Multiplet structure of highly excited states in ^{15}N and ^{15}O

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A series of calculations were performed which support the α-particle core excitation model as a possible explanation of the structure of certain highly excited states seen in ^{15}N and ^{15}O. The features of interest are the apparent compression of the core states and the multiplet structure. The compression is semiquantitatively accounted for by an enlargement of the single particle well due to the presence of the α particle. The standard core excitation model fails to account for the multiplet structure. An operator resulting from the nucleon-nucleon spin-orbit interaction is found to account successfully for the multiplet structure. In these calculations, the model of the core is that of a $p_{3/2}$ hole coupled to the states of ^{12}C. Two different models of the ^{12}C states are used and both give good results.

[NUCLEAR STRUCTURE ^{15}O, ^{15}N. Calculated levels, J^{π}. α-nucleon spin-orbit interaction.]

I. INTRODUCTION

In a recent note, Weller[1] has summarized a series of reaction studies in light nuclei in which resonance features are seen in highly excited states of the compound system. Since these resonances have large α decay widths and occur near the threshold for α particle emission, he has suggested that they could be interpreted as arising from an α particle core excitation model with the core in its lowest states of excitation. The purpose of the present paper is not to justify the existence of such a model but, given the model, to investigate to what extent it can successfully reproduce the observed resonant energy spacings and spins and parities. Particular emphasis is given to the problem of choosing the proper interaction between the core and the α particle.

Specifically, we have considered a model of the highly excited states of ^{15}N and ^{15}O as being an α particle in an $L=1$ single particle state coupled to the ground and first few excited states of ^{11}B and ^{11}C. The predicted multiplets of energy levels bear a striking correspondence to the observed resonances in the 12 to 15 MeV excitation region which have strong α decay rates as Weller has shown. (See Fig. 1 of Ref. 1, for example.) There are, however, certain aspects of the proposed structure which require consideration:

(a) There is a definite compression of levels in the $A=15$ nuclei compared to the assumed corresponding levels of the $A=11$ nuclei. Moreover, the ^{15}N energies are compressed more than the ^{15}O energies.

(b) The observed multiplets are approximately consistent with an $\vec{I}\cdot\vec{L}$ interaction wherein the mass 11 target has angular momentum I and the α particle has angular momentum L, with $L=1$. However, an inversion of level ordering occurs between the ground and the first and second excited state multiplets.

(c) The multiplet splitting decreases with increasing excitation.

Weller[1] suggests that the compression mentioned in (a) might be due to a broadening of the core potential well by the presence of the α particle and that the inversion referred to in (b) might be due to core deformation. He also suggested that the decreased splitting (c) may indicate a weakening of the interaction with higher target excitation.

Our prime concern in this paper is with aspects (a) and (b). We will focus mainly on the lowest energy states of the model where the decreased splitting with excitation in the mass 15 nuclei is less pronounced. We consider, in the next section, several possible explanations of the compression effect. The normal core excitation model as it applies in this case was tried and found lacking, as shown in Sec. III. Two reasonably successful calculations, using a microscopic dipole interaction, are described in Sec. IV. Our conclusions are presented in the final section, Sec. V.

II. COMPRESSION EFFECT

In standard applications of the core excitation model, the energies and wave functions of the core have been identified with those of the free core. This approach has proven to be successful in many instances when the remaining part of the problem was a single particle or hole. It seems clear however, that when the satellite particle is an α

particle that the simple approach may no longer be possible. This effect should be most obvious in the light nuclei where the mass of the α particle in the combined system is a significant fraction of the entire mass and where α clusters are known to be an important element in the structure of the nucleus.

The simplest effect to imagine is that the spacing of single particle energies is reduced. The harmonic oscillator energies $\hbar\omega$, which gives the separation of the major shells, are expected to be given approximately by $41(A+4)^{-1/3}$ MeV instead of $41A^{-1/3}$ MeV, where A is the mass number of the core. For $A=11$ this a reduction of 10%. The same effect is seen in single particle energies of other models such as the Woods-Saxon potential.[2]

It is also possible to understand how a compression of levels could occur in other circumstances. This will be done specifically for models which have been applied to the mass 11 nuclei. They have been studied in the strong coupling model[3] and in the core excitation model[4] as well as in extensive shell model calculations of p shell nuclei.[5] The discussion below will be limited to the strong coupling and core excitation models.

In the strong coupling case, the lowest negative parity states of the mass 11 nuclei are viewed as arising from a $K=\frac{3}{2}$ band built on the $\frac{3}{2}^-$ ground state and $K=\frac{1}{2}$ band built on the $\frac{1}{2}^-$ first excited state. In the adiabatic limit, the energy levels are given by

$$E_J(i,K)=\epsilon_i(K)+\frac{\hbar^2}{2I_K}\left[J(J+1)-2K^2+\delta_{K1/2}a(-1)^{J+1/2}(J+\tfrac{1}{2})\right], \qquad (1)$$

where $\epsilon_i(K)$ is essentially the single particle energy—or in the present case, a single hole in ^{12}C—as obtained from Nilsson energy diagrams. The second term in Eq. (1) gives the rotational band built on the single particle state and I_K is the moment of inertia of the K band.

