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Hypertriton as a test of theoretical hyperon-nucleon potentials*

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A separable potential approximation to recent meson theoretic hyperon-nucleon potentials is made such that the low energy scattering parameters are reproduced. The resulting charge asymmetric potentials describing the Λ-p and Λ-n singlet and triplet interactions are used in a Faddeev-type calculation of the hypertriton binding energy. One particular one-boson exchange potential model, having effective ranges greater than 3 fm, gives a reasonable binding.

[NUCLEAR STRUCTURE $^3_\Lambda$H, Y-N potentials, separable potential three-body calculation, B_Λ.]

I. INTRODUCTION

Recently Nagels, Rijken, and deSwart[1,2] have developed meson theoretic potentials to describe the low-energy nucleon-nucleon (N-N) and hyperon-nucleon (Y-N) scattering data in a multichannel Schrödinger equation formalism. Mass differences in the various isomultiplets and symmetry breaking exchanges were included in a combined analysis of the N-N, Λ-p, Σ^+-p, Σ^--p, etc., data. A determination of the resulting low-energy Λ-N scattering parameters, the scattering length a and the effective range r, showed the effects of a sizeable charge asymmetry. Model A, which included nonets of pseudoscalar and vector mesons and an uncorrelated 2π exchange, provided the following set of low-energy scattering parameters[1]:

$$a^s_{p\Lambda} = -2.16 \pm 0.26 \text{ fm} \quad r^s_{p\Lambda} = 2.03 \pm 0.10 \text{ fm}$$
$$a^t_{p\Lambda} = -1.36 \pm 0.07 \text{ fm} \quad r^t_{p\Lambda} = 2.31 \pm 0.08 \text{ fm}$$
$$a^s_{n\Lambda} = -2.67 \pm 0.35 \text{ fm} \quad r^s_{n\Lambda} = 2.04 \pm 0.10 \text{ fm}$$
$$a^t_{n\Lambda} = -1.02 \pm 0.05 \text{ fm} \quad r^t_{n\Lambda} = 2.55 \pm 0.10 \text{ fm}.$$

In contrast Model B, which was based on a one-boson exchange (OBE) model of the interaction, provided[2]:

$$a^s_{p\Lambda} = -2.11 \pm 1.23 \text{ fm} \quad r^s_{p\Lambda} = 3.19 \pm 0.65 \text{ fm}$$
$$a^t_{p\Lambda} = -1.88 \pm 0.57 \text{ fm} \quad r^t_{p\Lambda} = 3.16 \pm 0.38 \text{ fm}$$
$$a^s_{n\Lambda} = -2.37 \pm 1.53 \text{ fm} \quad r^s_{n\Lambda} = 3.09 \pm 0.63 \text{ fm}$$
$$a^t_{n\Lambda} = -1.66 \pm 0.48 \text{ fm} \quad r^t_{n\Lambda} = 3.33 \pm 0.40 \text{ fm}.$$

We shall use potentials reproducing these low-energy scattering parameters in hypernuclear binding energy calculations.

The hypertriton $^3_\Lambda$H is the lightest of the bound hypernuclei. As such it has long received considerable attention,[3] usually in the sophisticated formalism of a Faddeev-type calculation. The most prolific contributors have been Schick, Hetherington, and co-workers.[4-6] The level of complexity in the calculations has reached the inclusion of Λ-Σ coupling,[5] tensor forces, short-range repulsion,[6] etc., normally in a separable-potential approximation. It was pointed out in Ref. 1 that it has been all too easy to fit the low-energy Λ-p elastic scattering data. Therefore, we wish to examine the predictions of the new meson theoretic potentials in a calculation of the binding energy of the hypertriton, both to improve the theoretical estimate of the Λ-separation energy $[(B_\Lambda = B(^3_\Lambda\text{H}) - B(^2\text{H})]$ and to determine whether bound-state calculations can help differentiate between the two proposed models of the Y-N interaction.

