PHYSICAL REVIEW C VOLUME 10, NUMBER 2 AUGUST 1974

Pion-nucleon absorption operator ambiguity*

J. L. Friar
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 6 May 1974)

In agreement with recent results it is shown that the pion-nucleon absorption operator depends on the nucleon single-particle potential's Lorentz transformation properties. This is essentially the same result as for the nuclear magnetic moment problem, these terms being of relativistic order [i.e., $(v/c)^2$ corrections] in both problems. There exists a unitary equivalence of various forms of the absorption operator, but if handled consistently this leads to the same physical results.

Interest has been recently rekindled in the problem of the form of the pion- (single-) nucleon absorption operator. It was shown several years ago by Barnhill[1] and Cheon[2] that there was an ambiguity in the form of this operator. Recently Miller[3] and Bolsterli, Gibbs, Gibson, and Stephenson[4] showed that a definite result could be obtained using a straightforward procedure. The latter authors also showed that the operator depends on the form of the nucleon single-particle potential in a puzzling way. These authors also suggest several alternative conclusions that one may draw from their results. We have independently confirmed their results and would like to offer our own somewhat stronger conclusions as well as tie together some loose ends which relate all of the various approaches mentioned above. In particular, we would like to show that the somewhat puzzling result of Bolsterli *et al.* is to be expected.

Conventionally one adopts the model of the pion interacting with the nucleon through one of two forms of vertex arrived at by appeal to Lorentz invariance and symmetry arguments. These vertices are the *ps*-*ps* (γ_5) and the *ps*-*pv* ($\gamma_\mu\gamma_5$), while much of the interest has settled on the former.[1-5] The problem is to reduce a relativistic description to a "nonrelativistic" form, where complicated momentum-dependent operators together with nonrelativistic wave functions presumably provide an adequate description of the absorption (or emission) process. The procedure most often utilized is analogous to that used in developing effective charge and current operators for the electromagnetic interaction from the relativistic form of the nucleon current.[6,7] We adopt for simplicity a single-nucleon picture of the pion interaction with each nucleon in the nucleus and hope that some appropriate generalization of this picture is physically realistic. Each nucleon therefore is described by a Dirac equation interacting with fixed potentials V_s (a scalar) and V_v (the fourth component of a vector, like the Coulomb case) and a pion field $\vec{\phi}(t)$ (an isotopic vector)

$$\frac{i\partial}{\partial t}\psi=[\,\vec{\alpha}\cdot\vec{p}+\beta m+\beta V_s+V_v+ig\beta\gamma_5(\vec{\tau}\cdot\vec{\phi})]\,\psi \qquad (1)$$

using the conventions of Ref. 6.

We could possibly add additional potential terms, but this would not add much to the argument. The usual procedure[1,2] is to omit all potential terms. (The present author is guilty of this omission in another context.[8]) We will include them, however, and keep all terms in the reduction of (1) to nonrelativistic (two-component) form up to order $(1/m)^3$. In our convention we regard the potentials as being of order $(1/m)$ since the nucleus is weakly bound. We also regard all time derivatives of the pion field as order $(1/m)$, since energy conservation demands that this essentially equals the energy difference of the final and initial nuclear states, and each of these is order $(1/m)$. For this convention to make sense, we must rule out absorption of extremely energetic pions. Proceeding in the usual fashion (Ref. 6) we perform a Foldy-Wouthuysen[9] reduction of (1) to order $(1/m)^3$, keep only terms of order g, and using $\phi\equiv\vec{\tau}\cdot\vec{\phi}$ obtain

$$\begin{aligned}
&\frac{i\partial}{\partial t}\,\psi'=H\psi'\,;\\
&H=H'+m+\frac{p^2}{2m}-\frac{p^4}{8m^3}+V\equiv H'+H_0\,,\\
&H'=-\frac{g}{2m}\,\vec{\sigma}\cdot\vec{\nabla}\phi-\frac{g}{8m^2}\,\{\vec{\sigma}\cdot\vec{p},\,\mathring{\phi}\} \qquad (2)\\
&\qquad+\frac{g}{8m^3}\{\vec{p}^{\,2},\vec{\sigma}\cdot\vec{\nabla}\phi\}\\
&\qquad+\frac{g}{4m^2}\,[2\vec{\sigma}\cdot\vec{\nabla}(\phi V_s)-\phi\vec{\sigma}\cdot\vec{\nabla}V]\;,
\end{aligned}$$

where $V=V_s+V_v$ plus relativistic corrections to the potential of order V/m^2. The same procedure (assuming the nucleus is weakly bound or equivalently that the binding energy is much smaller than the nucleon mass) allows us to write for the eigenfunctions of $H_0(\psi_0')$ in terms of the eigenfunctions in (1) with $g=0$ (denoted ψ_0)