When the α particle is added to this system, two effects should occur. The single particle energies should be compressed, essentially as discussed above and the moment of inertia should increase due to the larger spatial extension of the wave functions. As a check on this latter point, a Hartree-Fock calculation for the mass 11 nuclei was done with the harmonic oscillator parameter $\alpha=(m\omega/\hbar)^{1/2}$ appropriate to ^{12}C, then with that appropriate to ^{16}O. The Rosenfeld force was used to obtain approximate moments of inertia for the $K=\frac{3}{2}$ and $K=\frac{1}{2}$ bands. The result was that $I_{1/2}$ and $I_{3/2}$ both increased by about 20%; that is, the moment of inertia is roughly proportional to α^{-2}. This verifies that the rotational bands should compress as well as the single particle energies if the effect of the α particle is, to the lowest order, to enlarge the potential well seen by the mass 11 nucleons. For later use, we define the ratio $R=\alpha_0^2/\alpha^2$ as the fractional compression.

Essentially the same results can be expected in the core excitation model of the mass 11 nuclei. In a companion study of the present one,[4] we have shown that the lowest negative parity states of ^{11}B and ^{11}C can be described as resulting from a $p_{3/2}$ hole in ^{12}C. The presence of the α particle will again have the effect of increasing the moment of inertia; this time, it is that of the ^{12}C core.

In Fig. 1 we show the effect on the mass 11 spectrum of compressing the energies of the first two excited states of the ^{12}C $K=0$ rotational band. Two calculations are shown. In one the interaction strengths between the $p_{3/2}$ hole and the core are held constant, and in the other the quadrupole interaction strength has been held constant while the dipole strength has been increased by making it inversely proportional to the R^2, where R is given above. The assumed centers of gravity of the multiplets found in ^{15}O and ^{15}N by Weller and the unperturbed energies of ^{11}C and ^{11}B are also shown.

It is obvious that the compression of the mass 11 energy spectrum is quite large in both cases. The ^{15}N case is particularly startling, exhibiting a compression of the centers of gravity of the multiplets to about one third the unperturbed mass 11 energies.

In both calculated spectra shown, the compression effect of the broadening of the ^{12}C well due to the addition of the α particle is obvious. The only exception is the $\frac{3}{2}^*$ state which is essentially unaffected by the core compression.

It seemed clear to us that the core-hole interaction ought to be affected by the core compression but of course the difficulty was to decide how. Our first choice was to assume that the change in the interaction would be small and so could be ignored. The results of the calculation leaving the strengths unchanged exhibit the compression effect but as is easily seen the $\frac{1}{2}$ level cannot possibly be brought down low enough to give the proper energy for the ^{15}N case for any reasonable compression of the core. Our second choice, that is varying the dipole strength inversely with the square of the compression, was arrived at completely empirically. The compression was found to be a relatively insensitive function of the quadrupole interaction strength, but it was quite sensitive to the dipole strength. Since we had only rather vague ideas of the origin of the dipole term, our approach was to simply vary it in a variety of ways with the compression and see what happened. The final result was to choose to increase the dipole strength of the interaction as $(-4/R^2)$ MeV, where the strength for $R=1$ was found in our mass 11 studies.[4] Calculations of the multiplet structure based on both models will be presented in Sec. IV.

III. α PARTICLE CORE EXCITATION MODEL OF ^{15}N

In this section we present the results obtained by attempting to describe the observed multiplet structure of ^{15}N using a standard core excitation model. This discussion is restricted to ^{15}N simply because the spins and parities are best known in this case. The similar multiplet structure observed in ^{15}O leads us to assume that analogous results should hold in that case also.

As Weller pointed out, the structure of the positive parity states in the 14 MeV excitation region of ^{15}N strongly suggests that those states might be approximately described as resulting from an $L=1$ α particle state coupled to the lowest negative parity states of ^{11}B through a dipole interaction term of the standard form used in core excitation calculations:

$$H_{\text{int}} = -\eta_I \vec{I}\cdot\vec{L}, \tag{2}$$

where $\vec{I}$ is the spin of the core state and $\vec{L}$ is the angular momentum of the α particle. This interaction is diagonal in the basis formed by vector coupling the core and α particle states to a total angular momentum J. The multiplet energies are

$$E_J(I) = E_I^0 - \tfrac{1}{2}\eta_I \times [J(J+1) - I(I+1) - L(L+1)], \tag{3}$$

where E_I^0 is the center of gravity of the multiplet of states built on the particular core state I. Assuming that this description applied to the three lowest multiplets in the region of interest in ^{15}N and that these multiplets are built on the $\frac{3}{2}^-$, $\frac{1}{2}^-$, $\frac{5}{2}^-$ lowest states of ^{11}B, we find the following values for the E_I^0 and the strengths η_I:

$$E_{3/2}^0 = 13.3\ \text{MeV}, \quad \eta_{3/2} = +0.18\ \text{MeV};$$

$$E_{1/2}^0 = 13.8\ \text{MeV}, \quad \eta_{1/2} = -0.04\ \text{MeV};$$

$$E_{5/2}^0 = 14.2\ \text{MeV}, \quad \eta_{5/2} = -0.05\ \text{MeV}.$$

These results were obtained from energies given by Ramirez, Blue, and Weller.[6]

In the standard applications, a single strength η has been used for all multiplets. It is clear that, in this case, no single value can be assumed. Note also the sign difference which is due of course to the inversion of levels referred to in the Introduction.