Because $^3_\Lambda$H is lightly bound and therefore a loose structure, it should not be particularly sensitive to the short-range (high-momentum) character of the Y-N force; the mean separation of either Λ-N subsystem should be as large as the range of the interaction. (A reasonably accurate binding energy for the triton can be obtained from simple, one-term separable N-N potentials fitted to the low-energy N-N scattering parameters.) Thus we propose to use s-wave separable potentials to describe the Y-N interaction in a calculation of the $^3_\Lambda$H binding energy. Furthermore, since the scattering lengths and effective ranges obtained by Nagels, Rijken, and deSwart contain the effects of Λ-Σ coupling, we propose to use "effective" Λ-N interactions without explicit coupling of the Λ to the Σ but fitted to the low-energy scattering parameters containing this information. We do not wish to underemphasize the importance of Λ-Σ coupling, which has

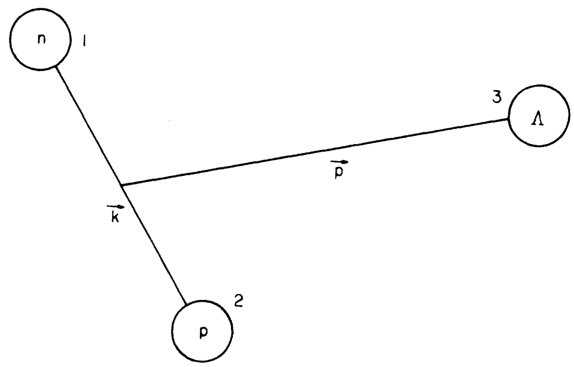

FIG. 1. Momentum space coordinate system used to describe the constituents of ${}^3_\Lambda$H.

been shown to provide significant effects in the s-shell hypernuclei.[7,8] However, our procedure will include the effects of the explicit Λ-Σ coupling which was included in the models of Refs. 1 and 2.

In Sec. II we describe the system of coupled equations that one obtains by means of a Schrödinger equation development of three-body theory. One advantage of this formulation of the problem is that one can obtain the wave function at the same time that one obtains the binding energy. In Sec. III we present results for the Λ-separation energy for both potential models A and B as well as some previously proposed Y-N separable potentials; both the ground state ($J=\frac{1}{2}$) and the possible excited state ($J=\frac{3}{2}$) are considered. A discussion of our conclusion appears in Sec. IV.

II. THEORETICAL DEVELOPMENT

The ground state of the hypertriton has quantum numbers ($T=0, J^\pi=\frac{1}{2}^+$), which in the context of the present work can be thought of as the coupling of a deuteron ($T=0, J^\pi=1^+$) with a Λ ($T=0, J^\pi=\frac{1}{2}^+$). For convenience we shall label the particles and momenta as indicated in Fig. 1. The three-body bound-state equation can then be written from the momentum representation of the Schrödinger equation:

$$\left[\frac{k^2}{2\mu}+\frac{p^2}{2\mu_\Lambda}+E_3\right]\psi(k,p) = -\int d^3k' V_{np}(k,k')\psi(k',p) - \int d^3k'_{31} V_{n\Lambda}(k_{31},k'_{31})\psi(k'_{31},p_2) - \int d^3k'_{23} V_{p\Lambda}(k_{23},k'_{23})\psi(k'_{23},p_1)\,, \tag{1}$$

where μ is the n-p reduced mass, μ_Λ is the reduced mass of the Λ-$2N$ system, and E_3 is the ${}^3_\Lambda$H binding energy.

Utilizing the notation of Danos[9] as elaborated upon by Lehman and O'Connell,[10] one can express the s-wave separable interactions as

$$\begin{aligned}
V_{np}(k,k') &= -\frac{\lambda_t}{2\mu} g_t(k) g_t(k') [\chi^{[1]}(12)\times\bar{\chi}^{[1]}(12)]^{[0]}\hat{1}\,,\\
V_{n\Lambda}(k,k') &= -\frac{\Lambda_t^n}{2\mu_{n\Lambda}} h_t^n(k) h_t^n(k') [\chi^{[1]}(31)\times\bar{\chi}^{[1]}(31)]^{[0]}\hat{1} - \frac{\Lambda_s^n}{2\mu_{n\Lambda}} h_s^n(k) h_s^n(k') [\chi^{[0]}(31)\times\bar{\chi}^{[0]}(31)]^{[0]}\hat{0}\,,\\
V_{p\Lambda}(k,k') &= -\frac{\Lambda_t^p}{2\mu_{p\Lambda}} h_t^p(k) h_t^p(k') [\chi^{[1]}(23)\times\bar{\chi}^{[1]}(23)]^{[0]}\hat{1} - \frac{\Lambda_s^p}{2\mu_{p\Lambda}} h_s^p(k) h_s^p(k') [\chi^{[0]}(23)\times\bar{\chi}^{[0]}(23)]^{[0]}\hat{0}\,,
\end{aligned} \tag{2}$$