$$\psi_0' = \left(\frac{1+\beta}{2}\right)(1+\vec{p}^2/8m^2)\psi_0\,, \tag{3}$$

so that only the upper two components contribute (we are dealing, of course, with positive energy solutions). The extra factor is needed to preserve a proper normalization. The same reduction to two-component form is possible by means of a Pauli reduction, but care must be taken to preserve the normalization and treat initial and final states symmetrically so that the Hermiticity of the effective (two-component) Hamiltonian is preserved. This has not always been done in the past.[10,11] The reduction (2) removes the antinucleon states from the effective Hilbert space of the problem, so that positive energy nucleon states are the only ones we need in constructing intermediate states in perturbation theory. Since the Foldy-Wouthuysen transformation leads to operators which do not mix upper and lower components (called "even"), starting with a positive energy state (3) cannot lead to a negative energy state (with only lower components). The *effect* of such states is included in the complicated effective Hamiltonian (2). In particular, the potential (V_s or V_v) can excite a nucleon-antinucleon pair which is deexcited by the external pion field. The effect of this, when expanded in terms of powers of $(1/m)$, is to lowest order just the ϕV terms in (2). Similar things occur in the electromagnetic problem.[12]

Although H' in (2) depends on V in an obvious way, it also unfortunately depends on V_s, as shown in Ref. 4. Such a result for the nuclear electromagnetic current operator has been known for over 20 years and has severely hampered analysis of the deuteron magnetic moment.[13-16] The nuclear magnetic moment μ which is proportional to $(1/m)$ in the nonrelativistic approximation has relativistic corrections of order $(1/m)^3$ which depend on the potential in general and on the *type* of potential as well, just as in (2). That the magnetic moment depends on V is not surprising since the relativistic corrections to the potential are of order V/m^2 and are momentum dependent, leading to a current in the usual way. The reason why the dependence of μ on the potential is not the same for V_s as for V_v was pointed out by Lipkin and Tavkhelidze.[17] We follow them and write the Dirac equation for a particle interacting with a vector electromagnetic potential $\vec{A}$ and *constant* scalar and vector nucleon potentials V_s^* and V_v^*

$$\vec{\alpha}(\vec{p}-e\vec{A})\psi + \beta(m+V_v^*)\psi = (E-V_v^*)\psi\ . \tag{4}$$

The result is an equation for a particle of mass $m+V_s^*$ and energy $E-V_v^*$. Performing a nonrelativistic reduction,[6] we see that V_v^* does not contribute to the current, while V_s^* merely changes the nucleon mass to an effective mass. The magnetic moment thus changes from $\mu/2m$ to $(\mu/2m)(1-V_s^*/m)$. The identical effect is seen in (2) by neglecting derivatives of V_v and V_s, since the effective coupling constant $g/2m$ then changes to $(g/2m)(1-V_s/m)$.

Working within the framework of the Breit equation with two body interactions does not alter the situation as has been shown by Breit[14,16] and Sachs[15] and in an analogous fashion by Close and Osborn[18] and Faustov.[19] The additional contributions to the magnetic moment are more readily identifiable in terms of the type of exchanged particle, however.

There is an additional freedom in doing the Foldy-Wouthuysen (F-W) transformation as was pointed out by Barnhill[1] and Cheon.[2] The F-W transformation is not unique, since after transforming the original Hamiltonian to "even" form, a unitary transformation can be made of the new Hamiltonian which will also be even if the unitary operator is even. Using a transformation of the form

$$\tilde{H} - \frac{i\partial}{\partial t} = e^{i\mu U}\left(H - \frac{i\partial}{\partial t}\right)e^{-i\mu U} \tag{5}$$

with $U = g\beta\gamma_5\{\vec{\alpha}\cdot\vec{p},\,\phi\}/4m^2$, working to $O(1/m)^3$ and $O(g)$ yields a new *interaction* Hamiltonian H'' for positive energy nucleons

$$\begin{aligned} H'' &\cong H' + i\mu[U, H_0] - \mu\dot{U} \\ &= \frac{-g}{2m}\vec{\sigma}\cdot\vec{\nabla}\phi - \frac{(\mu+1/2)g}{4m^2}\{\vec{\sigma}\cdot\vec{p},\,\dot{\phi}\} \\ &\quad + \frac{g}{8m^3}\{\vec{p}^2, \vec{\sigma}\cdot\vec{\nabla}\phi\} + \frac{\mu g}{8m^3}\{\vec{\sigma}\cdot\vec{\nabla}[\vec{\nabla}^2, \phi]\} \\ &\quad + \frac{g}{2m^2}\vec{\sigma}\cdot\vec{\nabla}(\phi V_s) + \frac{g(\mu-1/2)}{2m^2}\phi(\vec{\sigma}\cdot\vec{\nabla})V\ . \end{aligned} \tag{6}$$

