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Comment on precise single-scattering optical-potential fit to 1 GeV p-^{4}He elastic scattering

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The assertion in a recent letter by Saperstein of the failure of the Glauber scattering formalism in describing forward angle p-^{4}He elastic scattering is pointed out to be incorrect. The single scattering optical potential described there is at best no better than the Glauber formalism in describing elastic p-nucleus scattering at high energies.

[NUCLEAR REACTIONS ^{4}He(p,p); E = 1 GeV, $\sigma(\theta)$. Watson single scattering potential and Glauber multiple scattering amplitude analyses equivalent.]

In a recent letter,[1] Saperstein fits the 1 GeV p-^{4}He elastic differential cross section using a scattering amplitude obtained from the Klein-Gordon equation with a potential which is the single scattering term in the Watson[2] multiple scattering optical potential. In the course of his discussion he refers to the, in comparison, "failure" of the Glauber single scattering approximation, states that in contrast to the optical potential fit the Glauber formalism[3] requires the specification of multi-nucleon correlations in order to reproduce the small angle scattering and conjectures that the reason the Glauber formalism fails is that an incorrect form of the nucleon-nucleon scattering amplitude is generally used. We would like to comment particularly on these sections of the letter.

Let us turn first to the use of the nucleon-nucleon amplitude in the Glauber formalism. The form generally used is taken from an empirical fit to the elastic cross section in the nucleon-nucleon center of mass frame (n-n) [we shall denote this frame by the superscript c and the nucleon-nucleus center of mass frame (n-N) by the superscript L]:

$$f^{(c)}(q)=(4\pi)^{-1}(i+\rho)k^{(c)}\sigma_T\exp(-\beta^2q^2), \qquad (1)$$

where q is the momentum transfer, ρ is the spin and isospin averaged ratio of the real to the imaginary part of the forward n-n amplitude, σ_T the total cross section, and $k^{(c)}$ the incident momentum in the (n-n) system. This amplitude enters into the nucleon-nucleus elastic amplitude $F(k',k)$, where k' is the momentum of the scattered particle, by way of the profile function. Consider the scattering from a nucleus containing A particles. In the L frame

$$F^{(L)}(k',k)=\frac{k^{(L)}}{2\pi i}\int e^{i(k-k')\cdot b}\left\{\int |u(r_1,\ldots,r_A)|^2\prod_{j=1}^{A}[1-\Gamma_j(b-s_j)]\,d^{(3)}r_j-1\right\}d^{(2)}b, \qquad (2)$$

where b is the impact parameter relative to the center of the nucleus and $(b-s_j)$ is the impact parameter relative to the jth nucleon, $u(r_1,\ldots,r_A)$ is the many-body ground state wave function and the profile function Γ_j, which depends only on the phase shifts, is related to the free scattering amplitude of the incident particle from the jth nucleon by

$$\frac{f_j^{(c)}(k',k)}{k^{(c)}}=\frac{i}{2\pi}\int e^{i(k-k')\cdot b}\,\Gamma_j(b)d^{(2)}b\ . \qquad (3)$$

The factor $k^{(L)}$ in Eq. (2) has nothing to do with the parametrization of the nucleon-nucleon amplitude, but rather is a result of the high energy approximation.[3] The profile function Γ_j is itself an invariant and the form determined in the c frame from nucleon-nucleon scattering may be used directly in Eq. (2). We might also note that since the right hand side of Eq. (3) is invariant under transformations from the c to the L frame, since the vector b is normal to the incident momentum, so also is the left hand side. Equation (2) is the one generally, and correctly, used when applying the Glauber formalism. This is of course in the context of the small angle approximation.

What is termed in Ref. 1 the "failure" of the Glauber single scattering approximation (SSA) at forward angles is then not due to the use of the wrong momentum in Eq. (1), but rather occurs because multiple scattering contributions to the amplitude are important even at low momentum transfers. Before elucidating this point, a note of caution is necessary. It is most important to keep straight whether one is talking about multiple scattering approximations to the *scattering amplitude* or to an *optical potential* and further whether one is considering scattering from a *potential* or from nucleons in many-body system. Otherwise there is confusion.

