific distortion effects do have a significant influence on the behavior of the $d+\alpha$ system. In particular, they exhibit an odd-even character; that is, they are quite important in even orbital angular-momentum states, but much less so in odd orbital angular-momentum states. Also, with the inclusion of these effects, the $d+\alpha$ experimental data can be well explained using a nucleon-nucleon potential which fits low-energy two-nucleon scattering data and which yields satisfactory agreement with the empirical $\alpha+\alpha$ phase-shift result.

The square-integrable functions used in TT were chosen to have a $d+\alpha$ cluster structure, with the distorted deuteron cluster having an rms radius either smaller or larger than that of a free deuteron and with the α cluster having the free α rms radius. No explicit provision for specific distortion of the α cluster was included, since it was felt that the high rigidity of the α particle renders its distortion quite small compared to that of the deuteron. A brief examination of this omission was discussed in TT; however, the exact extent to which α-particle specific distortion effects are important is not certain. Therefore, it is the purpose of the present investigation to examine these effects in detail in the $\alpha+\alpha$ system. We have chosen to examine these effects in this particular system rather than the $d+\alpha$ system, since the analysis in the $\alpha+\alpha$ system requires much less computational effort.

II. FORMULATION

The formulation of the present study is quite similar to that described in TT. The wave function of the $\alpha+\alpha$ system is assumed to be of the form

$$\Psi_N = \Psi_0 + \sum_{i=1}^{N} \mathcal{A}\left[\phi_{1i}\phi_{2i}G_i(\vec{R}_1 - \vec{R}_2)\xi(s,t)\right], \quad (1)$$

where $\vec{R}_1$ and $\vec{R}_2$ denote the c.m. coordinates of the two α clusters, respectively, and

$$\Psi_0 = \mathcal{A}\left[\phi_1\phi_2 F(\vec{R}_1 - \vec{R}_2)\xi(s,t)\right] \quad (2)$$

is the usual no-distortion wave function. In Eqs. (1) and (2), $\mathcal{A}$ is the antisymmetrization operator, $\xi(s,t)$ is an appropriate spin-isospin function, and ϕ_1 and ϕ_2 represent the spatial part of the free α-particle wave function. The function ϕ_1 is chosen as

$$\phi_1 = \exp\left[-\tfrac{1}{2}\alpha_1 \sum_{j=1}^{4} (\vec{r}_j - \vec{R}_1)^2\right] + c\exp\left[-\tfrac{1}{2}\alpha_2 \sum_{j=1}^{4} (\vec{r}_j - \vec{R}_1)^2\right] \quad (3)$$

and similarly for ϕ_2. The parameters α_1, α_2, and c are determined by minimizing the α-particle internal energy. For a nucleon-nucleon potential of the form

$$V_{ij} = \left(\frac{1+P^\sigma_{ij}}{2}V_t + \frac{1-P^\sigma_{ij}}{2}V_s\right)\left(\frac{u}{2} + \frac{2-u}{2}P^r_{ij}\right), \quad (4)$$

where

$$V_t = -V_{0t}\exp(-\kappa_t r_{ij}^2), \qquad V_s = -V_{0s}\exp(-\kappa_s r_{ij}^2), \quad (5)$$

with

$$V_{0t} = 66.92 \text{ MeV}, \quad \kappa_t = 0.415 \text{ fm}^{-2}, \qquad V_{0s} = 29.05 \text{ MeV}, \quad \kappa_s = 0.292 \text{ fm}^{-2}, \quad (6)$$

the values so determined are

$$\alpha_1 = 0.5248 \text{ fm}^{-2},$$
$$\alpha_2 = 1.2328 \text{ fm}^{-2}, \quad (7)$$
$$c = 6.2375.$$

These values yield an α-particle internal energy of -34.70 MeV and an rms radius of 1.21 fm.[2] The functions ϕ_{1i} and ϕ_{2i} are chosen as

$$\phi_{1i} = \exp\left[-\tfrac{1}{2}\tilde{\alpha}_i \sum_{j=1}^{4} (\vec{r}_j - \vec{R}_1)^2 \right],$$
$$\phi_{2i} = \exp\left[-\tfrac{1}{2}\tilde{\alpha}_i \sum_{j=5}^{8} (\vec{r}_j - \vec{R}_2)^2 \right], \quad (8)$$

with the choice of $\tilde{\alpha}_i$ to be discussed below. The function $F(\vec{R}_1 - \vec{R}_2)$ describes the relative motion of the clusters and can have either bound-state or scattering asymptotic boundary conditions. As for the function $G_i(\vec{R}_1 - \vec{R}_2)$ in the distortion-function term of Ψ_N, we make, as in TT, the following expansion:

$$G_i(\vec{R}_1 - \vec{R}_2) = \sum_{l=0}^{\infty} \tilde{A}_{li} \frac{1}{r} g_{li}(r) P_l(\cos\theta), \quad (9)$$

where

$$g_{li}(r) = r^{n+1} \exp(-\tilde{\beta}_i r^2) \quad (10)$$

with

$$n = 4 \text{ for } l = 0, 2,$$
$$n = l \text{ for } l \geq 4.$$

The choice of $\tilde{\beta}_i$ will also be discussed below.