The parameter μ is completely arbitrary. We can eliminate the potential term proportional to V by choosing $\mu=\frac{1}{2}$, at the cost of introducing a new term proportional to μ, which vanishes if we take $\mu=0$. This case ($\mu=\frac{1}{2}$) is very popular in the literature, since selective neglect of terms of order $(1/m)^3$ [i.e., order $(v/c)^2$ corrections] leads to the "Galilean-invariant" operator. To get this we neglect all terms in (6) but the first and second and evaluate the time derivative for a slowly moving meson. This gives for $\mu=\frac{1}{2}$

$$H'' \cong -\frac{ig}{2m}[\vec{\sigma}\cdot\vec{p}\phi - m_\pi\{\vec{\sigma}\cdot\vec{p},\,\phi\}/2m]\ . \tag{7}$$

We note that while the meson to be absorbed may be slowly moving, conservation of energy guarantees that the final nuclear state will be highly excited and the terms we have neglected in (7) could possibly be large. One can in fact entirely eliminate the time-derivative term by choosing $\mu = -\frac{1}{2}$. There is no reason *a priori* for believing that the terms of relativistic order should preserve or be of Galiliean-invariant form. In the electromagnetic problem, the three relativistic corrections of lowest order to the charge operator of nucleons are due to Zitterbewegung, Thomas precession, and the electric dipole interaction of a *moving* magnetic dipole. All these phenomena are manifestly relativistic in origin. It may appear on first sight that the second term in (7) is not a $(v/c)^2$ correction, since it involves the ratio of two masses. Energy conservation states that the pion energy (including its mass) equals the energy difference of the final and initial nuclear states, and this is adequately represented by a nonrelativistic treatment of the kinematics involved if the initial pion is not moving rapidly [i.e., it is of order $(1/m)$]. If the pion mass were much larger, the expansion scheme itself would break down entirely.

Our last remark is a fairly obvious one which seems not to have been made before in this problem. Although there is an "ambiguity" in the form of the interaction caused by the freedom of doing a unitary transformation, there is no ambiguity from this cause in the physical results. [The additional μ-dependent terms in (6) do not change the transition probability to order g, as one can see by doing first-order perturbation theory on these terms.] Thus, the unitary freedom to change the form of the operator does not change the physics so long as one includes the potential in the commutator in (6). If this is left out and the transition probability is then calculated using wave functions which depend on V, the results will indeed depend on μ. This was in fact the case in some previous analyses.[1,2] The use of the Pauli reduction leads to an unambiguous result for H' (which can then be unitarily transformed). If one does the Pauli reduction care must be exercised in preserving normalization and Hermiticity.

In summary we draw the following conclusions. Assuming the γ_5 interaction of pions with nucleons we have calculated the $(v/c)^2$ corrections to the lowest order $(g/2m)$ operator describing pion absorption on a single nucleon. This operator depends on the form of the nucleon's potential (i.e., its Lorentz-transformation properties) in a nontrivial way in agreement with Ref. 4. This unpleasant result is to be expected. There is, furthermore, no reason to expect Galilean invariance if one includes the $(v/c)^2$ terms. Resorting to a model which uses two-body potentials rather than effective scalar and vector single-particle interactions will not affect the result.[14-16,18,19] There is an ambiguity in the form of the interaction due to a unitary equivalence, but this will not affect the physics. A similar reduction of the (ps-pv) interaction yields (6) with $\mu = \frac{1}{2}$ and without the ϕV_s term. One will undoubtedly have to live with this situation until more is known about the pion-nucleon interaction.

---

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AEC AT(11-1)-3235.

[1] M. V. Barnhill, III, Nucl. Phys. A131, 106 (1969).
[2] Il-T. Cheon, Prog. Theor. Phys. Suppl., Extra No. (1968).
[3] G. A. Miller, Nucl. Phys. A224, 269 (1974).
[4] M. Bolsterli, W. R. Gibbs, B. F. Gibson, and G. J. Stephenson, Jr., LASL Report, 1974 (to be published).
[5] J. M. Eisenberg, R. Guy, J. V. Noble, and H. J. Weber, Phys. Lett. 45B, 93 (1973).
[6] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[7] J. L. Friar, Ann. Phys. (N.Y.) 81, 332 (1973).
[8] J. L. Friar, Nucl. Phys. 87, 407 (1966).
[9] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).
[10] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic, New York, 1957).
[11] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968).
[12] D. O. Riska and G. E. Brown, Phys. Lett. 38B, 193 (1972).
[13] G. Breit, Phys. Rev. 71, 400 (1947).
[14] G. Breit and I. Bloch, Phys. Rev. 72, 135 (1947).
[15] R. G. Sachs, Phys. Rev. 72, 91 (1947).
[16] G. Breit and R. M. Thaler, Phys. Rev. 88, 1214 (1952); 89, 1177 (1953).
[17] H. J. Lipkin and A. Tavkhelidze, Phys. Lett. 17, 331 (1965).
[18] F. E. Close and H. Osborn, Phys. Lett. 34B, 400 (1971).
[19] R. Faustov, Nuovo Cimento 69A, 37 (1970).