Neutron resonance spectroscopy. XV. The separated isotopes of Cd†

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The results of neutron time of flight spectroscopy measurements for the separated Cd isotopes (110, 112, 114, 116) and for natural Cd, using the Nevis synchrocyclotron, are given. Transmission and self-indication measurements were made for a range of sample thicknesses for natural Cd, and for each of the above even A separated isotopes. Resonance parameters, E_0 and $(g)\Gamma_n^0$, are given to ~ 10 keV for 110, 112, 114, 116, and to 2.3 keV for 111 and 113. The ^{111}Cd level assignment is from recent published measurements at the Oak Ridge electron linear accelerator using a sample enriched in ^{111}Cd. Levels in ^{113}Cd are those stronger ones seen in natural Cd, but not identified as due to other isotopes. We also obtained level parameters to ~ 10 keV for 181 levels which have no isotopic assignment. Many resonance Γ_γ and J values were obtained for 111 and 113, and a few Γ_γ values for even A isotopes, with $\langle\Gamma_\gamma\rangle$ of each isotope being between 100 and 110 meV. The $10^4 S_0$ values are 0.50 ± 0.10, 0.38 ± 0.06, 0.53 ± 0.09, 0.43 ± 0.07, 0.70 ± 0.19, and 0.20 ± 0.06 for 110, 111, 112, 113, 114, and 116, respectively. Comparison of the $(g\Gamma_n^0)^{1/2}$ distributions with Porter-Thomas theory for 110, 112, 114, and 111 shows that many p levels are observed. An estimate of the level detection sensitivity and a Bayes's theorem analysis permit us to identify the detected p levels, and to determine the p strength function with a relatively small uncertainty. Few expected missing weak s levels for each of these isotopes were then added to their p level subtracted sets. Various statistical tests were applied, showing that the final s level selection choices for 110, 112, 114 and 111 are in good agreement with the orthogonal ensemble theory. The $10^4 S_1$ values for 110, 112, 114, and 111 are 2.8 ± 0.6, 2.5 ± 0.5, 3.2 ± 1.0, and 3.5 ± 0.7. The $\langle D\rangle$ values for s population are (174 ± 18) eV (110); (24.0 ± 1.5) eV (111); (137 ± 8) eV (112); (22.1 ± 3.8) eV (113); (183 ± 29) eV (114); and (264 ± 38) eV (116).

[NUCLEAR REACTIONS ^{110,111,112,113,114,116}Cd(n, n), (n, γ), $E = 0$–10 keV; measured $\sigma_t(E)$; deduced E_0, $g\Gamma_n$, Γ_γ, J, S_0, $\langle D_0\rangle$, S_1; O.E. theory test.]

I. INTRODUCTION

This is one of a series[1-7] of papers reporting the results of high resolution pulsed neutron time of flight spectroscopy using the Columbia University Nevis synchrocyclotron as a source. In this paper, we present the results of measurements using various thickness samples of natural Cd, and of samples enriched in ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{116}Cd, the major abundant even A isotopes. Natural Cd consists of 1.22% ^{106}Cd, 0.89% ^{108}Cd, 12.43% ^{110}Cd, 12.86% ^{111}Cd, 23.79% ^{112}Cd, 12.34% ^{113}Cd, 28.81% ^{114}Cd, and 7.66% ^{116}Cd. Transmission measurements were made using our 202.05 m flight path with 16 000 detector timing channels of 40 ns width above 1280 eV, and with 2, 4, 8, ... times 40 ns widths at lower energies. A "self-indication" detector was used with a 39.57 m flight path. This detector used thin Cd samples at the detector, counting capture γ rays, with or without another Cd sample present in a transmission position nearer to the source. By methods described in earlier papers, we analyze the observed resonance transmission dips (200 m detector) or resonance capture peaks (40 m detector) vs neutron energy to determine the resonance parameters of observed levels.

The present measurements provide a large factor increase in the number of levels for which parameters are established for the even A separate Cd isotopes. In addition to the resonance energy, E_0, we always establish the reduced neutron partial width, $\Gamma_n^0 \equiv \Gamma_n(1\text{ eV}/E_0)^{1/2}$ for even A isotope resonances, or $g\Gamma_n^0$ for odd A isotope resonances. The spin statistical weight factor $g = \frac{1}{4}$ or $\frac{3}{4}$ for compound nuclear spin $J = 0$ or 1, respectively, since ^{111}Cd and ^{113}Cd both have $I = \frac{1}{2}(+)$ ground state. The total level width $\Gamma = \Gamma_n + \Gamma_\gamma$. In some favorable cases, we also establish the capture width, Γ_γ, and for some odd A resonances, the level spin J. The binding energy for an extra neutron is 7.93, 7.38, 6.98, 9.40, 6.54, 9.05, 6.14, and 5.76 MeV, respectively, for ^{106}Cd, ^{108}Cd, ^{110}Cd, ^{111}Cd, ^{112}Cd, ^{113}Cd, ^{114}Cd, and ^{116}Cd.

