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Comment on Pauli blocking in pion-nucleus scattering*

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A modified solution is presented for the Chew-Low equations in nuclei, in the presence of Pauli blocking. The resulting scattering amplitude has more satisfactory behavior at low energies (for pionic atoms), while supporting use of a blocked amplitude in the region of the 3,3 resonance.

[NUCLEAR SCATTERING $A\ (\pi,\pi), E\sim 150–250$ MeV; calculated scattering amplitudes. Role of Pauli principle, dispersion theory.]

Much attention[1,2] has recently been devoted to the effects of Pauli blocking in pion-nucleus scattering. In particular, one may consider[3] modifying the Chew-Low formalism[4] for the description, in the nuclear medium, of the pion-nucleon scattering in the 3, 3 channel, which is important from threshold on up through a few hundred MeV. The Born term (for $T=\frac{3}{2}, J=\frac{3}{2}$) is then suppressed[1,3] due to the exclusion principle,[5,6] although the numerical consequences of this suppression are relatively small[7] inasmuch as high intermediate pion momenta predominate in the dispersion integral of Chew-Low theory. Previously,[3] we discussed this effect by considering the Chew-Low equation for the 3, 3 channel amplitude $H(\omega)$,

$$H(\omega)=\frac{\lambda F}{\omega}+\frac{1}{\pi}\int_{\mu}^{\infty} d\omega_p p^3 v^2(p)\frac{|H(\omega_p)|^2}{\omega_p-\omega-i\epsilon} + \text{crossed term}, \tag{1}$$

where λ is the coupling constant, $v^2(p)$ is the nucleon cut-off function, and F is the Pauli suppression factor, taken in Refs. 1 and 3 as

$$F=\begin{cases}\frac{3}{4}\frac{q}{p_F}\left[1-\frac{1}{12}\left(\frac{q}{p_F}\right)^2\right], & q\leq 2p_F\\ 1, & q>2p_F\end{cases} \tag{2}$$

for pion momentum $q=(\omega^2-\mu^2)^{1/2}$, and Fermi momentum p_F; the crossed term will generally be ignored in the following. Although H depends on both ω and q, unfortunately the Chew-Low approach does not allow inclusion of this separate dependence; we thus approximate F also by its on-shell value. Equation (1) was then solved treating F as a regular function, which we feel to be an acceptable rough approximation in the 3, 3 region where we are far above threshold. The solution may be written as

$$H(\omega)=N(\omega)/D(\omega), \tag{3}$$

where

$$N(\omega)=N_0(\omega)\equiv\lambda F/\omega, \tag{4}$$

and

$$D(\omega)=D_0(\omega)\equiv 1-\frac{\lambda\omega}{\pi}\int_{\mu}^{\infty}\frac{d\omega_p p^3}{\omega_p^2}\frac{v^2(p)F}{\omega_p-\omega-i\epsilon}. \tag{5}$$

The factor F in Eq. (4) clearly precludes[7] the use of this result near threshold, since F vanishes there.

In order to construct a solution which is valid near threshold, the singularities of F can no longer be ignored. Near threshold, F of Eq. (2) involves branch cuts starting at $\omega=\pm\mu$; the right-hand cut coincides with the physical region, and the left-hand cut we approximate by a pole, as in the usual treatment of effective range approximations in dispersion theory. It is convenient to use as simple a parametrization for the blocking factor as possible, for example

$$F(\omega)=\frac{\omega-\mu}{\omega+M},\quad 0\leq M\lesssim\mu, \tag{6}$$

which embodies the usual constraints that $F(\mu)=0$ and $F(\omega\gg 2p_F)\approx 1$, since fortuitously $p_F\sim 2\mu$ (but tends to over-block). The N/D formalism gives a solution of the form of Eq. (3) with

$$N(\omega)=\frac{1}{\pi}\int_{-\infty}^{0^+} d\omega_p\frac{\phi(\omega_p)D(\omega_p)}{\omega_p-\omega-i\epsilon} \tag{7}$$

and

$$D(\omega)=1-\frac{\omega}{\pi}\int_{\mu}^{\infty} d\omega_p p^3 v^2(p)\frac{N(\omega_p)}{\omega_p(\omega_p-\omega-i\epsilon)}, \tag{8}$$

where $\phi(\omega_p)$ is defined with reference to

$$H(\omega)=\frac{1}{\pi}\int_{-\infty}^{0^+} d\omega_p\frac{\phi(\omega_p)}{\omega_p-\omega-i\epsilon}+\frac{1}{\pi}\int_{\mu}^{\infty} d\omega_p\frac{p^3|H(\omega_p)|^2 v^2(p)}{\omega_p-\omega-i\epsilon}. \tag{9}$$

Then, for the $F(\omega)$ of Eq. (6),

$$\phi(\omega_p)=\frac{\pi\lambda\mu}{M}\delta(\omega_p)-\pi\lambda(1+\mu/M)\delta(\omega_p+M), \tag{10}$$

TABLE I. Amplitude for 3,3 channel, in units of μ^{-3}, as a function of pion total lab energy.

