Isospin-forbidden alpha decay of the 15.11-MeV state in ^{12}C†

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The branching ratio $\Gamma_\alpha/\Gamma_{total}$ has been measured for the 15.11-MeV 1^+ $T=1$ state in ^{12}C. The result is $\Gamma_\alpha/\Gamma_{total} = (4.1 \pm 0.9)\%$, in agreement with one previous measurement and in disagreement with another. The result was obtained by direct observation of the decay α particles from the reaction $^{13}C(^3He, \alpha)^{12}C^* \to 3\alpha$. The data are consistent with a sequential decay via the first excited state of ^8Be. The 3α breakup of the 1^+ $T=0$ state at $E_x = 12.71$ MeV was also studied. The data in both cases were fitted with a model in which the α-α final state interaction was treated as a simple Breit-Wigner resonance; order-of-emission interference effects were included. If the (isospin-forbidden) α width is interpreted as resulting from mixing between the two 1^+ states via a charge-dependent interaction, a large isospin-violating matrix element is involved.

[NUCLEAR REACTIONS $^{13}C(^3He, \alpha)^{12}C^*(\alpha_1)^8Be^*(\alpha_2)\alpha_3$, $E=15$ MeV, α-α coincidence, deduced $\Gamma_\alpha/\Gamma_{total}$ for 15.11-MeV level of ^{12}C.]

I. INTRODUCTION

A number of workers [1-7] have considered the question of isospin mixing in the two lowest 1^+ states in ^{12}C recently. These two states (1^+ $T=0$ at 12.71 MeV, 1^+ $T=1$ at 15.11 MeV) form a particularly good case for the study of isospin mixing for a number of reasons. First, the states themselves are believed to be describable in terms of simple configurations; this means that the effects of the Coulomb force can be estimated in a reasonably straightforward way; this is clearly essential if we are attempting to isolate an isospin-violating component in the *nuclear* force. An additional motivation for studying these states stems from the fact that no other 1^+ states are known in ^{12}C. One therefore has the hope that the problem can be treated within the framework of two-state mixing. This assumption means that we treat the physical 1^+ states as linear combinations of two different eigenstates of a charge-independent Hamiltonian, say $|T=0\rangle$ and $|T=1\rangle$. Then

$$\begin{aligned} |15.11\rangle &= (1-\beta^2)^{1/2}|T=1\rangle + \beta|T=0\rangle, \\ |12.71\rangle &= -\beta|T=1\rangle + (1-\beta^2)^{1/2}|T=0\rangle, \end{aligned} \tag{1}$$

where β is related to the charge-dependent matrix element connecting the two states:

$$\beta = \frac{\langle T=1|H_{CD}|T=0\rangle}{E_1 - E_0} \ . \tag{2}$$

It is important to realize that the charge-dependent matrix element can only be extracted from a measurement of the $T=0$ impurity in the 15.11-MeV state if the assumption (1) is made. As an illustration of why this assumption is important one might point out that the second $T=1$ state in ^{12}C ($E_x = 16.11$ MeV, $J^\pi = 2^+$) has been known[8] for some time to have a width for α decay of 6 keV. It is difficult to interpret this result, however, without knowing the location and α widths of all the 2^+ $T=0$ states in ^{12}C. In the case of the two 1^+ states in ^{12}C the assumption of two-state mixing has always been made[1,2,4,6]; however, recent experiments[9] have suggested the existence of a third broad 1^+ state at an excitation energy of about 19 MeV. If this suggestion proves correct, it may be necessary to include the effects of this state in any discussion of the isospin purity of these states.

The problem of determining the $T=0$ admixture in the 15.11-MeV state has been studied experimentally by several methods. Recently Braithwaite, Cecil, Bussoletti, and Garvey[1] excited the two 1^+ states as final states in the (d, d') reaction. They interpreted the small yield to the 15.11-MeV state as evidence for a $T=0$ impurity in that state. The further assumption that mixing with the 12.71-MeV state is responsible for the isospin violation leads to a charge-dependent matrix element connecting the two states of 250 keV. This is to be compared with an estimate of 50 keV based on pure Coulomb forces (see Ref. 4). This result, of course, assumes an isospin-conserving reaction mechanism. It has recently been pointed out by Iachello and Singh[10] that, in second order, the (d, d') reaction can proceed via processes like $(d, t)+(t, d)$ or $(d, ^3He)+(^3He, d)$. The Coulomb force introduces an asymmetry between these two processes and consequently an isospin-changing amplitude can exist without the necessity to invoke isospin mixing in either the initial or final states. While no completely realistic calculations of the effect have been performed to date, an estimate[10] based on δ-function interactions produced an effect comparable to that reported in Ref. 1. In addition, a search[11] for the population of the 15.11-MeV state in the ^{14}N(d, α)-^{12}C reaction at $E_d = 40$ MeV yielded a negative result with a limit about half the size of the effect reported in Ref. 1.

Braithwaite *et al.* also looked at the single-nucleon transfer spectroscopic factors leading to two members of the $T=1$ multiplet using the ^{13}C(d, t)^{12}C and ^{13}C(d, ^{3}He)^{12}B reactions. The attractive feature of this method is that interference in the pickup reaction between the $T=0$ and $T=1$ components of the state in ^{12}C leads to an effect which is linear in the mixing amplitude β. If one assumes that the distorted-wave Born approximation is capable of handling the kinematic differences between the two states (as well as all charge-dependent effects in the form factors and distorted waves) then any remaining departure from the predicted ratio of two for the (d, ^{3}He) and (d, t) cross sections leading to the ground state of ^{12}B and the 15.11-MeV state of ^{12}C respectively can be attributed to admixture of a $T=0$ impurity in the latter state. Braithwaite *et al.* do indeed observe such a residual discrepancy in the spectroscopic factors; their analysis implies a charge-dependent matrix element of about 250 keV, in agreement with the (d, d') result. However, it should be pointed out there are still a number of questions relating to the model dependence of this result. For example, Cotanch and Philpott[12] have calculated the ratio of (d, ^{3}He) and (d, t) cross sections leading to analog states in the mass-18 system and have found that effects of the same order of magnitude as those found by Braithwaite *et al.* can be obtained by including coupled channels effects using the Lane $\vec{t}\cdot\vec{T}$ potential. They further conclude from additional microscopic calculations that subtleties of nuclear structure can play an important role here.