Let us examine the SSA to the elastic amplitude in the Glauber formalism. In Eq. (2) the product can be expanded

$$\prod_{j=1}^{A}(1-\Gamma_j)=1-\sum_{j=1}^{A}\Gamma_j+\sum_{i\neq j}\Gamma_i\Gamma_j+\cdots+(-)^A\prod_{j=1}^{A}\Gamma_j. \quad (4)$$

If we keep only the terms linear in Γ we have the SSA,

$$F^{(1)}(k',k)=\sum_{j=1}^{A}f_j(k',k)\int e^{i(k-k')\cdot s_j}\rho(r_j)d^{(3)}r_j, \quad (5)$$

where $\rho(r)$ is the single particle ground state density,

$$\rho(r_j)=\int |u(r_1,\ldots,r_A)|^2 d^{(3)}r_1\cdots d^{(3)}r_{j-1}\times d^{(3)}r_{j+1}\cdots d^{(3)}r_A$$

and further in the forward direction

$$F^{(1)}(k,k)=\sum_{j\neq 1}^{A}f_j(k,k)\ . \quad (6)$$

These last two equations clearly describe a situation where the incident particle scatters from a single nucleon only. Further, this term in the amplitude depends in principle *linearly* on the *single particle* density only (no assumptions have been made as to whether nucleon-nucleon correlations are important or not). This again points up the fact that it is a single scattering term.

Correlations can only be detected if the incident particle scatters more than once—or more accurately with more than one target particle (excluding the so called self-correlations described in Ref. 4. Now, as is seen in Eqs. (2) and (4), the Glauber double scattering amplitude is proportional not to the two-nucleon correlations [i.e., $\rho(1,2)-\rho(1)\rho(2)$, which vanishes for completely independent particles], but to the two-particle density $\rho(1,2)$. So even for uncorrelated systems [i.e., where $\rho(1,2)=\rho(1)\rho(2)$] there can be (and is) a significant contribution from the double scattering part of the amplitude even at forward angles. It is possible to have multiple scattering with no correlations if only those collisions that leave the nucleus in its ground state contribute. This is in contrast to the multiple scattering contributions to the optical potential, as has been recognized by Watson[2] and emphasized by Glauber[3] and by Feshbach *et al.*[4] What is significant about, for example, the second term in the multiple scattering expression for the optical potential is not so much that it is a "double scattering" term but rather that, except for terms $O(1/A)$, it is proportional to the two-body *correlations* and thus is small when the correlation is small.

When a potential is put in a wave equation and a scattering amplitude obtained numerically, as is done in Ref. 1, contributions from multiple scattering of *all orders* from the *potential* are being included. That this is not the case for the single scattering Glauber amplitude will be made clear. What is the relation between a many-body description of scattering and the optical potential description? The amplitude for elastic scattering from a many-nucleon system is given by Eq. (2). Further, the Glauber amplitude for scattering from a potential $V(r)$ is

$$F_{\text{POT}}(k',k)=\frac{k}{2\pi i}\int e^{i(k-k')\cdot b}(e^{i\chi(b)}-1)d^{(2)}b\ , \quad (7)$$

where the phase shift function is defined to be

$$\chi(b)=-(\hbar v)^{-1}\int_{-\infty}^{\infty}V(b^2+z^2)^{1/2}dz \quad (8)$$

and v is the incident velocity. By definition the optical potential is that potential which gives the same elastic amplitude as that for scattering from a many-particle system, i.e., for which

$$F(k',k)=\frac{k}{2\pi i}\int e^{i(k-k')\cdot b}(e^{i\chi_{\text{opt}}(b)}-1)d^{(2)}b\ , \quad (9)$$

where $F(k',k)$ is given by Eq. (2) and

$$\chi_{\text{opt}}(b)=-(\hbar v)^{-1}\int_{-\infty}^{\infty}V_{\text{opt}}(b^2+Z^2)^{1/2}dz\ . \quad (10)$$