Cd is in a mass region of a minimum of the s strength function S_0, but near a split maximum of the p strength function, S_1. Thus, we tend to observe a mixed s and p resonance population for each isotope, with a poor ability to distinguish individual weak levels as s or p. The results for the even A isotopes of Cd thus are not favorable for tests of theories of single (statistical) level spacing systematics of the type which we made for the even A isotopes of mass $150 < A < 190$, where essentially all s levels are seen, but no p

levels are included over an energy region containing ~20 to 100 *s* levels per isotope.

Historically, Cd has been of interest to neutron physics for its large thermal neutron cross section due to the 0.178 eV resonance in ^{113}Cd. It is used as a thermal neutron filter.

Although many lower resolution studies have been made previously using samples enriched in particular Cd isotopes, no resonance assignments to date have been made for the low abundant ^{106}Cd and ^{108}Cd isotopes. We also assign no levels to these isotopes.

Previously, results[8] have been obtained for ^{110}Cd only for levels at 89.6 and 372 eV. In this paper, we give resonance parameters for 79 levels in ^{110}Cd to 10 keV.

The previous resonance parameter results for ^{112}Cd were only for the levels at 67 and 227 eV until the separated Cd isotope results of Shepkin, Adamchuk, Danelyan, and Muradyan[9] were presented at a 1966 Nuclear Data Conference in Paris. They give resonance parameters for eight levels in ^{112}Cd to 1450 eV. In this paper, we present results for 98 levels in ^{112}Cd to 11.5 keV.

Results for only two ^{114}Cd levels (59 and 121 eV) were known before Shepkin *et al.*[9] presented results for five more levels in ^{114}Cd to 1107 eV. In this paper, we give results for 54 levels to 10 keV.

The only previous resonance results for ^{116}Cd are those in Ref. 9 for a level at 29.3 eV. In this paper, we give results for 21 levels in ^{116}Cd to 9 keV.

The main previous results for the odd A isotopes, ^{111}Cd and ^{113}Cd, are those of Wasson and Allen[10] for ^{111}Cd using the Oak Ridge electron linear accelerator (ORELA) for capture measurements, and those of the Ref. 9. Wasson and Allen give level parameters for a large number of *s* and *p* levels to 2300 eV using a sample enriched in ^{111}Cd. The Shepkin *et al.* measurements[9] had much lower resolution. They list 21 levels in ^{111}Cd to 625 eV and 22 levels in ^{113}Cd to 858 eV. We observe resonances in natural Cd, but not in the enriched 110, 112, 114, or 116 Cd isotope samples which are at the positions of resonances identified as being due to ^{111}Cd by Wasson and Allen, or being due to ^{113}Cd by Shepkin *et al.* While some ^{111}Cd or ^{113}Cd resonances in natural Cd are shielded by strong levels in 110, 112, 114, or 116, we observe over half of the ^{111}Cd levels given by Wasson and Allen, with only very weak levels missed. The relatively poor energy resolution of Shepkin *et al.* resulted in their inability to resolve close levels in the upper parts of their energy regions. We observe all but one of their reported ^{113}Cd levels.

The following comments refer to resonances seen in our natural Cd samples, but not in our 110, 112, 114, and 116 Cd isotope samples. After subtracting levels to 2300 eV which are identified as due to ^{111}Cd by Wasson and Allen,[10] the remaining stronger levels to 2300 eV are considered to be due to ^{113}Cd if they are too strong to be due to ^{106}Cd or ^{108}Cd. Altogether, we obtain results for 181 more levels[11] to 10 keV for which an isotope assignment could not be given, but for which values of $(ag\Gamma_n^0)$ were obtained, where a is the abundance of the (unknown) responsible isotope in natural Cd. Above 2300 eV, the stronger of these levels are due to ^{111}Cd or ^{113}Cd. We present level parameters for 98 levels in ^{111}Cd to 2300 eV and for 37 levels in ^{113}Cd to 2300 eV. There is some evidence that one or more strong levels in 106 or 108 may coincide with some weak levels in ^{111}Cd to cause us to obtain too large $g\Gamma_n^0$ values. An example is the ^{111}Cd level at 311.4 eV where our $g\Gamma_n^0$ is ~20 times those of the ORELA and Shepkin *et al.* values. Our $g\Gamma_n^0$ values for ^{111}Cd are systematically larger than those of Wasson and Allen, but are compatible with the Shepkin *et al.* results. Our results for the 110, 112, 114, and 116 Cd isotopes are more reliable than for ^{111}Cd and ^{113}Cd.