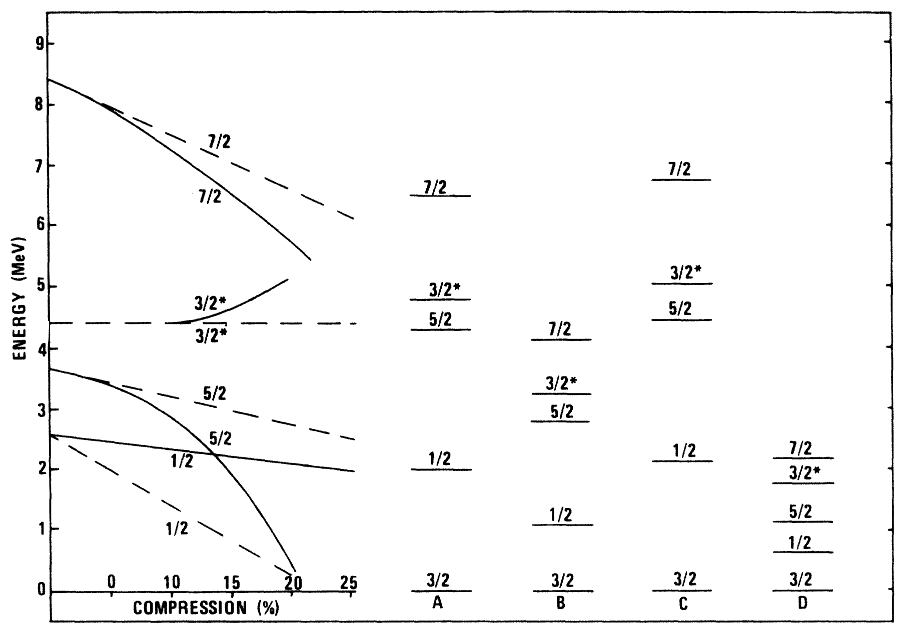

FIG. 1. At the left the compression of the target states as a function of the compression of the ^{12}C core is shown. The solid lines refer to the calculation in which the core-hole interaction was held constant. The dashed lines refer to the calculation in which the core-hole dipole interaction was inversely proportional to the compression squared. Spectra A and C are the unperturbed ^{11}C and ^{11}B level schemes. Spectra B and D are the presumed centers of gravity of the multiplets in ^{15}O and ^{15}N. Energies, spins, and parities of the mass 15 states used in these calculations are taken from Refs. 1, 6, and 13.

The question arises, could the inversion be accomplished by including the effects of a deformed α particle potential which Weller suggested, assuming the deformation to be a quadrupole one. We attempted, then to reproduce the observed structure with an interaction of the form

$$H_{\text{int}} = -\eta \vec{I} \cdot \vec{L} - \chi \underline{Q}(c) \cdot \underline{Q}(\alpha), \quad (4)$$

where $\underline{Q}(\alpha)$ is the single α particle quadrupole operator. The quadrupole matrix elements of $\underline{Q}(c)$, the core mass quadrupole operator, were taken from our previous studies of the mass 11 nuclei. We were unable to reproduce the observed ordering of energy levels with any value of the strength parameters η and χ. Our conclusion is that the inversion is not caused by a deformation; at least, not one of the quadrupole variety.

It seemed unlikely at the outset that the differences in sign of the approximate dipole strength seen above could be caused by a quadrupole interaction. The splitting of each multiplet seemed too patently dipole in character. This property led us to look for some dipole operator which would give a state dependent strength. The results of calculations using such an operator are presented in the following section.

IV. MICROSCOPIC VECTOR OPERATOR APPROACH

Rawitscher[7] has shown that the nucleon-nucleon spin-orbit potential will give rise to a dipole interaction H_d between an α particle with angular momentum $\vec{L}$ and the nucleon i of the target nucleus. This interaction, Rawitscher's equation (10), can be written as

$$H_d = \vec{V} \cdot \vec{L}, \quad \vec{V} = \sum \vec{V}_i, \quad (5)$$

where

$$\vec{V}_i = \frac{v_0(r_i, R)\vec{s}_i - r_i\, v_1(r_i, R)/R(\vec{s}_i + 2\pi \vec{T}_i)}{4\pi\mu}. \quad (6)$$

In this equation, r_i and $\vec{s}_i$ are the nucleon radial coordinate and spin, $\vec{T}_i$ is a spin-angle vector operator whose spherical tensor description is

$$T_\mu = \sum_m (2\, m\, 1(\mu - m)\,|\,1\,\mu)\; Y_{2m}(\hat{r}) s_{\mu - m}, \quad (7)$$

R is the raidal coordinate of the α particle, and μ is the reduced mass of the α particle and target nucleus. In obtaining the above result, reasonable approximations were made about the structure of the α particle. Also, a multipole expansion of the nucleon-α spin-orbit potential was made; the quantities v_0 and v_1 are the monopole and dipole coefficients resulting from this expansion. In his applications, Rawitscher used a free nucleon-α spin-orbit potential obtained by Morgan and Walter.[8] We follow his example.

Among the applications of this formalism which Rawitscher considers is a brief comment on the present case. He notes that if the $\frac{3}{2}^-$ ground state and $\frac{1}{2}^-$ first excited state of ^{11}B are taken to be single particle states and if the α particle is in an $L = 1$ state, then the observed inversion of level ordering is predicted by the interaction H_d given above. We were encouraged by this result to attempt a more detailed calculation, using this method and what we believe to be reasonable models of the low-lying negative parity states of the mass 11 nuclei. Two such calculations are described below, both of which treat the mass 11 nuclei as a $p_{3/2}$ hole coupled to the ^{12}C core. They differ in the manner in which the ^{12}C core states are treated.