where λ_t and Λ_j^i are the strengths of the interactions, the μ's are the appropriate two-body reduced masses; the form factors

$$\begin{aligned}
g_t(k) &= (\beta_{np}^{t\,2}+k^2)^{-1}\,,\\
h_t^n(k) &= (\beta_{n\Lambda}^{t\,2}+k^2)^{-1}\,, \text{ etc.},
\end{aligned} \tag{3}$$

which are the simplest one-parameter forms satisfying threshold and asymptotic constraints in momentum space, contain the interaction ranges β, $[\chi^{[J]}\times\bar{\chi}^{[J]}]^{[0]}$ are the spin-projection operators, and $\hat{J}=(2J+1)^{1/2}$.

The resulting integral equation for the (12)-permutation of the complete wave function is

$$\begin{aligned}
\psi(k,p) &= \lambda_t g_t(k) F_\Lambda(p) [\chi^{[1]}(12)\times\chi^{[1/2]}(3)]^{[1/2]}\\
&+ \Lambda_t^n h_t^n(k_{31}) F_t^p(p_2) [\chi^{[1]}(31)\times\chi^{[1/2]}(2)]^{[1/2]}\\
&+ \Lambda_s^n h_s^n(k_{31}) F_s^p(p_2) [\chi^{[0]}(31)\times\chi^{[1/2]}(2)]^{[1/2]}\\
&+ \Lambda_t^p h_t^p(k_{23}) F_t^n(p_1) [\chi^{[1]}(23)\times\chi^{[1/2]}(1)]^{[1/2]}\\
&- \Lambda_s^p h_s^p(k_{23}) F_s^n(p_1) [\chi^{[0]}(23)\times\chi^{[1/2]}(1)]^{[1/2]}\,,
\end{aligned} \tag{4}$$

where the minus sign before Λ_s^p is required by symmetry of the wave function under interchange of neutron and proton. In Eq. (4) we have defined the spectator functions:

$$F_\Lambda(p)[\chi^{[1]}(12)\times\chi^{[1/2]}(3)]^{[1/2]}=\hat{1}[\chi^{[1]}(12)\times\bar{\chi}^{[1]}(12)]^{[0]}\int d^3k' g_t(k')\psi(k',p)\,, \tag{5a}$$

$$F_t^p(p_2)[\chi^{[1]}(31)\times\chi^{[1/2]}(2)]^{[1/2]}=\frac{\mu}{\mu_{n\Lambda}}\,\hat{1}[\chi^{[1]}(31)\times\bar{\chi}^{[1]}(31)]^{[0]}\int d^3k'_{31}h_t^n(k'_{31})\psi(k'_{31},p_2)\,, \tag{5b}$$

$$F_s^p(p_2)[\chi^{[0]}(31)\times\chi^{[1/2]}(2)]^{[1/2]}=\frac{\mu}{\mu_{n\Lambda}}\,\hat{0}[\chi^{[0]}(31)\times\bar{\chi}^{[0]}(31)]^{[0]}\int d^3k'_{31}h_s^n(k'_{31})\psi(k'_{31},p_2)\,, \tag{5c}$$

$$F_t^n(p_1)[\chi^{[1]}(23)\times\chi^{[1/2]}(1)]^{[1/2]}=\frac{\mu}{\mu_{p\Lambda}}\,\hat{1}[\chi^{[1]}(23)\times\bar{\chi}^{[1]}(23)]^{[0]}\int d^3k'_{23}h_t^p(k'_{23})\psi k'_{23},p_1)\,, \tag{5d}$$

$$F_s^n(p_1)[\chi^{[0]}(23)\times\chi^{[1/2]}(1)]^{[1/2]}=\frac{\mu}{\mu_{p\Lambda}}\,\hat{0}[\chi^{[0]}(23)\times\bar{\chi}^{[0]}(23)]^{[0]}\int d^3k'_{23}h_s^p(k'_{23})\psi(k'_{23},p_1)\,. \tag{5e}$$