The variational amplitudes $\tilde{A}_{li}$ as well as the variational function $F(\vec{R}_1 - \vec{R}_2)$ are determined by solving the equation

$$\langle \delta\Psi_N | H - E' | \Psi_N \rangle = 0, \quad (11)$$

where E' is the total energy of the system composed of the internal energies of the two α particles and their relative energy E in the c.m. frame. The Hamiltonian in Eq. (11) has the form

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^{8} \nabla_i^2 + \sum_{i<j=1}^{8} V_{ij} - T_{\text{c.m.}}, \quad (12)$$

where $T_{\text{c.m.}}$ is the center-of-mass kinetic-energy operator and V_{ij} is given by Eq. (4).

It should be noted from Eq. (4) that the Coulomb interaction has been omitted in this calculation in order to reduce the large amount of computer time required to handle the exchange Coulomb potential between the clusters. However, we feel that, because the Coulomb interaction is long ranged and weak compared to the nuclear interaction, it cannot have any significant influence on specific distortion effects in the $\alpha + \alpha$ system. Rather, its omission will merely result in a shift of the energy scale such that, for example, the $l = 0$ ground state of ^{8}Be will appear bound rather than as a resonance state.

III. RESULTS AND DISCUSSION

The procedure for choosing the nonlinear parameters $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ in the distortion functions is the same as that discussed in TT. Briefly, what we do is to examine the $l = 0$ partial wave with a single distortion function and determine the $(\tilde{\alpha}_1, \tilde{\beta}_1)$ value which yields the greatest separation energy in the ground state of ^{8}Be. A value of $u = 0.87$ is used in the nucleon-nucleon potential of Eq. (4), since a no-distortion calculation [Ψ_0 only in Eq. (1)] yields a separation energy for the ^{8}Be ground state of 2.14 MeV with this value for u. This is roughly the separation energy one would expect in the absence of the Coulomb interaction. Using this value of u and $N = 1$ in Eq. (1), we find two local maxima corresponding to $(\tilde{\alpha}_1, \tilde{\beta}_1)$ values of (0.55, 0.70 fm^{-2}) and (1.20, 0.75 fm^{-2}) and separation energies of 2.19 and 2.41 MeV, respectively.

In the case where more than one distortion function is used, it is a tedious matter to vary all the nonlinear parameters. Therefore, as in TT, we use the procedure of choosing a set of $(\tilde{\alpha}_i, \tilde{\beta}_i)$ values such that regions near the one-distortion-function maxima are covered. We have examined many such sets to make certain that the resultant choice is in fact appropriate, and we did find that when enough distortion functions are used, the particular choice of $(\tilde{\alpha}_i, \tilde{\beta}_i)$ set is not critical. In Table I, we have listed the $(\tilde{\alpha}_i, \tilde{\beta}_i)$ values for four different sets or configurations which we examined. Configurations III and IV are five-distortion-function configurations chosen to favor the one-distortion-function maxima at (0.55, 0.70 fm^{-2}) and (1.20, 0.75 fm^{-2}), respectively. Configuration II is a nine-distortion-function configuration chosen to

TABLE I. Values of parameters $\tilde{\alpha}_i$ and $\tilde{\beta}_i$, in fm^{-2}, in various distortion-function configurations.

	Distortion-function configuration							
Index	I		II		III		IV	
i	$\tilde{\alpha}_i$	$\tilde{\beta}_i$	$\tilde{\alpha}_i$	$\tilde{\beta}_i$	$\tilde{\alpha}_i$	$\tilde{\beta}_i$	$\tilde{\alpha}_i$	$\tilde{\beta}_i$
1	1.50	0.80	1.20	0.75	0.65	0.70	1.50	0.80
2	0.90	0.80	0.90	0.75	0.45	0.70	0.90	0.80
3	1.20	0.65	1.20	0.60	0.55	0.55	1.20	0.65
4	1.20	0.95	1.50	0.75	0.55	0.85	1.20	0.95
5	1.20	0.80	1.20	0.90	0.55	0.70	1.20	0.80
6	0.65	0.70	0.90	0.90				
7	0.45	0.70	0.90	0.60				
8	0.55	0.55	1.50	0.60				
9	0.55	0.85	1.50	0.90				
10	0.55	0.70						

favor the one-distortion-function maximum at ($1.20\ \text{fm}^{-2}$, $0.75\ \text{fm}^{-2}$), while configuration I is a ten-distortion-function configuration chosen to favor both one-distortion-function maxima. The value of the separation energy E_B in the ^{8}Be ground state (using $u = 0.87$) corresponding to each of these configurations is given in Table II. One can see from this table that, except for configuration III, the value of E_B is rather insensitive to the choice of which particular one of these configurations is used.