Additional evidence concerning isospin mixing in the mass-12 system is provided by the recent measurement by Cecil *et al.* of the $M1$ matrix element connecting the 12.71-MeV $T=0$ state with the ground state.[2] The strength of this matrix element is too large to be accounted for by the isoscalar $M1$ transition probability calculated from intermediate coupling wave functions.[13] These calculations reproduce other electromagnetic transition probabilities in the $0p$ shell quite well. If the extra $M1$ strength is attributed to mixing of the 12.71 MeV state with the 15.11 MeV $T=1$ 1^+ state (which has a very strong isospin-allowed $M1$ decay to the ^{12}C ground state) then the observed matrix element can be used to infer the amount of mixing. Because of a phase uncertainty two solutions are obtained from this (model-dependent) argument. One of these ($H_{\rm CD} = 120 \pm 24$ keV) is significantly smaller than the result of Braithwaite *et al.*, whereas the other solution ($H_{\rm CD} = 456 \pm 24$ keV) is significantly larger.

The problem of the isospin purity of these two 1^+ states in ^{12}C is further complicated by the results of Adelberger *et al.*,[3] who have searched for the isospin forbidden proton decay ^{13}N ($E_x = 15.1$, $T=\frac{3}{2}$) $\rightarrow p + {}^{12}$C($E_x = 12.71$). The presence of this decay would indicate a $T=1$ admixture in the 12.71-MeV state; if one accepts the idea of two-state mixing this constitutes a direct measurement of β. Accordingly, the absence of this decay implies a limit on β; the limit obtained in Ref. 3 indicates that β^2 is less than $\frac{1}{3}$ of the value obtained by Braithwaite *et al.*

An additional method for investigating the isospin purity of these levels is to examine their particle decays. The 12.71-MeV state decays 97% into the isospin-allowed channel $\alpha + {}^8$Be(2^+).[4] The absolute width for this decay is determined as (14.18 $\pm$ 2.8) eV from the recent measurement[2] of the ground state $M1$ decay in combination with known[8] branching ratios; this α width is extremely small compared to a single-particle estimate. This is basically understood[14] to be the result of the orbital symmetry of the wave function. (The 1^+ state can only decay to the component of the ^{8}Be 2^+ state with the [31] orbital symmetry.) Despite the small α width of the 12.71-MeV level, in the absence of other known 1^+ states it is tempting to associate any (isospin-forbidden) α decay from the 15.11-MeV level with mixing between the two states as a result of charge-dependent forces.

The study of the α decay of the 15.11-MeV state is also important because of the presence in the literature of a number of contradictory experimental results. The α decay of this level was first investigated by Alburger and Donovan in 1965.[5] No branch was observed, and an upper limit of 5% was obtained for any decay involving an α particle in the final state. In 1970 the α decays of the 12.71- and 15.11-MeV states were studied by Reisman, Connors, and Marion.[4] The states were populated with the ^{13}C(^{3}He, α)^{12}C reaction at a bombarding energy of 2.5 MeV; the decay α particles were observed in time coincidence. These workers also investigated the radiative decays of both states. A knowledge of the radiative decays of the 15.11-MeV state is essential for interpreting the spectrum of α particles resulting from its decay because of the existence of a γ transition connecting the 15.11-MeV state to the 12.71-MeV state. Consequently the α particles from the breakup of the 12.71-MeV state are in prompt coincidence with any radiations forming the 15.11-MeV state, and their presence must be taken into account.

Reisman *et al.* obtained values of $(1.2 \pm 0.7)\%$ and $(0.7 \pm 0.4)\%$ for the branching ratios of the 15.11-MeV level for α decay and γ decay to the 12.71-MeV state, respectively. More recently, the γ-decays of the 15.11-MeV state have been reinvestigated by Alburger and Wilkinson.[7] They obtained a value of $(1.4 \pm 0.4)\%$ for the decay to the 12.71-MeV state, a factor of 2 larger than that found by Reisman *et al.*,[4] although still in agreement within the quoted experimental errors. However, if the result of Alburger and Wilkinson is correct, the resulting increase in the α-particle background from the decay through the 12.71-MeV level is sufficient to call into question the existence of the direct branch observed in Ref. 4.

The experimental results of Reisman *et al.* are also in contradiction with the work of Artemov *et al.*,[6] who found that the decay of the 15.11-MeV level produced an α particle with a branching ratio of $(6 \pm 2.5)\%$. This is to be compared with the value of $(1.9 \pm 0.8)\%$ obtained in Ref. 4 for the sum of the direct α branch and the decay through the 12.71-MeV level.

A primary motivation of the present work was to resolve these contradictions by making a more precise measurement of the fractional α-particle decay width (Γ_α/Γ) for the 15.11-MeV level. The method employed is similar to that of the earlier workers. The state was formed using the $^{13}C(^3He, \alpha)^{12}C$ reaction; one of the three α particles resulting from the breakup of the excited ^{12}C nucleus was detected in time coincidence with the α particle signaling the formation of the state. The experimental procedure is described in detail in Sec. II.

The spectrum of α particles resulting from the decay of either of the excited ^{12}C 1^+ states exhibits a complex shape which, in general, depends on the mechanism of the decay. For simultaneous three-body breakup this spectrum would reflect the three-body phase space; if a sequential decay through the 2^+ state in ^8Be is assumed (the decay to the ^8Be ground state is forbidden by conservation of parity) the spectrum depends on the parameters describing the final state interaction (width and energy of the 2^+ state in ^8Be and angular momentum quantum numbers).

While in the present study the primary interest is focused on the α particle branching ratio of the 15.11-MeV level, it was also felt that an attempt to understand the observed spectrum shapes was desirable. In particular, a method of calculating the expected spectrum shape is necessary if the effect of the indirect branch through the 12.71-MeV state is to be accounted for. It would be incorrect, for example, to use the experimentally observed spectrum for the decay of the 12.71-MeV state as a correction to the data for the decay of the 15.11-MeV state. This is a consequence of the fact that when the 12.71-MeV state is populated using the $(^3He, \alpha)$ reaction at a bombarding energy of 15 MeV the recoiling ^{12}C has more than twice the kinetic energy of a ^{12}C produced in the 15.11-MeV state. Those α decays of the 12.71-MeV state which result from γ feeding from the higher 1^+ state thus have considerably more center-of-mass motion; this can only be correctly accounted for if one has a model describing the decay process. In addition, it is clear that a necessary condition that the α width of the 15.11-MeV state be attributable to mixing with the 12.71-MeV state is that both decays proceed via the same mechanism. It is equally clear that this condition alone is not sufficient.