In the first calculation, the ^{12}C core is assumed to have two states. The 0^+ ground state is treated as a particle-hole vacuum and the 2^+ excited state is assumed to be a one particle-one hole state as given by Goswami and Pal.[9] Antisymmetrization between the hole of the core in the excited state and the $p_{3/2}$ hole is ignored, however, It is perhaps worth pointing out that our previous study of the two core level model did not require a detailed model of these two states, but some model is required here to obtain matrix elements of the dipole operator H_d.

In the second calculation, we make explicit use of the rotational model of the ^{12}C core. It is a three core state model in which we assume that the 0^+, 2^+, 4^+ states are members of a $K = 0$ band, which eliminates contribution of the core to dipole matrix elements and permits the core to be treated in general terms.

A. Particle-hole model

In the calculation described in this section we again adopt the viewpoint of the core excited model. We imagine that the states of interest in ^{15}O and ^{15}N result from the interaction of an α particle in an $L = 1$ state with the appropriate mass 11 core. This interaction is envisioned in terms of a multipole expansion, the main contributors being the monopole and dipole parts.

The monopole part of the interaction is assumed to have two aspects. It effectively binds the α particle to the mass 11 core, thus allowing these continuum states to be treated as bound states. For convenience of calculation we take the potential to be a three dimensional harmonic oscillator with an oscillator parameter $\alpha = (m\omega/\hbar)^{1/2}$ of 0.62 fm^{-1}, the same as for nuclei in the mass 15 region.

Secondly, the monopole part is assumed to cause a compression of the core states as discussed in

Sec. II. Since our interest in this section is focused on the multiplet structure of the mass 15 states, we have allowed the amount of compression of the mass 11 core states to be somewhat adjustable but to coincide roughly with the center of gravity values quoted in Sec. III. It is also assumed that the core configuration amplitudes are unaffected by the compression but that the single particle wave functions are those appropriate to the mass 15 nucleus.

The dipole part of the interaction is taken to be that given in Eq. (5). It is convenient to rewrite H_d as a product of factors operating on the core and α particle separately so that standard techniques[10] can be used to evaluate the matrix elements. For this purpose, the α particle radial coordinate is integrated out. The basis states of the mass 15 system, in this model, are vector coupled core states ϕ_{IM_I} and α particle states $\phi_{NL} = R_{NL}(R)Y_{LM_L}$:

$$\Phi_{JM}(I, L) = [\phi_I \times \phi_{NL}]_{JM} = R_{NL}[\phi_I \times Y_L]_{JM} . \qquad (8)$$

Matrix elements of H_d can then be written as

$$\langle (I'L)JM|\vec{V}\cdot\vec{L}|(IL)JM\rangle = \Big((I'L)JM|\vec{v}\cdot\vec{L}|(IL)JM\Big) , \qquad (9)$$

where

$$\vec{v} = \sum_i \vec{v}_i$$

and

$$\vec{v}_i = \int_0^\infty R_{NL}{}^2 \vec{V}_i R^2 dR . \qquad (10)$$

Thus $\vec{v}_i$ depends only on the target nucleon's coordinate r_i and spin $\vec{s}_i$. The rounded brackets have been used to indicate that the α particle radial coordinate R has been integrated out. The matrix element in Eq. (9) can then be written in terms of the separate reduced matrix elements $\langle I'\|\vec{v}\|I\rangle$ and $\langle L\|\vec{L}\|L\rangle = 6^{1/2}$, for $L = 1$. To evaluate $\langle I'\|\vec{v}\|I\rangle$, we used the two core state wave functions obtained in our study of mass 11 nuclei. In this study, the low-lying negative parity states are constructed from the combination of a $p_{3/2}$ hole and the 0^+ and 2^+ states of ^{12}C. The resulting mass 11 states are:

$$|\tfrac{3}{2}(\text{g.s.})\rangle = A\,|(0^+p_{3/2}^{-1})\tfrac{3}{2}\rangle - (1-A^2)^{1/2}|(2^+p_{3/2}^{-1})\tfrac{3}{2}\rangle ,$$

$$|\tfrac{3}{2}^*\rangle = (1-A^2)^{1/2}|(0^+p_{3/2}^{-1})\tfrac{3}{2}\rangle + A\,|(2^+p_{3/2}^{-1})\tfrac{3}{2}\rangle ,$$

$$|I\rangle = |(2^+p_{3/2}^{-1})I\rangle , \quad I = \tfrac{1}{2}, \tfrac{5}{2} ,$$

and where A was found to be 0.741.

Since $\vec{v}$ is a sum of one body operators, it can be viewed as a sum of two terms $\vec{v}_h + \vec{v}_c$, with $\vec{v}_h$ acting on the $p_{3/2}$ hole and $\vec{v}_c$ acting on the ^{12}C core. The term involving $\vec{v}_h$ can be simplified by angular momentum and particle-hole conjugation methods to finally yield a reduced matrix element $\langle p_{3/2}\|\vec{v}_i\|p_{3/2}\rangle$ involving only one $p_{3/2}$ particle. In the evaluation of the core term involving $\vec{v}_c$, the only nonvanishing terms occur when the core is in the 2^+ state since a vector operator cannot connect the 0^+ state to itself or to the 2^+ state. Thus angular momentum simplification yields the single core reduced matrix element $\langle 2\|\vec{v}_c\|2\rangle$ to be evaluated.