Comparing with Eq. (2) we see

$$\chi_{\text{opt}}(b)=-i\ln\left\langle\prod_{j=1}^{A}[1-\Gamma_j(b-s_j)]\right\rangle. \quad (11)$$

Expansion of the logarithm yields the multiple scattering expression for the phase shift function (and indirectly the optical potential). Assuming the profile functions Γ_j are the same for all the nucleons, Glauber[3] obtains

$$\chi_{\text{opt}}(b) = iA \int \rho(r)\Gamma(b-s)d^{(3)}r - \tfrac{1}{2}i \int [A(A-1)\rho(1,2) - A^2\rho(1)\rho(2)]\,\Gamma(b-s_1)\Gamma(b-s_2)\,d^{(3)}r_1 d^{(3)}r_2 + \cdots . \tag{12}$$

If we neglect correlations,

$$\chi_{\text{opt}}(b) \cong iA \int \rho(r)\Gamma(b-s)d^{(3)}r . \tag{13}$$

The Born term in Eq. (9) is obtained by expanding the second exponential and keeping only the term linear in χ_{opt}. Using Eq. (13)

$$F_B(k',k) = \frac{ikA}{2\pi} \int\int e^{i(k-k')\cdot b}\rho(r)\Gamma(b-s) \times d^{(3)}r d^{(2)}b \tag{14}$$

which is just the SSA. Thus this term in the Glauber amplitude describes single scattering from either a many-particle system *or* an independent particle optical potential and that it does not reproduce the data as well as when scattering of all orders from the potential is taken into account is not surprising. Rather if one compares the optical potential of Ref. 1 with that obtained by inverting Eq. (10) using χ_{opt} as given in Eq. (13), it is easy to show, making the same assumptions as in Ref. 1, that they are precisely the same. Indeed the close relationship between the Watson and Glauber formalisms at high energy is known[5] and it is to be expected that the results obtained in Ref. 1 not be essentially different from those of, say, Bassel and Wilkin,[6] who use the full elastic Glauber amplitude.

Finally we would like to make a short comment on the role of the factor ρ which appears in Eq. (1). We agree with Saperstein in questioning the significance of the information about nucleon-nucleon scattering that can be gotten from a value of ρ obtained from a phenomenological fit to nucleon-nucleus data, if for no other reason than the dramatic effect spin and isospin have on the cross sections and shown especially clearly by Lambert and Feshbach.[7] We feel therefore that the value for ρ at 600 MeV obtained in just this way in Ref. 1 is subject to suspicion.

Much of our argument is not new and has already been stated by Glauber in Ref. 3. We put it forth again for purposes of clarification and to point out that the results of Ref. 1 unfortunately offer us no new information.

[1]A. M. Saperstein, Phys. Rev. Lett. 30, 1257 (1973).

[2]See A. L. Fetter and K. M. Watson, Advan. Theor. Phys. 1, 115 (1965).

[3]R. J. Glauber, *Lectures in Theoretical Physics* (Interscience, New York, 1959), Vol. 1, pp. 315–414.

[4]H. Feshbach and J. Hüfner, Ann. Phys. (N.Y.) 56, 268 (1970); H. Feshbach, A. Gal, and J. Hüfner, Ann. Phys. (N. Y.) 66, 20 (1971); H. Feshbach and J. J. Ullo, Massachusetts Institute of Technology Center for Theoretical Physics Report No. 374 (to be published).

[5]J. M. Eisenberg, Ann. Phys. (N.Y.) 71, 542 (1972) and references cited there.

[6]R. H. Bassel and C. Wilkin, Phys. Rev. 174, 1179 (1968).

[7]E. Lambert and H. Feshbach, Phys. Lett. 38B, 487 (1972); Ann. Phys. (N. Y.) 76, 268 (1973); V. Franco, Phys. Rev. Lett. 21, 1360 (1968).