During the analysis of the data it was recognized that the angular momenta involved in the sequential decay through the ^8Be 2^+ state $(1^+ \rightarrow 2^+ \rightarrow 0^+)$ provide an unusually clear-cut example for the study of final state interactions in the 3α system; this results both from the unique orbital angular momenta involved in each decay step and the ability to form the initial state with known alignment in a very clean way.

An example of the kind of information one might hope to acquire concerning the dynamics of the decay process is provided by the work of Aaron and Amado,[15] who have pointed out that conventional analyses of three-body decays involving interfering isobar pairs violate unitarity. When the 3α decay channel is not inhibited it is possible to measure the energy spectra of the decay products with sufficient precision to test various descriptions of the decay process such as the one proposed in Ref. 15. A modest attempt in this direction has been made in the present work in that the symmetrization of the final state wave function has been included in the analysis. The specific effects suggested by the work of Ref. 15 (essentially an energy dependence of the reduced width dictated by the requirement of unitarity) have been neglected. These effects are expected to be small in the present case (they are essentially proportional to the order-of-emission interference effects described in Sec. III), and will certainly not change any of the conclusions concerning the total α-decay width of the 15.11-MeV state, which is the main focus of attention here. However, more experiments are currently planned as part of our investigation of the mechanism of the decay; a detailed account of this aspect of the present work will be deferred to a future publication.

II. EXPERIMENTAL METHOD

The method used in the present work consisted of studying the reaction

$$^{13}C(^3He, \alpha_0)^{12}C^* \to \alpha_1 + \alpha_2 + \alpha_3 .$$

Experimentally α_0 and any of α_1, α_2, or α_3 were detected in time coincidence. The reaction ^{13}C-(^3He, α)^{12}C was chosen because it strongly populates the 1^+ states at 12.71 and 15.11 MeV at a convenient bombarding energy (15 MeV was used). The large cross sections to these states result from the fact that the dominant configuration of both states is a $p_{3/2}$ hole coupled to a $p_{1/2}$ particle. The (^3He, α) reaction thus picks up a neutron from the filled $p_{3/2}$ shell and a large cross section results. The kinematics of the (^3He, α) reaction imply that the angular distribution of α particles from the (^3He, α) reaction will be strongly forward peaked. Since in the present experiment we are searching for a small decay branch from the 15.11-MeV state it is important that the state be prepared in as background-free a way as possible. In the present work a position-sensitive detector located in the focal plane of a magnetic spectrometer was used to detect the α particles from the $^{13}C(^3He, \alpha)^{12}C$ reaction (α_0) at zero degrees with respect to the beam. The magnetic spectrometer makes possible a high resolution energy measurement at forward angles (where the direct yield is large) without the complication of a large flux of elastically scattered particles. The good energy resolution of the magnetic spectrometer system is quite important in this experiment since the natural width of the 15.11-MeV level is much less than the experimental energy resolution. Since it is essential to distinguish the α decay of the level itself from the α decay of the underlying continuum background, an improvement in the energy resolution in the α_0 spectrum directly results in a corresponding increase in the signal-to-noise ratio.

In addition to good energy resolution the magnetic spectrometer has a comparatively large solid angle ($\Omega = 8 \times 10^{-3}$ sr). Since we are searching for a small decay branch, the coincidence yield should be optimized by using the maximum feasible solid angle for both the α_0 detector and the (moveable) detector which records α_1, α_2, or α_3. This second detector consisted of a large area surface-barrier detector with a diameter of 1.90 cm which was placed 4.8 cm from the target. The solid angle subtended by this detector was thus almost 1% of the full sphere. The apparatus itself is illustrated in schematic form in Fig. 1. The ^{13}C target was a self-supporting foil of thickess 150 $\mu g/cm^2$. The measured energy resolution for α_0 was approximately 40 keV.

Timing information was derived from the 0° position-sensitive detector using a two-level leading edge discriminator; for the moveable detector constant-fraction timing was used. The magnetic field in the spectrograph was adjusted to position the group of interest (either the 12.71- or 15.11-MeV state) near the end of the detector in order to minimize the pulse risetime and optimize the timing. The resolution obtained in the time spectrum was 6 nsec full width at half-maximum (FWHM).

For each coincidence event words corresponding to α_0 energy, α_0 momentum (position), α energy in the moveable detector, and the output of a time-to-amplitude converter were written on magnetic tape using a PDP-9 computer. Data reduction was performed off-line using the same computer. In addition to the coincidence events, the computer stored all events which produced a pulse in the zero degree detector (zero degree singles). The branching ratio can thus be obtained from the ratio of coincidence events to singles events independent (to first order) of dead time.

III. CALCULATION OF THE BREAKUP SPECTRA

In this section we discuss the complications which are introduced into the analysis as a result of the fact that the isospin-violating decay of the 15.11-MeV state leads to a three-body final state. Because we detect only one of the three α particles resulting from the ^{12}C breakup, the experiment is not kinematically complete. In other words,

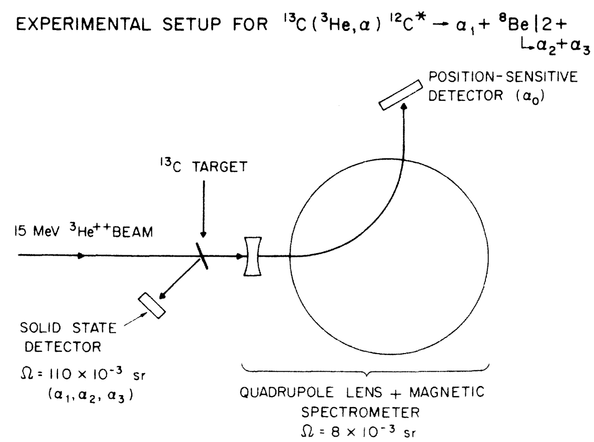

FIG. 1. Schematic diagram of the experimental apparatus used for the coincidence measurements.

the observed α spectrum contains contributions from a range of relative energies of the undetected pair. For purposes of describing the ^{12}C breakup it is convenient to begin in the system in which the ^{12}C is at rest (system center of mass, abbreviated SCM). In this system one of the α particles can have a kinetic energy ranging from 0 to $\frac{2}{3}$ of the available energy, which is given by

$$E_T = E_x(^{12}\mathrm{C}) - S_{3\alpha} ,$$

where $S_{3\alpha} = 7.275$ MeV is the energy at which ^{12}C becomes unbound to 3α decay. The decay into 3 α particles of states in ^{12}C above the proton threshold has been extensively studied by the Rice group[16] and others[17] using the $^{11}\mathrm{B} + p \rightarrow {}^{12}\mathrm{C}^* \rightarrow 3\alpha$ reaction. Sequential decays through states in ^{8}Be was the dominant mechanism observed in all of these studies. The basic ideas involved in analyzing data of this kind have been discussed by several authors.[18-20] The coordinate system in which the recoiling ^{8}Be is at rest will be denoted recoil center of mass (RCM). In the present case the decay to the ground state of ^{8}Be is forbidden by conservation of parity [since the initial states of interest in ^{12}C have $J^\pi = 1^+ \neq (-)^J$]. Consequently only decay through the first excited state of ^{8}Be ($J^\pi = 2^+$, $E_x \cong 2.9$ MeV) will be considered.