So far, no details of the core 0^+ and 2^+ states have been required. However, in order to evaluate the quantity $\langle 2\|\vec{v}_c\|2\rangle$, some structure model of the 2^+ state is necessary. For this purpose, we employed the wave functions given by Goswami and Pal[9] for this state. The general form is

$$|2^+\rangle = \sum_{j,j'} B_{j,j'}\,|(j^{-1}j')2\rangle , \qquad (11)$$

where the ^{12}C 0^+ state is taken to be the particle-hole vacuum. The hole states $1s_{1/2}$, $1p_{3/2}$ and particle states $1p_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $1f_{5/2}$, $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$ are used for j, j', respectively, in combinations yielding $I^\pi = 2^+$.

The coefficients $B_{jj'}$ are given in Sec. 3.3 of Ref. 9. The operator $\vec{v}_c$ is again written as a sum of two terms; one operating on the hole, the other on the particle. Again standard angular momentum and particle-hole conjugation methods are used to separate the core reduced matrix elements into terms involving only single particle reduced matrix elements, $\langle j_1\|\vec{v}_c\|j_2\rangle$. The sets of single particle states involved are those listed after Eq. (11).

The net result is that the matrix elements of H_d are ultimately expressible in terms of reduced matrix elements of the single particle operators $\vec{s}$ and $\vec{T}$ in Eq. (6) and radial integrals over the nucleon, as well as the α particle, of $v_0(r, R)$ and $v_1(r, R)$. The main computational problem, then, is to obtain the radial integrals. As indicated earlier, we used the free nucleon-α particle spin-orbit potential obtained by Morgan and Walter.[8] The detailed definitions of the nucleon-α spin-orbit potential used and the integrals for the multipole expansion coefficients are given by Rawitscher[7] and need not be repeated here. The geometrical parameters of the derivative Woods-Saxon form used are reasonably well known but there is an element of ambiguity in the over-all strength parameter. The problem is that this strength is found to be somewhat energy dependent in the free nucleon-α case and it is not clear what relative energy value to use. We return to this point later.

The integrals giving the multipole expansion coefficients v_0 and v_1 as well as the integrations over the α particle and nucleon radial coordinates are all evaluated numerically. We used harmonic oscillator wave functions with oscillator parameter $\alpha = 0.62$ fm^{-1} in each radial integral, including the α particle. This should be approximately the correct value for mass 15 nuclei. The integrals were found to be quite sensitive to changes in this value, as one might expect.

The α particle orbital was assumed to be a $1p$ state in the results shown below. Simple cluster model arguments indicate that a more appropriate orbital would be the $3p$ state. However, when this function was used the integrals obtained were quite small due to poor overlap of the various radial functions and multipole moment coefficients. The small integrals required an increased strength (see below) beyond what one could reasonably expect to exist.

As mentioned above, the main item of uncertainty in the interaction is the strength V_0 of the nucleon-α spin-orbit potential. Rawitscher used a value of 4.31 MeV in the cases he considered but found that the theoretical α-nucleus $\vec{I}\cdot\vec{L}$ interaction strengths thus obtained were smaller than the ones obtained phenomenologically by a factor of 10 or more. Our decision was to look at the multiplet structure of the mass 15 states as a function of this strength parameter. Since it occurs as an over-all multiplier we, in effect, are considering it to be the strength parameter of the interaction H_d.

The calculated energies of the five states of ^{15}O and ^{15}N built on the $\frac{3}{2}^-$ ground state and $\frac{1}{2}^-$ first excited state of ^{11}C and ^{11}B respectively are shown in Fig. 2 as a function of the spin-orbit strength V_0. We have shown only these two multiplets because we known from our mass 11 studies that the two core state model does not give an accurate picture of the higher excited states. We expect our wave functions for the mass 11 system to be even poorer in the compressed case due to increased mixing of omitted configurations. The curves were obtained by diagonalization of the energy matrix although clearly first order perturbation theory would suffice for strengths up to around 20 MeV.

In Fig. 3, the calculated results for $V_0 = 20$ MeV are compared with the observed positive parity states in ^{15}N and ^{15}O in the 13 MeV energy region. It is clear that the inversion of levels is obtained and that the sign of the interaction is also correctly predicted since the opposite sign for the strength parameter would have reversed the ordering in each multiplet. A strength parameter of 20 MeV is perhaps not unreasonable. This point is discussed further in the final section.