In Fig. 1, the convergence property of configuration II is illustrated. This figure shows E_B as a function of the number N of distortion functions used. The values of E_B for $N = 0, 3, 6, 9$ are equal to 2.136, 2.416, 2.431, 2.438 MeV, respectively, thus indicating that the use of these nine distortion functions is enough and the addition of more distortion functions will have a very minor effect. Such an examination for configuration I has also been made and a similar result has been obtained.

Our discussion of distortion-function configurations is based on the results of $l = 0$ bound-state calculations. However, the relative insensitivity of these results to the particular nature of the configuration indicates to us that positive-energy ($E > 0$) calculations with l different from zero will also be satisfactory, if the distortion configuration spans a large enough region of the $(\tilde{\alpha}, \tilde{\beta})$ space. In fact, using configuration I, we have examined the behavior of the $l = 0$, 2, and 4 phase shifts as a function of the number of distortion functions, and have found a convergence behavior quite similar to that obtained for the $l = 0$ bound-state calculation. Consequently, we have decided to use configuration I to study α-particle specific distortion effects for all states in the $\alpha + \alpha$ system.

In Fig. 2, we have plotted the $l = 0$, 2, and 4 $\alpha + \alpha$

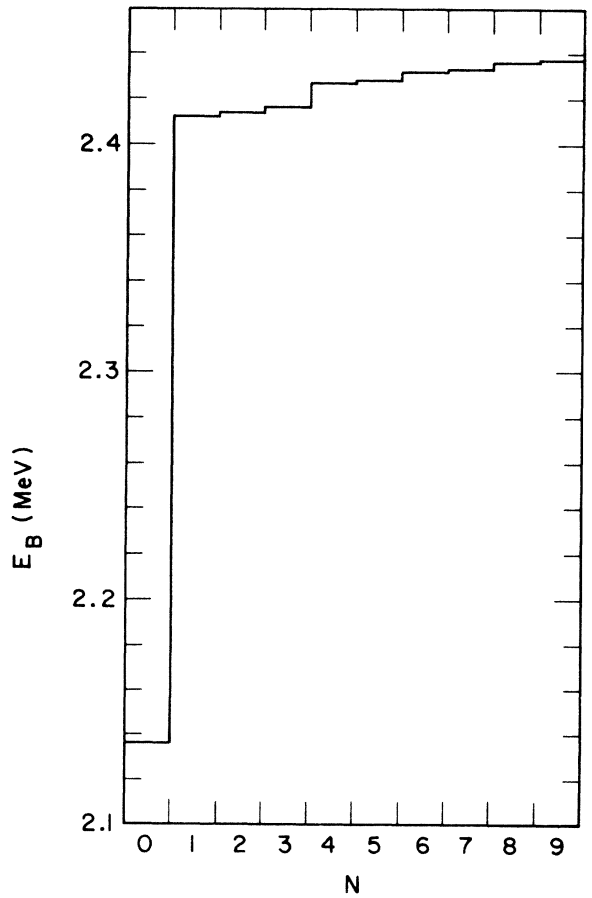

FIG. 1. $\alpha + \alpha$ separation energy as a function of the number of distortion functions in configuration II.