A kinematically complete description of the 3α breakup of ^{12}C requires the specification of five scalar variables; these can be conveniently considered to be the angles of emission of two of the α particles and the kinetic energy of one of them. Experimentally this would involve placing detectors at, say, (Θ_1, Φ_1) and (Θ_2, Φ_2) and requiring a coincidence between two of the three outgoing α particles. The spectrum observed in the present study can be considered as the integral over all allowed values of (Θ_2, Φ_2) of the decay probability $W(E_1, \Theta_1, \Phi_1, \Theta_2, \Phi_2)$. The expression for the decay probability contains all the dynamics of the sequential decay, including the angular correlation between α_1 and α_2 as well as the final state interactions between the two α particles comprising the ^{8}Be. In the present work the interaction in the α-α channel was taken to be a simple Breit-Wigner d-wave resonance. The parameters describing the resonance were taken from the α-α scattering data of Tombrello and Senhouse[21] ($R = 3.5$ fm, $\gamma^2 = 3.36$ MeV, $E_0 = 3.276$ MeV) where the resonance energy is measured with respect to the α-α system. None of the conclusions reached in this work depend on the details of this parametrization, however. The amplitude for the emission of α_1 at SCM angles (Θ_1, Φ_1) and α_2 at RCM angles (θ_2, φ_2) is proportional to

$$f = \sum_{m_b} (l m_a - m_b j_b m_b | j_a m_a) Y_l^{m_a - m_b}(\Theta_1, \Phi_1) Y_{l'}^{m_b}(\theta_2, \varphi_2) \frac{[(\Gamma_1/\sqrt{E_1})(\Gamma_2/\sqrt{E_{23}})]^{1/2} e^{i(\omega_l - \varphi_l)} e^{i(\omega_{l'} - \varphi_{l'})}}{E_0 - \gamma_2^2[S_{l'}(E_{23}) - S_{l'}(E_0)] - E_{23} - \frac{1}{2} i \Gamma_2} , \quad (3)$$

where the following notation has been employed:

j_a = spin of state in ^{12}C

j_b = spin of ^{8}Be resonance

l = orbital angular momentum in the decay $^{12}\mathrm{C} \rightarrow \alpha + {}^8\mathrm{Be}$

l' = orbital angular momentum in the decay $^8\mathrm{Be} \rightarrow 2\alpha$

Γ_1 = partial width for the decay $^{12}\mathrm{C} \rightarrow \alpha + {}^8\mathrm{Be}$

Γ_2 = partial width for the decay $^8\mathrm{Be} \rightarrow 2\alpha$

E_1 = kinetic energy of α_1 in SCM

E_{23} = relative kinetic energy of α_2 and α_3

E_0 = resonance energy of the 2^+ state in ^{8}Be

$\omega_l - \varphi_l$ = Coulomb minus hard sphere phase shift

$S_{l'}$ = R-matrix shift function.

The partial widths Γ_1 and Γ_2 were assumed to be given by the R-matrix expression $\Gamma = 2P_l(E)\gamma^2$ where $P_l(E)$ is the penetrability describing the appropriate relative motion (either α-^{8}Be or α-α) and γ^2 is an energy-independent reduced width which describes the nuclear overlap. The partial widths are divided by the appropriate momenta to guarantee the correct threshold dependence. The R-matrix quantities P_l, $P_{l'}$, $S_{l'}$, ω_l, φ_l, $\omega_{l'}$, $\varphi_{l'}$ were calculated as a function of the appropriate energy in 200 keV increments and stored in a table. The appropriate values were then calculated as required using logarithmic interpolation. The α-^{8}Be penetrability was calculated using a radius of 5 fm. The reduced width for $^{12}\mathrm{C} \rightarrow \alpha + {}^8\mathrm{Be}$ (which serves simply as an over-all multiplicative factor) was arbitrarily set to unity. The amplitude (3) is transformed entirely into the SCM by multiplying by the appropriate Jacobian (see Ref. 18).

The amplitude must now be symmetrized in the coordinates of the three α particles. These order-of-emission interference effects have been well known in the ^{12}C system for some time and have been extensively studied by the Rice group and others.[16,17] Basically the point is that even a kinematically complete experiment cannot distinguish which of the three α particles is emitted first. For a given pair of angles $(\Theta_1\Phi_1)(\Theta_2\Phi_2)$ each of the three possible choices for the first emitted particle corresponds to a different relative energy E_{23} in the ^{8}Be system. Because the 2^+ state in ^{8}Be is wide, more than one of these amplitudes can have an appreciable value; because the α particles are bosons the symmetrized amplitude is obtained by summing the expressions (3) corresponding to the three possibilities illustrated in Fig. 2. The square of the resulting expression must then be summed over the initial spin projections in ^{12}C(m_a) weighted by the population parameters describing the alignment of the initial state. The calculated spectra are quite insensitive to this alignment; however, in the present work the alignment of the initial state in ^{12}C was determined by a separate measurement of the ground state γ-ray angular correlation from both the 12.71- and 15.11-MeV states at the same bombarding energy. Hence the correct value was used in all cases.

In order to obtain the spectrum as a function of E_1 for given $\Theta_1\Phi_1$ it is necessary to integrate over the unobserved quantities $\theta_2\varphi_2$. In the present work a computer program was written in which the decay probability was numerically integrated over the available phase space. The kinematically allowed region of angles was divided into an equal number of angular steps in θ_2 and φ_2 and the integration was performed using Simpson's rule. Confining the integration to the kinematically allowed region is important; near the endpoint of the spectrum of α_1 the recoiling the α_2-α_3 system is confined to a narrow cone. If the integral were carried out over all angles in coarse steps, severe numerical problems would result. The initial motion of the ^{12}C was taken into account by transforming to the ^{12}C rest system (SCM) before performing the integration. The final calculated spectra were normalized to the experimentally observed number of counts at the appropriate angle.