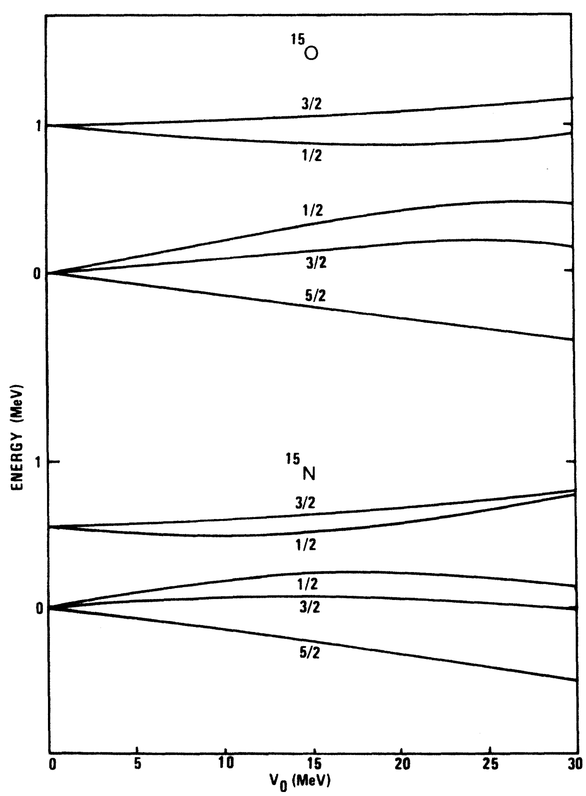

FIG. 2. Multiplet splitting as a function of spin-orbit interaction strength in the particle-hole core model.

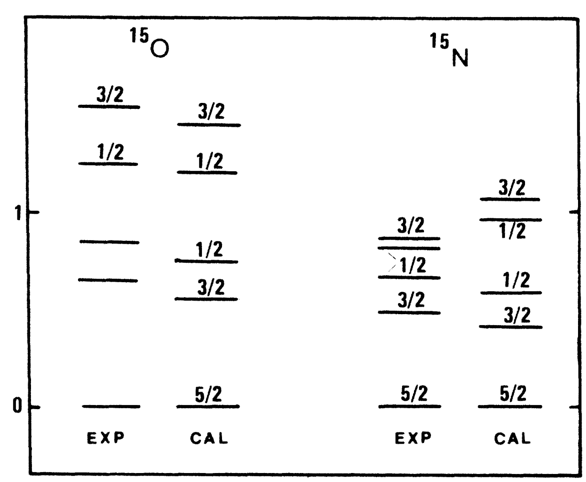

FIG. 3. Comparison between the calculated multiplet structure and the theoretical multiplet structure of ^{15}O and ^{15}N for a spin-orbit interaction strength of 20 MeV.

We view the weaknesses of this approach to be that antisymmetrization between the $p_{3/2}$ hole of the mass 11 model and the hole in the Goswami and Pal wave function was ignored, that the specific compression of the target states used was arbitrarily chosen to be about the multiple center of gravity values, and finally only the two lowest multiplets of states were reasonably well described by the calculation. The latter two objections are dealt with in the calculations presented in Sec. IV B.

B. Rotational model

In this section we will describe calculations based on a model which describes the target states as arising from the coupling of a $p_{3/2}$ hole to the first three states of the $K=0$ rotational band of ^{12}C. Our picture of the mass 15 states under consideration will be quite similar to that of the preceding section in that an α particle in an $L=1$ state is coupled to the low-lying negative parity states of the mass 11 nuclei through the dipole operator in Eq. (5) above.

The model chosen for the mass 11 greatly simplifies the calculation of the interaction between the α particle and the target. The simplification results from the assumption that the interaction is a sum of one body operators. A monopole operator of this type cannot change the relative spacing of the ^{12}C states from a $K=0$ rotational band.[11] The entire interaction between the α particle and the target depends, in this picture, only on the interaction between the α particle and the $p_{3/2}$ hole.

We will present two calculations below based on the two models of the compression discussed in