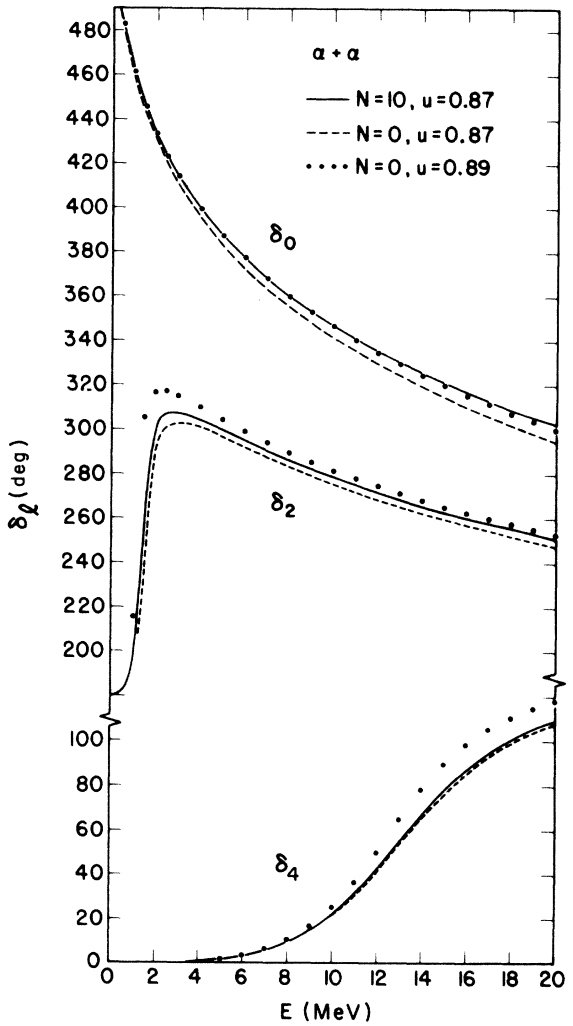

FIG. 2. $\alpha + \alpha$ phase shifts as a function of energy for $l = 0$, 2, and 4. The solid curves show phases calculated using ten distortion functions of configuration I and $u = 0.87$. The dashed curves and the solid dots show phases calculated using no distortion and $u = 0.87$ and 0.89, respectively.

TABLE II. Variation of E_B with various distortion-function configurations.

Distortion-function configuration	E_B (MeV)
I	2.44
II	2.44
III	2.21
IV	2.43

phase shifts for c.m. energies from 0 to 20 MeV. The solid curves show the results calculated with the ten-distortion-function configuration I and $u = 0.87$. The dashed curves and the solid dots represent the no-distortion results with $u = 0.87$ and 0.89, respectively. One can see from this figure that the inclusion of distortion functions into the $\alpha + \alpha$ calculation with $u = 0.87$ yields results approximately equivalent to those obtained in a no-distortion calculation with $u \approx 0.89$ for $l = 0$, $u \approx 0.88$ for $l = 2$, and $u \approx 0.875$ for $l = 4$. The difference between these "effective" u values and the value of $u = 0.87$ is a rough measure of the importance of specific distortion in the $\alpha + \alpha$ system. In this respect, it is interesting to note that in the $d + \alpha$ calculation of TT, a typical value for this difference was 0.2 for $l = 0$, which is much larger than the value of 0.02 for $l = 0$ in the present case. For $l > 0$, the effect of distortion is even smaller, with essentially no effect in $l = 4$ states.

One may argue that the reason for these rather small distortion effects is related to the fact that our choice of the α-particle wave function ϕ_1 yields an rms radius which is substantially smaller than the experimental value. To make sure that this is not the case, we have made a similar calculation using a two-nucleon potential with the parameters in Eq. (5) given by

$$V_{0t} = 37.22 \text{ MeV}, \quad \kappa_t = 0.20 \text{ fm}^{-2},$$
$$V_{0s} = 29.05 \text{ MeV}, \quad \kappa_s = 0.292 \text{ fm}^{-2}. \tag{13}$$

This potential is designed to yield an α-particle rms matter radius of 1.48 fm, which is consistent with experimental electron-scattering data.[3] Using this potential and a two-Gaussian spatial function[4] having the form of Eq. (3), we obtain a minimum internal energy $E_\alpha = -28.58$ MeV with the parameters

$$\alpha_1 = 0.3638 \text{ fm}^{-2},$$
$$\alpha_2 = 0.8278 \text{ fm}^{-2}, \tag{14}$$
$$c = 6.4981.$$

The procedure discussed in the preceding paragraphs was then repeated using this particular potential with $u = 0.87$. The results obtained were very similar to those obtained using the more realistic nucleon-nucleon potential of Eqs. (4)–(6). For example, the ^{8}Be separation energy was found to be 2.47 MeV using ten distortion functions, as compared to 2.20 MeV with no distortion. Thus, we feel that our findings regarding specific distortion effects in the $\alpha + \alpha$ system are reliable.

In conclusion, we have found from this investigation that α-particle specific distortion effects are relatively unimportant in the $\alpha + \alpha$ system[5] and, therefore, the neglect of such effects is substantially justified in other light nuclear systems, such as the $d + \alpha$ system.

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[1]D. R. Thompson and Y. C. Tang, Phys. Rev. C 8, 1649 (1973).

[2]By choosing the function ϕ_1 as the sum of three Gaussian functions, we find only a 0.07 MeV decrease in internal energy and a negligible change in rms radius. Therefore, the choice of a two-Gaussian function for ϕ_1 or ϕ_2 is sufficient.

[3]For a discussion of the extraction of the matter radius from electron-scattering data, see Appendix A in J. A. Koepke, R. E. Brown, Y. C. Tang, and D. R. Thompson, Phys. Rev. C 9, 823 (1974).

[4]For this potential we find that the use of a two-Gaussian function for ϕ_1 or ϕ_2 is again sufficient.

[5]Specific distortion effects in $l = 0$ states have also been examined by L. C. Niem, P. Heiss, and H. H. Hackenbroich, Z. Phys. 244, 346 (1971), and a similar conclusion has been reached.