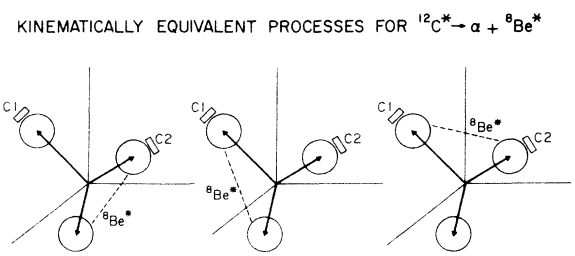

FIG. 2. Diagrams representing the three possible amplitudes which must be coherently added in calculating the breakup spectra.

IV. RESULTS

A. 12.71-MeV state

The α decay of this state was investigated primarily to test the over-all experimental system using a case where the branching ratio is well known.[4] In addition, the measurement served as a test of the model used to predict the observed α spectra. This latter aspect will not be stressed in the present work, although we will demonstrate that the model gives a reasonable account of the data. Because the decay of the 12.71-MeV state to the 2^+ state in ^{8}Be is the dominant decay for that state it was possible in a reasonable time period to accumulate data at several angles of observation of the decay α particles. In this connection, it is perhaps worthwhile to mention an interesting accident concerning the angular correlation of the α particles resulting from the ^{12}C breakup. Suppose we write the decay

$$^{12}\mathrm{C} \to \alpha_1 + {}^8\mathrm{Be}\,|\,2^+ \to \alpha_2 + \alpha_3$$

and suppose for the moment we treat the α particles as distinguishable. If we denote by $W(\hat{k}_1)$ the energy-integrated angular correlation of α_1 when α_2 and α_3 are unobserved, and $W(\hat{k}_2)$ = energy-integrated angular correlation of α_2 (in the SCM) when α_1 is unobserved, it happens that for the spin sequence $1^+ \to 2^+ \to 0^+$ the functions $W(\hat{k}_1)$ and $W(\hat{k}_2)$ are the same. This is a consequence of the fact that the $1^+ \to 2^+$ transition requires $L = 2$, which happens to equal the orbital angular momentum involved in the ^{8}Be breakup. If we now treat α_1

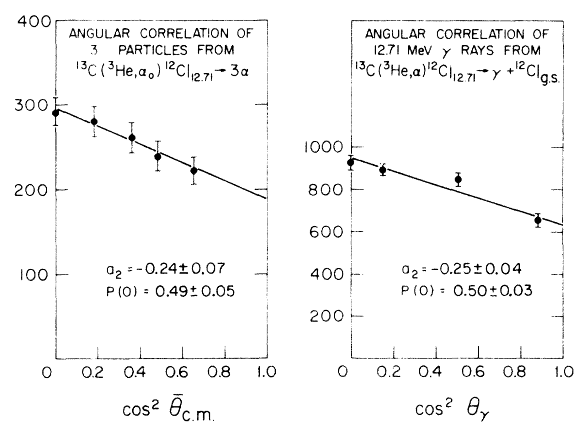

FIG. 3. Left: Angular correlation of all α particles emitted in coincidence with α_0 observed at 0° populating the 12.71-MeV state. Right: Angular correlation of 12.71-MeV γ rays in coincidence with α_0 observed at 0°. Note: The fact that the A_2 coefficients are equal in the two cases is an accident of angular momentum algebra.

and α_2 as indistinguishable we must add the amplitudes for these processes. But these amplitudes have the same angular dependence. Consequently order-of-emission interference effects cannot affect the angular correlation. This means that we can simply integrate the entire α spectrum as a function of angle; the resultant angular correlation should be given by what one would predict for a two-body final state involving the decay $1^+ \to 2^+$ via $L = 2$.

The only assumption involved in the foregoing is the existence of a sequential decay through the 2^+ state in ^{8}Be. For the 12.71-MeV state we have tested this idea. The results are shown in Fig. 3, where the total number of α particles observed as a function of angle are plotted versus angle in the center-of-mass system. The energy dependence of the lab $\to$ c.m. transformation was taken into account. The observed angular correlation depends only on the alignment of the initial state. A value of $P(0) = 0.49 \pm 0.05$ was obtained from a fit to the data. For comparison, the same quantity was extracted from a fit to the angular correlation for the γ decay of the 12.71-MeV state to the ^{12}C ground state. The ^{13}C(^{3}He, $\alpha\gamma$)^{12}C angular correlation was measured in a separate experiment at the same bombarding energy. The outgoing α particles were detected in the magnetic spectrometer; the γ rays were detected with an array of four 7.62 $\times$ 10.16 cm NaI (Tl) crystals. This γ ray has pure $M1$ character so the $\alpha_0 - \gamma$ angular correlation depends only on the alignment of the initial state. The result, $P(0) = (0.50 \pm 0.03)$ is in excellent agreement with that obtained from the α decay to the ^{8}Be 2^+ state.

The intensity of the α-decay branch was deduced from the zero-order coefficient obtained from fitting the angular correlation shown in Fig. 3 to the formula

$$W(\theta) = A_0[1 + a_2 P_2(\cos\theta)] .$$

The branching ratio is given by

$$\frac{\Gamma_\alpha}{\Gamma} = \frac{A_0}{\epsilon N_s} . \quad (4)$$

Here N_s is the number of singles, i.e. the number of times the 12.71-MeV state was populated. ϵ is an over-all efficiency which includes the detector solid angle and an estimate for coincidence losses resulting from pulse pileup. The size of the correction for coincidence losses (10–15%) was determined from a separate experiment in which the α-decay branching ratio was measured for the 7.20-MeV state in ^{20}Ne, which was populated using the ^{19}F(^{3}He, d)^{20}Ne reaction. In that case $\Gamma_\alpha/\Gamma = 1.0$ and ϵ can be determined from the measured values of A_0 and N_s in (4).