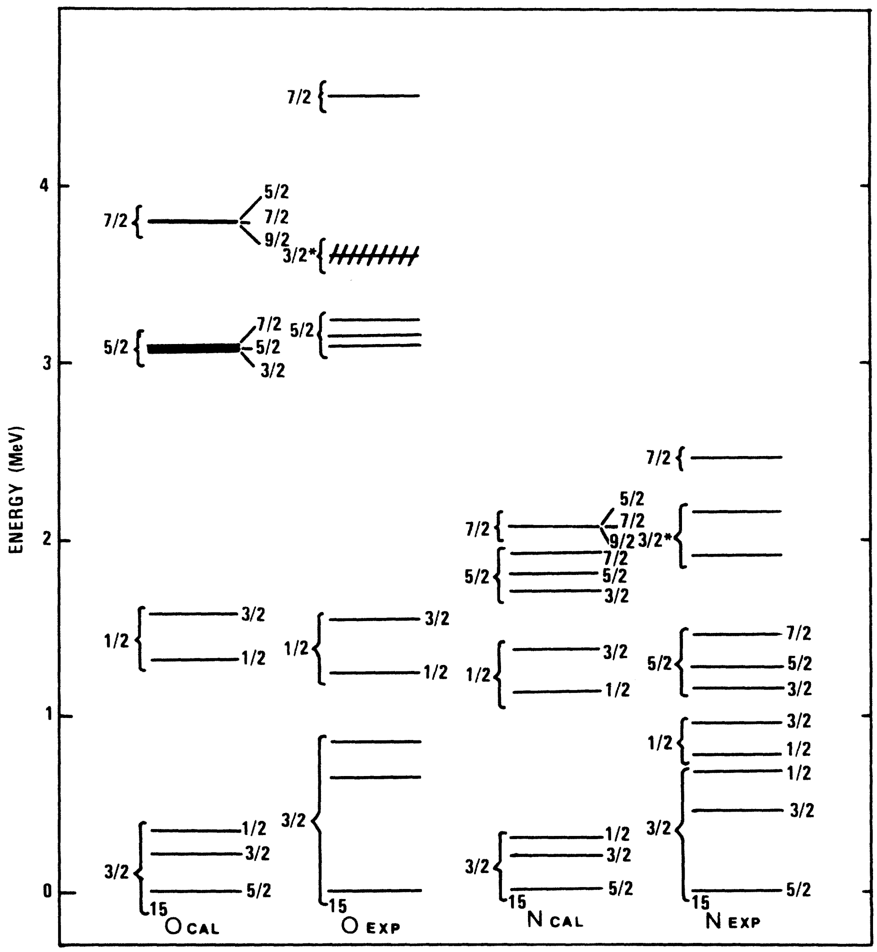

FIG. 4. Experimental and calculated multiplet structure of ^{15}O and ^{15}N using the rotational core model. The spin-orbit interaction strength was 25 MeV and the core-hole interaction strengths in the target have been held constant. The number next to the braces in the calculated spectra denote the mass 11 parent state of each multiplet and in the experimental spectra, the assumed parent.

Sec. II above.

First we calculated the multiplet structure using mass 11 wave functions obtained by compressing the ^{12}C core energies but leaving the interaction strengths between the hole and the core unchanged. Since the unperturbed calculated mass 11 energies are, particularly in the $\frac{1}{2}^-$ case, somewhat different from the experimental energies we have used the energy differences caused by compression and subtracted them from experimental mass 11 energies. The results are shown in Fig. 4. The spectra are obtained by compressing the core states by 30% in the ^{15}O and 50% in the ^{15}N case. The interaction strength V_0 was 25 MeV and a $1p$ harmonic oscillator wave function was used to evaluate the radial integrals. First order perturbation theory was used to calculate the multiplet splitting since our calculation in the previous section shown for these strengths diagonalization did not produce significant changes in the splittings.

The results are obviously quite good producing inversions in the dipole ordering where appropriate. It should also be noticed that the dipole splitting appears to fall off in energy as is found experimentally [see Sec. I(c)].

A second version of the compression allowed the dipole strength of the interaction between the $p_{3/2}$ hole and the ^{12}C core, which in our picture form the target, to vary. Here we have used the actual calculated energies rather than energy differences and experimental energies. The ^{15}O calculation presented in Fig. 5 was calculated assuming that the compression of the core was predicted from the simple relationship $R = \alpha_0^2/\alpha^2$ obtained in Sec. II yielding a core compression of 12%. The reduced matrix element $\langle \frac{3}{2}^{-1} \| \vec{v}_i \| \frac{3}{2}^{-1} \rangle$ for the $p_{3/2}$ hole was evaluated using the oscillator parameter appropriate to the compression. The interaction strength used was $V_0 = 20$ MeV.

In order to obtain the ^{15}N spectrum shown in Fig. 5 a further core compression was required. The compression used was 18%; i.e., the compressed ^{12}C core energies are 18% of the free core values. Again the matrix elements were calculated using the oscillator parameter appropriate to this compression of 18% and a strength V_0 of 20 MeV.

Neither the $\frac{7}{2}^-$ nor $\frac{3}{2}^{-*}$ multiplets appear in

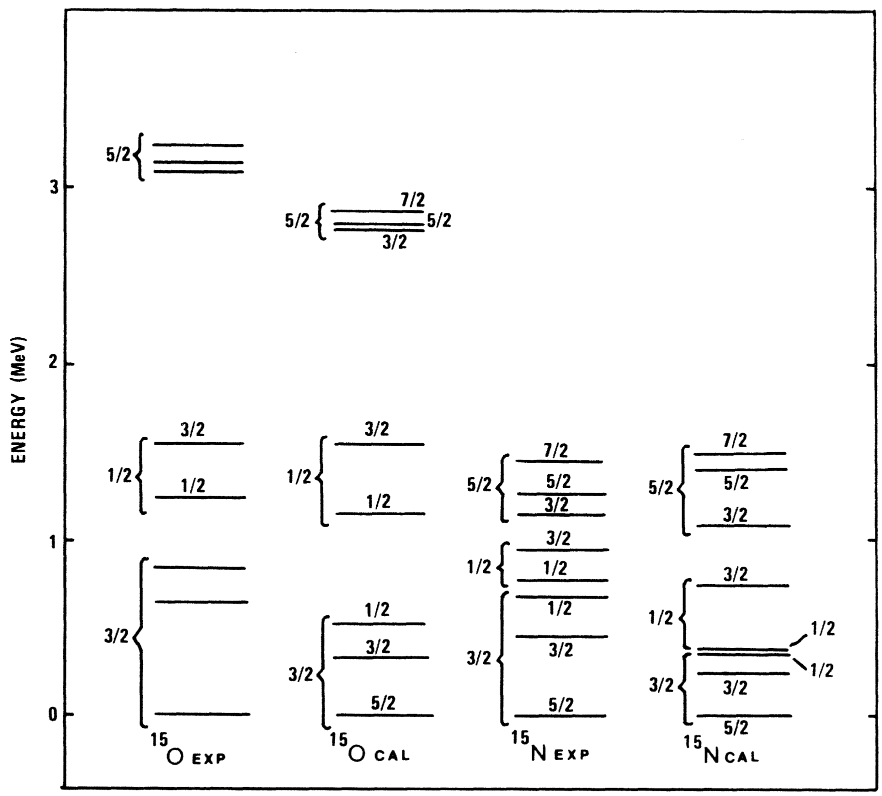

FIG. 5. Experimental and calculated multiplet structure of ^{15}O and ^{15}N using the rotational core model. The spin-orbit interaction strength was 20 MeV and the dipole core-hole interaction strength in the target has been varied inversely with the square of the core compression. The braces in the calculated spectra denote the mass 11 parent state of each multiplet and in the experimental spectra, the assumed parent.