Inserting Eq. (4) into the original expression in Eq. (1) leads one to the desired set of integral equations that determine the spectator functions:

$$\begin{aligned}\Big\{1-\lambda_t\int d^3k\,g_t^2(k)/\Delta\Big\}F_\Lambda(p)=&-\tfrac{1}{2}\sqrt{3}\,\Lambda_s^n\int d^3k\,g_t(k)h_s^n(k_{31})F_s^p(p_2)/\Delta-\tfrac{1}{2}\Lambda_t^n\int d^3k\,g_t(k)h_t^n(k_{31})F_t^p(p_2)/\Delta\\&-\tfrac{1}{2}\sqrt{3}\,\Lambda_s^p\int d^3k\,g_t(k)h_s^p(k_{23})F_s^n(p_1)/\Delta-\tfrac{1}{2}\Lambda_t^p\int d^3k\,g_t(k)h_t^p(k_{23})F_t^n(p_1)/\Delta\,,\end{aligned} \tag{6a}$$

$$\begin{aligned}\Big\{\frac{\mu_{p\Lambda}}{\mu}-\Lambda_s^p\int d^3k_{23}[h_s^p(k_{23})]^2/\Delta\Big\}F_s^n(p_1)=&-\tfrac{1}{2}\sqrt{3}\,\lambda_t\int d^3k_{23}h_s^p(k_{23})g_t(k)F_\Lambda(p)/\Delta\\&+\tfrac{1}{2}\sqrt{3}\,\Lambda_t^n\int d^3k_{23}h_s^p(k_{23})h_t^n(k_{31})F_t^p(p_2)/\Delta\\&+\tfrac{1}{2}\Lambda_s^n\int d^3k_{23}h_s^p(k_{23})h_s^n(k_{31})F_s^p(p_2)/\Delta\,,\end{aligned} \tag{6b}$$

$$\begin{aligned}\Big\{\frac{\mu_{p\Lambda}}{\mu}-\Lambda_t^p\int d^3k_{23}[h_t^p(k_{23})]^2/\Delta\Big\}F_t^n(p_1)=&-\tfrac{1}{2}\lambda_t\int d^3k_{23}h_t^p(k_{23})g_t(k)F_\Lambda(p)/\Delta-\tfrac{1}{2}\Lambda_t^n\int d^3k_{23}h_t^p(k_{23})h_t^n(k_{31})F_t^p(p_2)/\Delta\\&+\tfrac{1}{2}\sqrt{3}\,\Lambda_s^n\int d^3k_{23}h_t^p(k_{23})h_s^n(k_{31})F_s^p(p_2)/\Delta\,,\end{aligned} \tag{6c}$$

$$\begin{aligned}\Big\{\frac{\mu_{n\Lambda}}{\mu}-\Lambda_s^n\int d^3k_{31}[h_s^n(k_{31})]^2/\Delta\Big\}F_s^p(p_2)&\\=-\tfrac{1}{2}\sqrt{3}\,\lambda_t\int d^3k_{31}h_s^n(k_{31})g_t(k)F_\Lambda(p)/\Delta&+\tfrac{1}{2}\sqrt{3}\,\Lambda_t^p\int d^3k_{31}h_s^n(k_{31})h_t^p(k_{23})F_t^n(p_1)/\Delta\\&+\tfrac{1}{2}\Lambda_s^p\int d^3k_{31}h_s^n(k_{31})h_s^p(k_{23})F_s^n(p_1)/\Delta\,,\end{aligned} \tag{6d}$$

$$\begin{aligned}\Big\{\frac{\mu_{n\Lambda}}{\mu}-\Lambda_t^n\int d^3k_{31}[h_t^n(k_{31})]^2/\Delta\Big\}F_t^p(p_2)=&-\tfrac{1}{2}\lambda_t\int d^3k_{31}h_t^n(k_{31})g_t(k)F_\Lambda(p)/\Delta-\tfrac{1}{2}\Lambda_t^p\int d^3k_{31}h_t^n(k_{31})h_t^p(k_{23})F_t^n(p_1)/\Delta\\&+\tfrac{1}{2}\sqrt{3}\,\Lambda_s^p\int d^3k_{31}h_t^n(k_{31})h_s^p(k_{23})F_s^n(p_1)/\Delta\,,\end{aligned} \tag{6e}$$

where

$$\Delta = \Delta(k, p, E_3)$$
$$= k^2 + \frac{\mu}{\mu_\Lambda} p^2 + 2\mu E_3 .$$