For the 12.71-MeV state in ^{12}C the above analysis procedure yielded a value

$$\left.\frac{\Gamma_\alpha}{\Gamma}\right|_{12.71} = 1.02 \pm 0.1 .$$

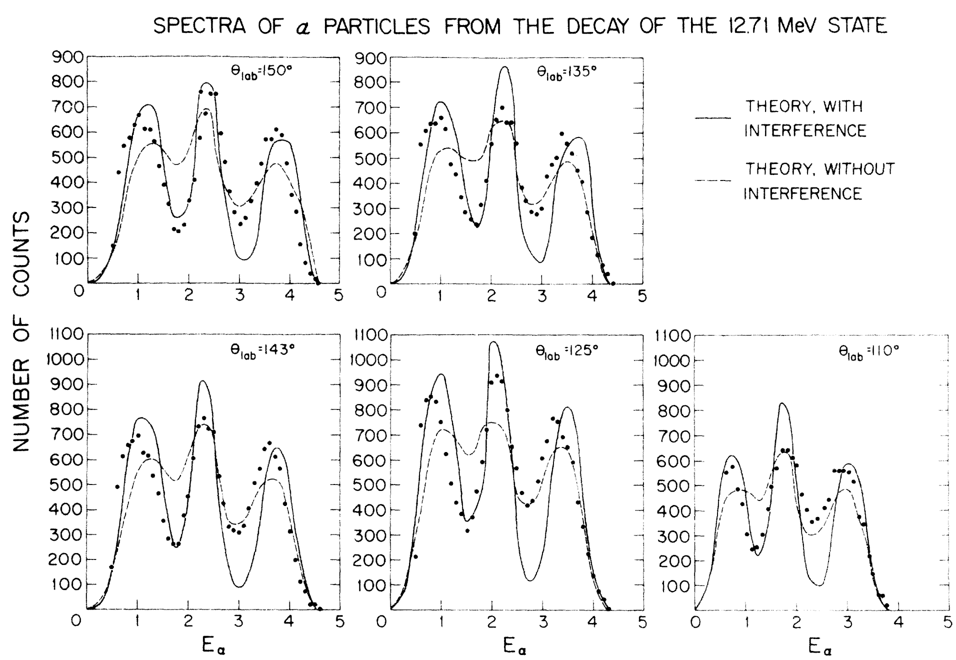

FIG. 4. α particle coincidence spectra obtained with the moveable detector at the angles indicated.

This is in good agreement with the previously measured value[4]

$$\frac{\Gamma_\alpha}{\Gamma}\bigg|_{12.71} = 0.971 \pm 0.003$$

which was obtained by measuring the competing γ-decay branch.

The experimental spectra obtained at laboratory angles of 110, 125, 135, 143, and 150° are compared with the predictions of the model described in Sec. III in Fig. 4. The solid curves represent the calculations using the correctly symmetrized amplitude. The dashed curves show the same calculation with the order of emission interference terms omitted. The difference between the two predictions is an indication of the size of the order-of-emission interference effects. Of course only the solid curve can be meaningfully compared with the data. The calculation is essentially parameter-free; all necessary information concerning the 2^+ state in ^{8}Be has been taken from α-α scattering data. In any case, calculations with a different set of parameters ($E_0 = 3.03$ MeV, $\gamma^2 = 0.92$ MeV, $R = 4.5$ fm) produced results quite similar to those of Fig. 4. The agreement is seen to be satisfactory; the theoretical curves appear to overestimate the destructive interference producing the minimum at about $E_\alpha = 2.5$–3 MeV.

We have not considered it to be worthwhile to attempt a systematic variation of parameters in order to improve the fit shown. Our reasons for this decision result from the fact that the curves displayed in Fig. 4 are the result of a complicated numerical integration over the three-particle phase space. Consequently the effects of the various dynamical assumptions made in the calculation are not likely to appear in a straightforward way. We are at present planning a series of triple coincidence experiments in which the decay of an aligned state in ^{12}C can be studied in a kinematically complete fashion. When these new experimental data are available a more detailed comparison with the model will be attempted.

B. 15.11-MeV state

We now come to a discussion of the decay of the 15.11-MeV state, the primary object of interest from the standpoint of isospin mixing. The analysis proceeds in a similar fashion to that of the 12.71-MeV state except for two complications.

The first of these is the fact that, because the α-decay branch is relatively weak, the influence of the strongly α-decaying continuum underneath the 15.11-MeV state must be considered. The second complication results from the presence of α particles in the spectrum resulting from the indirect branch

$$^{12}\mathrm{C}|_{15.11} \rightarrow {}^{12}\mathrm{C}|_{12.71} + \gamma \; . \quad (^{12}\mathrm{C}|_{12.71} \rightarrow 3\alpha)$$

A convenient way to examine the relative contributions of the 15.11-MeV state and the continuum background to the coincidence spectra is to display the spectra of α particles emitted at zero degrees in coincidence with all α particles recorded in the moveable detector.

Such a spectrum is shown in Fig. 5. The center of the moveable detector was located at a laboratory angle of 129.5°. [This angle corresponds to the zero of $P_2(\cos\Theta_{\mathrm{c.m.}})$ with the center-of-mass angle calculated for a two-body final state in ^{8}Be having an excitation energy of 2.9 MeV.] The top portion of Fig. 5 consists of all events falling within the true peak in the time spectrum, and hence corresponds to the sum of true plus accidental coincidences. The middle spectrum in Fig. 5 was obtained by setting an equivalent window on the flat portion of the time spectrum corresponding to accidental coincidences. The zero-degree spectrum associated with the accidental coincidences

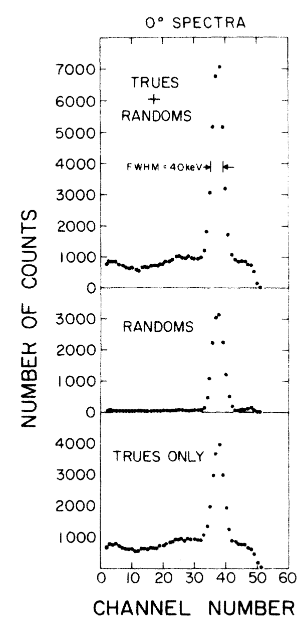

FIG. 5. Top: Spectrum of α particles observed at 0° to the beam direction in coincidence with all α particles emitted at $\Theta_\alpha = 129.5°$. This spectrum combines both true and accidental coincidences. Middle: Spectrum obtained simultaneously as above, but with a time window of equal width selecting accidental coincidences. Bottom: The difference of the two spectra shown above, thus containing true coincidences only.

is an image of the singles spectrum and shows a peak: valley ratio of about 100:1. The strong continuum background present in the top spectrum of Fig. 5 results from the fact that the α decays from the continuum are considerably stronger than the α-decay branch of the 15.11-MeV state.