Fig. 5 since the energy of both is too high although as can be seen from Fig. 1 the $\frac{7}{2}^-$ state will have come down considerably in energy with the compressions of 12 and 18%. Although not shown, we found that the splitting of the $\frac{7}{2}^-$ multiplet is again very small in agreement with experiment and again, the ordering of states is correctly given by the calculation.

In both calculations of this section, the compression of the mass 11 states has been obtained more naturally than was done in the particle-hole model of the previous subsection. Moreover, good agreement between calculation and experiment is obtained for almost all multiplets considered by Weller. The multiplet built on the $\frac{3}{2}^*$ is an exception, however. This was to be expected since the dipole properties of this one state are not well described by our mass 11 model.[4]

Two further points should be made with respect to the calculations discussed in this subsection. First, the energy spectra obtained do not depend strongly on the specific nature of the dipole operator; just that it operates on the $p_{3/2}$ hole only. Hence any one body vector operator with an adjustable over-all strength would suffice. The operator which we used, however, was satisfying in that its microscopic origin is clear and the sign and approximate strength were known from outside considerations. Secondly, as a bonus in these calculations, we also observe a decrease in splitting of the multiplets with excitation. This decrease is observed in the experimental spectrum as noted in Sec. I(c).

It is perhaps also worth noting that although the centers of gravity of the multiplets were obtained somewhat differently in the results shown in Figs. 4 and 5, there is very little difference in the results.

V. SUMMARY AND CONCLUSIONS

As noted in the Introduction, we set as a goal to investigate further the α particle core excitation model proposed by Weller[1] to explain the multiplet structure observed in certain resonances in highly excited states of light nuclei, notably ^{15}N and ^{15}O. The two essential features of interest are the apparent compression of the centers of gravity of the observed multiplets compared to the corresponding target states and the absolute and relative splitting of these multiplets.

Weller suggested that the compression was due to an expansion of the nuclear well due to the presence of the α particle. We feel that a convincing case has been made in Sec. II supporting this view. We have used the standard harmonic oscillator parameters $\hbar\omega = 41A^{-1/3}$ and $\alpha = (m\omega/\hbar)^{1/2}$ as a guide. When the α particle is added to the target of mass number A, the effect is to replace A by $A+4$ in these parameters thus giving increased ranges, α^{-1}, for the single particle wave functions and an increased moment of inertia for $I/I_0 = \alpha_0^2/\alpha^2$ for rotational states. The result is a compression of the target states.

In considering the multiplet structure, it was found that the standard core excitation model, discussed in Sec. III, was inadequate to produce the inversion of levels seen in the mass 15 states. The standard model is essentially a macroscopic model in which details of the target are not taken into account.

Good results have been obtained for the multiplet structure, however, using a microscopic operator model developed by Rawitscher. In this picture, the multiplet structure is presumed to arise from the nucleon-nucleon spin-orbit interaction averaged over the target and α particle coordinates. In the calculations based on this approach, the target mass 11 states were considered to be formed by coupling of a $p_{3/2}$ hole to ^{12}C core states. This led to two calculations, one assuming a spherical ^{12}C ground state and a first excited state generated by particle-hole excitation, and a second calculation assuming the ^{12}C core to be a rotator. Good results for the level ordering and splitting were obtained in both cases in straightforward calculation. The latter case has the virtue of making fewer assumptions and giving more of the observed spectrum.

It would be very helpful if the spins and parities of states in ^{15}O and ^{15}N were better known, particularly in the ^{15}O case. Recently experimental results have been published[12] on the states above 16 MeV excitation in ^{15}O although no spins and no definite parities were assigned. The spins and parities used in this paper were taken from the work of Weller and collaborators.[1,6,13] Similarly, it would be helpful if the multiplet structure were seen in other nuclei. The model has been applied to resonances seen in ^{10}B[14] and ^{19}Ne[15] but in these cases the α particle appears to be in an $L=0$ state and so there is no multiplet structure.

The main item of uncertainty throughout Sec. IV is the strength of the dipole operator which is the strength of the nucleon-α particle spin-orbit potential in the Rawitscher approach. We found that a spin-orbit strength of around 20 MeV gave good results. We attempted to understand this as follows.

Morgan and Walter[8] found that the energy dependence of this interaction strength could be expressed as $(3.95+0.144E)$, where E is the relative energy of the α particle and nucleon. Using a harmonic oscillator model for both the α particle in a $1p$ orbit and a nucleon in a $1p$ orbit, we estimate the relative kinetic energy to be about 13 MeV. Using this as E in the above formula gives $V_0 = 6$ MeV which is too small by a factor of 3 or 4. It seems quite likely, however, that the free nucleon-α interaction must be modified for use in bound states calculations. Also, it seems likely that dipole operators of similar nature to that in Eq. (5) could arise from other sources. (See Sec. II and Ref. 7).

Finally, we have commented that antisymmetrization effects have not been included in the mass 11 states. This is expected to be a major factor in the description of the higher excited states of the target and hence of the multiplet structure in the mass 15 cases built on these states. Perhaps all of these effects work together to account for the required additional strength.

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