One need only transform the variables under the integral on the right-hand side of Eq. (6) such that the argument of the spectator functions becomes the variable of integration. The integral equations are then easily solved by iteration or inversion. A 16 point Gegenbauer integration scheme was used, the Gegenbauer weights and abscissas being optimal for the functional form of the separable potentials chosen.[11]

It should be noted that for the separable potentials defined in Eq. (3) one has the following relationship between the scattering length, effective range, interaction strength λ, and interaction range β:

$$\frac{1}{a} = \frac{-\beta^4}{2\pi^2\lambda} + \frac{\beta}{2} ,$$
$$r = \frac{2\beta^2}{\pi^2\lambda} + \frac{1}{\beta} .$$

In addition the deuteron binding energy $B(^2\mathrm{H})$ is related to the scattering length and effective range by

$$\gamma = \frac{1}{a} + \frac{1}{2} r \gamma^2 ,$$

where $\gamma^2 = 2\mu B(^2\mathrm{H})$. In the present calculations $a^t_{np} = 5.423$ fm and $r^t_{np} = 1.761$ fm were assumed; the corresponding deuteron binding energy is 2.225 MeV.

III. NUMERICAL RESULTS

Using the scattering lengths and effective ranges of the meson exchange potential model A of Ref. 1 to determine the parameters of our Λ-N separable interactions, we obtain a Λ-separation energy B_Λ of some 0.7 MeV. This is a disturbingly large value, especially since variation of either the scattering lengths or effective ranges to the limits of their uncertainties cannot bring the calculated value of B_Λ into agreement with the latest experimental value[12] of 0.15 ± 0.08 MeV.

In contrast, using the scattering lengths and effective ranges of the OBE potential model B of Ref. 2 to determine the parameters of our Λ-N separable interactions, we obtain a value of 0.28 MeV for B_Λ, reasonably close to the experimental value. The decreased binding compared to model A is due to the increase in the effective ranges in model B. (This dependence of the three-body binding energy on the effective range was recently discussed by Gibson and Stephenson[13] in a study of effects of a possible charge asymmetry in the N-N interaction on the $^3\mathrm{H}$-$^3\mathrm{He}$ binding energy difference.) The uncertainties associated with the effective ranges in model B make the experimental value of B_Λ easily attainable within the quoted errors.

We point out that in either model the fact that the triplet Λ-N interaction is so much weaker than the singlet ensures that the $J = \frac{3}{2}$ state of $^3_\Lambda\mathrm{H}$ will not be bound; i.e., the total binding would be less than that of the deuteron. (Only the triplet Λ-N interaction contributes to the binding of the $J = \frac{3}{2}$ state.) For model A one can obtain some rather narrow resonances, similar to that seen in the "singlet deuteron" where one is unbound by only 60 keV. However, for model B this phenomena does not occur.

In the table we summarize our results for these two calculations along with $^3_\Lambda\mathrm{H}$ binding energy results for several sets of scattering lengths and effective ranges quoted by Choudhury and Gautam.[14] In none of these cases[4, 15-17] was charge symmetry breaking considered; i.e., the same singlet and triplet Λ-N scattering parameters were assumed to describe both the Λ-p and Λ-n interactions. We do not believe these potentials to be as good as those obtained from the low-energy scattering parameters of Refs. 1 and 2, but we include them for completeness in comparison with previous experimental analyses of Λ-p elastic scattering data and in order to compare with some of the previous

TABLE I. Binding energy of the hypertriton and corresponding Λ-separation energy for various sets of Λ-N scattering lengths and effective ranges.

Ref.	a^s_p (fm)	r^s_p (fm)	a^t_p (fm)	r^t_p (fm)	a^s_n (fm)	r^s_n (fm)	a^t_n (fm)	r^t_n (fm)	$B(^3_\Lambda\mathrm{H})$ (MeV)	B_Λ (MeV)
1	−2.16	2.03	−1.32	2.31	−2.67	2.04	−1.02	2.55	2.92	0.70
2	−2.11	3.19	−1.88	3.16	−2.47	3.09	−1.66	3.33	2.50	0.28
15	−2.46	3.87	−2.07	4.50					2.39	0.16
16	−1.80	2.80	−1.60	3.30					2.35	0.12
17	−2.76	3.05	−1.96	3.50					2.74	0.51
4	−1.80	2.06	−0.40	4.00					2.23	0.004

theoretical assumptions. The small differences in the calculated values of the ${}^3_\Lambda$H binding energies appearing in the table and those deduced from the values of B_Λ quoted in Ref. 14 arise primarily from differences in the assumed values of the scattering parameters used to determine the n-p triplet interaction.