This effect is even more exaggerated in the bottom of Fig. 5, which is the difference of the spectra discussed above and thus contains true coincidences only. The presence of a peak in this spectrum corresponding to the 15.11-MeV state demonstrates that we are indeed observing α particles resulting from the decay of the 15.11-MeV state. It should be noted, however, that slightly more than $\frac{1}{3}$ of the events with α_0 energies corresponding to pulses between channels 33 and 41 result from the decay of the continuum background. It should be emphasized that the events which are labeled "background" above do not necessarily arise from α-decaying states in ^{12}C near 15.11-MeV excitation. These α particles can arise from a number of processes, including sequential decays in which the roles of the two α particles are reversed.[22] In the absence of a rigorous prescription for accounting for these events, an empirical approach was adopted.

In Fig. 6 are shown α spectra obtained with the moveable detector in coincidence with particles in the 0° detector having energies just above or just below the peak corresponding to the 15.11-MeV state. The two spectra are quite similar; it would appear that these events largely result from sequential decays to the ^8Be ground state and first excited state. These two spectra were averaged and the result was normalized to an estimate of the total number of background counts underneath the peak corresponding to the 15.11-MeV state in the 0° spectrum. The resulting spectrum was subtracted from the spectrum of α particles in coincidence with particles in the zero degree detector with energies corresponding to the 15.11-MeV peak itself.

Once this "continuum background" has been approximately removed we must consider the effect of the γ decay of the 15.11-MeV state to the 12.71-MeV state. The total number of α particles in the moveable detectors resulting from this indirect process is known, since the singles 0° spectrum tells us the number of times the 15.11-MeV state was formed, and the branching ratios $\Gamma(15.1 \rightarrow 12.71)/\Gamma_\gamma(\text{total}) = 0.014 \pm 0.004$ and $(\Gamma_\alpha/\Gamma)_{12.71} = 0.971 \pm 0.003$ are known.[4,7] The energy distribution of these events can be calculated with the model described in Sec. III. The alignment of the initial state is known, and the alignment of the 12.71-MeV state can be calculated if we assume that the unobserved γ ray has pure $M1$ character. [The calculated spectrum is quite insensitive to the alignment because of the position of the counter near the zero of $P_2(\cos\theta)$.]

The background-subtracted spectrum from the moveable detector is shown in Fig. 7. The smooth curve represents the contribution to the spectrum of the indirect branch described above. It is clear that a considerable direct α decay is present. Figures 8 and 9 show the results of two different procedures used to determine the magnitude of the direct α-particle decay branch. The first procedure used was to assume that the number of indirect events is given as described above; the number of direct events can then be calculated,

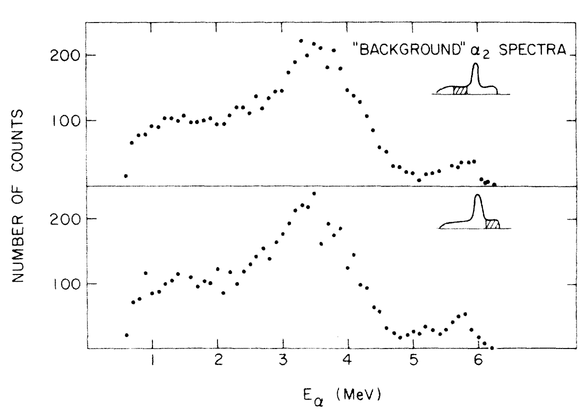

FIG. 6. Spectrum of α particles obtained at an average angle of 129.5° in coincidence with particles at 0° with energies slightly lower (top) or slightly lower (bottom) than those corresponding to the 15.11-MeV state.

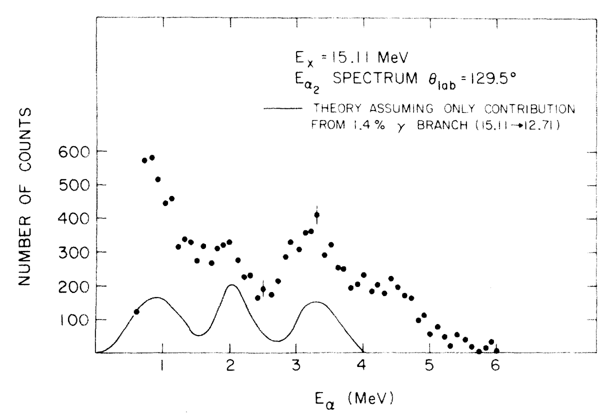

FIG. 7. Spectrum of α particles emitted at an average angle of 129.5° in coincidence with α particles leading to the 15.11-MeV state. Both random coincidences and an estimate of the zero-degree background have been subtracted as discussed in the text. The smooth curve represents the theoretical spectrum if only indirect feeding via the γ decay 15.11→12.71 MeV is present.

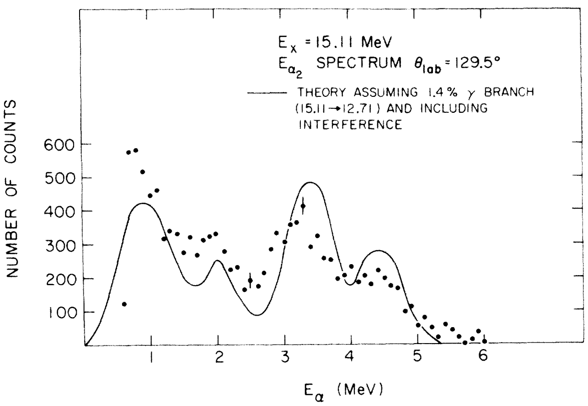

FIG. 8. Same data as Fig. 7, with the theoretical fit determined assuming both direct α decay and a 1.4% γ-ray branch feeding indirect α decay.

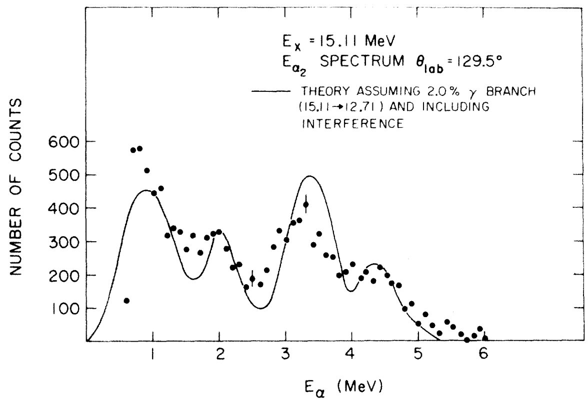

FIG. 9. Same as Fig. 8 except we normalize the direct decays to $E_\alpha > 4$ MeV (where the indirect decays cannot contribute). To give then the correct total number of counts, we require a 2% γ-branch feeding the indirect α decay.