ω (MeV)	Unblocked ($F\equiv 1$) ReH	ImH	Blocked [a] ($C=0.28$) ReH	ImH	Original [b] F ReH	ImH	Blocked [a] ($C=0$) ReH	ImH	Blocked [c] ReH	ImH
200	0.176	0.034	0.106	0.012	0.079	0.007	0.041	0.002	0.128	0.018
250	0.149	0.118	0.139	0.089	0.123	0.062	0.062	0.013	0.144	0.102
300	0.039	0.135	0.019	0.144	0.036	0.136	0.068	0.047	0.034	0.138
350	−0.006	0.082	−0.023	0.076	−0.015	0.080	0.029	0.070	−0.009	0.082
400	−0.011	0.050	−0.019	0.044	−0.016	0.047	0.008	0.052	−0.012	0.049

[a] These results refer to the use of $F(\omega)$ of Eq. (6) in Eqs. (11)–(13).
[b] This column is based on Eq. (2), as used in Ref. 3.
[c] As given by Eqs. (2) and (14)–(16).

and Eqs. (7) and (8) yield

$$N(\omega)=N_0(\omega)+\frac{\lambda C}{\omega+M}, \tag{11}$$

$$D(\omega)=D_0(\omega)-\frac{\lambda C\omega}{\pi}\times\int_\mu^\infty \frac{d\omega_p p^3}{\omega_p}\,\frac{v^2(p)}{(\omega_p+M)(\omega_p-\omega-i\epsilon)}, \tag{12}$$

where

$$C\equiv\frac{\lambda}{\pi}(M+\mu)\int_\mu^\infty \frac{d\omega_p p^3}{\omega_p^2}\,\frac{v^2(p)F(\omega_p)}{\omega_p+M}\times\left[1-\frac{\lambda(M+\mu)}{\pi}\int_\mu^\infty\frac{d\omega_p}{\omega_p}\,\frac{p^3v^2(p)}{(\omega_p+M)^2}\right]^{-1}. \tag{13}$$

Equations (11) and (12) exhibit the corrections brought about by the singularity structure of F. The quantity C, which measures these corrections, is $C=0.28/\mu^3$ for the case $M=0$, which will be pursued in the numerical results presented below. For that case, the net effect of the correction is to replace $F(\omega)$ by $F(\omega)+C$.

We note that one can rewrite Eqs. (11)–(13) so as to eliminate the specific one-pole form in favor of functional dependence of $F(\omega)$ only, that is

$$N(\omega)=N_0(\omega)+\lambda C'(1-F(\omega)), \tag{14}$$

$$D(\omega)=D_0(\omega)-\frac{\lambda C'\omega}{\pi}\int_\mu^\infty\frac{d\omega_p p^3}{\omega_p}\,\frac{v^2(p)(1-F(\omega_p))}{\omega_p-\omega-i\epsilon}, \tag{15}$$

and

$$C'\equiv\frac{\lambda}{\pi}\int_\mu^\infty\frac{d\omega_p p^3}{\omega_p^2}\,v^2(p)F(\omega_p)\frac{F(\omega_p)-F(\mu)}{\omega_p-\mu}\times\left[1-\frac{\lambda}{\pi}\int_\mu^\infty\frac{d\omega_p p^3}{\omega_p}\times v^2(p)(1-F(\omega_p))\frac{F(\omega_p)-F(\mu)}{\omega_p-\mu}\right]^{-1}. \tag{16}$$

Due to the linear dependence of $D(\omega)$ on ω, these equations preserve a useful feature of Eqs. (3)–(5) in that they exhibit explicitly the change in position and width of the resonance. Equations (14)–(16) can then be used with nonanalytic parametrizations of F, such as that of Eq. (2), in order to estimate the consequences of more realistic forms which do not block as drastically as that of Eq. (6).

Table I shows the consequences of evaluating[8] the 3, 3 channel amplitude for various cases. The simplistic choice of F in Eq. (6) leads to overly drastic blocking from threshold through the 3, 3 region. Nonetheless, one can infer from it the consequences of including or omitting the corrections due to the singularity structure of F (i.e., $C\neq 0$ versus $C=0$), and they are sizable. It emerges, however, that the blocked case, with $F(\omega)$ of Eq. (6) and $C=0.28/\mu^3$, bears close resemblance[9] to our original, approximate solution.[3] Very near to threshold ($\omega-\mu<10$ MeV), the overly-strong blocking, arising from Eq. (6), is again evident in a reduction of Re$H_{3,3}$ by a factor of about 3, but, of course, the solution is nonzero there; this would not be the case for the approximate solution of Eqs. (3)–(5), which is directly proportional to F.

With Eqs. (2) and (14)–(16), one obtains the results shown in the last column of Table I, which we consider to be the most realistic approximation from the viewpoint of the degree of blocking. Again, the main blocking effect occurs below the resonance energy. At threshold, the amplitude is reduced by a factor of only about 2, in contrast to the situation arising from Eq. (6).

In summary, we believe that our original approach[3] of Eqs. (4) and (5) overestimated the blocking effects on the low-energy side of the resonance. However, due to high absorption in the nuclear interior, the pion scattering is mostly sensitive to the nuclear surface, where effective densities are low.[3,10] Therefore, in the situation of real nuclei, the blocking, as well as effects[3,10,11] of pion quenching and changes in the pion momentum in the nuclear medium, all prove to have rather small numerical consequences.

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[8]In this evaluation, no attempt is made to distinguish between an internal and external pion momentum.
[9]We note also that we have explicity inserted the solution based on Eqs. (11)–(13) and that deriving from Eqs. (4) and (5) into the right-hand side of Eq. (1) in order directly to verify the numerical quality of these solutions. The degree to which Eq. (1) is satisfied is comparable for these two cases, giving the position of the zero in $\mathrm{Re}H(\omega)$ to within about 10 to 15 MeV.
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