IV. DISCUSSION AND CONCLUSIONS

It should be noted that the Λ-separation energy coming from the low-energy scattering parameters of model B is close to the value obtained in the case of the charge symmetric potentials determined by the scattering parameters from the analyses of the Λ-p elastic scattering data by Alexander *et al.*[15,16] In those cases, the singlet effective ranges are comparable to those of model B. (The singlet interaction accounts for $\frac{3}{4}$ of the total Λ-N interaction in the triton.) The increased scattering length in Ref. 15 is compensated for by the larger effective range, and the opposite is true in Ref. 16. Thus the similarities in the hypertriton binding energy results are understood.

The potentials used in the earlier variational work of Herndon and Tang[17] produce overbinding just as we obtain in the case of model A. However, the characteristics of their interactions are quite different from those of model A: the ranges are large which tends to produce low ${}^3_\Lambda$H binding, but the singlet scattering length is quite large also which counteracts this tendency; the singlet potential of Ref. 17 is a much stronger two-body interaction than that of model A. In contrast, the *ad hoc* potentials of Hetherington and Schick[4] which do provide a small B_Λ do so because of the abnormally weak triplet contribution to the three-body binding. The small scattering length and large effective range both tend to reduce the ${}^3_\Lambda$H binding energy.

From the results of our calculations, we conclude that the OBE potential model B of Nagels, Rijken, and deSwart provides a better value of the hypertriton binding than does model A. The larger effective ranges of model B lead to the smaller value of B_Λ, one closer to the experimental value. Therefore, it appears that the potential model described in Ref. 2 gives a better representation of the low-energy Y-N data.

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[1]M. M. Nagels, T. A. Rijken, and J. J. deSwart, Ann. Phys. (N.Y.) 79, 338 (1973).

[2]M. M. Nagels, T. A. Rijken, and J. J. deSwart, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Šlaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1972), pp. 41–45; and to be published.

[3]R. H. Dalitz and B. W. Downs, Phys. Rev. 110, 958 (1958).

[4]J. H. Hetherington and L. H. Schick, Phys. Rev. 139, B1164 (1965).

[5]L. H. Schick and A. J. Toepfer, Phys. Rev. 170, 946 (1968).

[6]L. H. Schick and J. H. Hetherington, Phys. Rev. 156, 1602 (1967); B. G. Kashef and L. H. Schick, Nuovo Cimento 50B, 397 (1967).

[7]B. F. Gibson, A. Goldberg, and M. S. Weiss, Phys. Rev. C 6, 741 (1972); 8, 836 (1973).

[8]J. Dabrowski and E. Fedorynska, Nucl. Phys. A210, 509 (1973).

[9]M. Danos, Ann. Phys. (N.Y.) 63, 319 (1971).

[10]D. R. Lehman and J. S. O'Connell, National Bureau of Standards Monograph No. 136, 1973 (unpublished).

[11]T. Brady, M. Fuda, E. Harms, J. S. Levinger, and R. Stagat, Phys. Rev. 186, 1069 (1969).

[12]M. Juric, G. Bohm, J. Klabuhn, U. Krecker, F. Wysitzki, G. Coremans-Bertrand, J. Sacton, G. Wilquet, T. Cantwell, F. Esmael, A. Montwill, R. H. Davis, D. Kielczeaska, T. Pniewski, T. Tymieniecka, and J. Zakrzewski, Nucl. Phys. B52, 1 (1973).

[13]B. F. Gibson and G. J. Stephenson, Phys. Rev. C 8, 1222 (1973).

[14]H. R. Choudhury and V. P. Gautam, Phys. Rev. C 7, 74 (1973).

[15]G. Alexander, O. Bernary, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, A. Fridman, and B. Schiby, Phys. Lett. 19, 715 (1966).

[16]G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth and W. Lughofer, Phys. Rev. 173, 1952 (1968).

[17]R. C. Herndon and Y. C. Tang, Phys. Rev. 159, 853 (1967).