since the total number of events in the spectrum is known. The energy spectrum of the direct events was calculated using the procedure of Sec. III, with the same parameters describing the 2^+ state in ^{8}Be. The results are shown in Fig. 8. An alternative analysis procedure is to use the fact that the high energy region of the spectrum ($E_\alpha > 4$ MeV) is free of contamination by the indirect decays. It is therefore reasonable to normalize the direct decays to give a good fit in this region. If this is done the number of indirect events must be adjusted to reproduce the correct total number of counts. The resulting "branching ratio" in the 15.11 → 12.71 decay is about 2%, which is not grossly different from the measured[7] value of (1.4 ± 0.4)%. The results of this second procedure are shown in Fig. 9. The fits certainly reproduce the main features of the data. The α-particle branching ratio of the 15.11-MeV state, as deduced from the two procedures described above, is

$$\frac{\Gamma_\alpha}{\Gamma} = 0.036 \pm 0.005 \quad \text{(First method)},$$

$$\frac{\Gamma_\alpha}{\Gamma} = 0.045 \pm 0.007 \quad \text{(Second method)},$$

where the errors are estimated from the statistics and the uncertainties concerning the 0° background. A "best" estimate of the best value of Γ_α/Γ has been obtained by averaging the above results and adding the errors in quadrature. Our result is then

$$(\Gamma_\alpha/\Gamma)_{15.11} = (4.1 \pm 0.9)\% .$$

This value is in strong disagreement with the results of Reisman, Connors, and Marion,[4] and in rather good agreement with those of Artemov *et al.*,[6] when the effects of the indirect branch are incorporated into the latter.

V. DISCUSSION AND INTERPRETATION

Within the spirit of the assumption of two-state mixing, the observed value of $(\Gamma_\alpha/\Gamma)_{15.11}$ can be used, in combination with other known information, to deduce the value of β. In Eq. (1) of Sec. I, only the state $|T=0\rangle$ is permitted to decay by α emission. The quantity β^2 is thus given directly by the ratio of the reduced widths for α emission from the physical states at $E_x = 15.11$ and 12.71 MeV,

$$\beta^2 = \frac{\gamma_\alpha^2(15.11)}{\gamma_\alpha^2(12.71)} .$$

TABLE I. Partial widths and branching ratios for 1^+ states in ^{12}C.

E_x (MeV)	$\Gamma_{\gamma 0}$ (eV)	$\frac{\Gamma_{\gamma 0}}{\Gamma_\gamma}$	$\frac{\Gamma_\alpha}{\Gamma}$	Γ_α (eV)
12.71	0.35 ± 0.05 [a]	0.83 ± 0.03 [b]	0.971 [b] ± 0.003	14.2 ± 2.8
15.11	37.0 ± 1.1 [c]	0.92 ± 0.02 [d]	0.041 ± 0.009 [e]	1.76 ± 0.33

[a] Reference 2.
[b] Reference 4.
[c] Reference 23.
[d] Reference 7.
[e] Present work.

The partial widths for α emission from the 15.11-MeV state can be obtained from the branching ratio determined in the present work in combination with other known quantities. The relevant data are given in Table I. In order to convert the results in the last column into a ratio of reduced widths it is necessary to allow for the effects of penetrability, i.e. the kinematic effects resulting from the different relative motion of α and ^{8}Be. It is perhaps worth mentioning that the ratio of these penetrabilities has been reported in the literature to have values ranging from 2 (Ref. 4) to 10.[6] In the present work the penetrabilities were extracted using the method described for calculating the energy spectrum. Specifically, for a ^{12}C decaying at rest with unit reduced width and available energies corresponding to excitation energies of 12.71 and 15.11 MeV the integrated yield was calculated. The parameters used were the same as those given in Sec. III. The ratio obtained corresponds to the desired relative penetrability, suitably integrated over the available phase space. The result is $P_{l=2}(E_x=15.11)/P_{l=2}(E_x=12.71)=6.0$ with an error of perhaps 25% resulting from the fact that even the ratio is a fairly sensitive function of the radius in the α-^{8}Be system. ($R=5.0$ fm was used.) Using this ratio and the assumption of two-state mixing we obtain

$$\frac{\gamma_\alpha^2(15.11)}{\gamma_\alpha^2(12.71)}=\frac{1}{6}\times\frac{\Gamma_\alpha(15.11)}{\Gamma_\alpha(12.71)}=0.021\pm0.007\ ,$$

which implies a 2% admixture in intensity of $T=0$ into the 15.11-MeV state. If we apply Eq. (2) of Sec. I, we obtain a charge-dependent matrix element $\langle T=1|H_{\text{CD}}|T=0\rangle=340\pm60$ keV. This is even larger than the value obtained by Braithwaite *et al.*[1] However, it must be strongly emphasized that deriving a value of β from the observed value of Γ_α depends critically on the assumption of two-state mixing. Furthermore, the fact that the 12.71-MeV state has such a small α width makes this assumption highly questionable. For example, the tentative 1^+ state studied in Ref. 9 has a width of ~600 keV. This is 40 000 times larger than the α width of the 12.71 MeV state, and the energy denominators are comparable. Consequently a matrix element of only a few keV would be enough to explain the α width observed in the present work. On the other hand, once more than two states are involved the entire problem becomes significantly more complicated, and it would be premature to speculate further along these lines.

Summarizing, the present work has demonstrated the existence of a $(4.1\pm0.9)\%$ α branch from the lowest $T=1$ state in ^{12}C. This α branch directly demonstrates the presence of a $T=0$ impurity in this state. However, it is difficult to relate the observed α width to a charge-dependent matrix element because the assumption of two-state mixing is probably invalid for the α decay as a result of the strongly inhibited α decay of the 12.71-MeV $T=0$ state. Clearly more experimental and theoretical work is desirable here. In particular, it would be nice if the method of Ref. 3 could be improved to get a value for the isospin-violating proton branch leading to the 12.71-MeV state. In addition, a unified theoretical treatment of the mass-12 problem which includes isospin mixing in both ^{12}C and ^{8}Be as well as the effects of the $(p_{3/2})^{-2}\,(p_{1/2})^2\ 1^+$ state (suggested by the experiments of Ref. 9 and the calculations of Ref. 13) would be most helpful in shedding further light on this problem.

Finally, the success of the simple model used to describe the α-particle energy spectra suggests that measurements of this kind can be useful as a proving ground for theoretical models of three-body final states. More work in this direction is presently under way in this laboratory.

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