We also give results for the total cross section vs energy for natural Cd from 15 to 200 eV, emphasizing the between resonance behavior. We give results for the average *s* level spacings and the *s* level strength functions for the isotopes 110 to 116. For 110, 111, 112, and 114, we give results for the *p* strength functions. We list values of $(g)\Gamma_n^0$ for all levels and the capture width Γ_γ for levels where it could be established. We list favored resonance J values for 11 levels in ^{111}Cd and 14 levels in ^{113}Cd.

The analysis of the Cd data is mainly due to Liou. All of the authors were involved in the operations to obtain the data and in some of the work in the initial stage of analysis.

II. EXPERIMENTAL CONDITIONS

The Cd measurements were made at the same time that samples of many other elements and isotopes were also studied. In addition to the details given above, a detailed description of the synchrocyclotron operation was given in Refs. 1 and 3. Time of flight measurements were made using ~46, 75, 40, and 17.5 g of enriched Cd (as CdO) for ^{110}Cd (96.68%), ^{112}Cd (98.15%), ^{114}Cd (98.57%), and ^{116}Cd (97.22%), respectively. Samples of ~32×127 mm area were used having principal isotope $(1/n)$ values (b/atom) of 166, 249, and 498 for ^{110}Cd, 102, 154, and 307 for ^{112}Cd, 193, 290, and 579 for ^{114}Cd, and 454 for ^{116}Cd. Nine different thicknesses of natural Cd metal

were used having ~76×202 mm area and $(1/n)$ values from ~5 to 380 b/atom (element). The impurity of other elements in any of the Cd samples is insignificantly small. We find no evidence that any observed level could be attributed to impurity elements other than Cd. A total counting time of ~40 min to 4 h was involved for each sample using each detector, corresponding to 1 to 5×10^7 total detector counts (all channels) for the 200 m transmission measurements, and 3 to 5×10^6 total detector counts for the 40 m self-indication measurements for each condition.

III. DATA ANALYSIS AND RESULTS

Our methods of data analysis have been described in earlier papers.[1-5] The determination of $g\Gamma_n^0$ or $ag\Gamma_n^0$ involves making a plot, for each resonance, of the implied functional relationship between $g\Gamma_n^0$ and Γ from the analysis of the ob-

TABLE I. Resonance parameters for ^{110}Cd. For $l=0$ levels $\Gamma_n^0 \equiv \Gamma\ (1\ \text{eV}/E)^{1/2}$. Levels having an a before the energy are believed likely to be p levels on the basis of a Bayes's theorem analysis. For these levels it is appropriate to use $g\Gamma_n^1 \approx \Gamma_n^0\ (459\ \text{keV}/E)$, the reduced p level neutron width (not given here).

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
89.52 ± 0.15	16 ± 1	3953.5 ± 1.4	16 ± 2
230.93 ± 0.16	0.55 ± 0.06	a 3980.9 ± 7.0	0.44 ± 0.17
a 339.66 ± 0.28	0.039 ± 0.005	a 4099.0 ± 1.4	0.87 ± 0.16
369.61 ± 0.20	1.2 ± 0.1	a 4161.5 ± 1.5	0.73 ± 0.15
a 505.34 ± 0.25	0.064 ± 0.008	a 4180.7 ± 1.5	1.7 ± 0.3
a 652.05 ± 0.37	0.15 ± 0.02	4242.8 ± 1.5	5.1 ± 0.8
a 761.78 ± 0.46	0.40 ± 0.04	4304.8 ± 1.6	2.1 ± 0.5
799.76 ± 0.50	14.5 ± 1.1	4402.3 ± 1.6	5.1 ± 0.8
a 824.08 ± 0.52	0.24 ± 0.03	a 4480.0 ± 8.4	1.6 ± 0.6
916.89 ± 0.31	0.66 ± 0.13	a 4661.1 ± 1.8	1.1 ± 0.2
920.85 ± 0.31	2.5 ± 0.2	a 4675.1 ± 1.8	1.3 ± 0.3
1115.9 ± 0.4	0.93 ± 0.12	4747.7 ± 1.8	2.1 ± 0.4
a 1135.3 ± 0.4	0.21 ± 0.04	4864.5 ± 1.9	3.7 ± 0.6
a 1241.5 ± 0.5	0.14 ± 0.03	5121.4 ± 2.0	10 ± 2
a 1318.1 ± 0.3	0.55 ± 0.06	5291.0 ± 2.1	7.4 ± 1.1
1346.7 ± 0.3	26 ± 2	5369.9 ± 2.2	2.2 ± 0.5
a 1685.8 ± 0.4	0.32 ± 0.05	5694.2 ± 2.4	4.1 ± 0.9
a 1809.5 ± 0.4	0.56 ± 0.12	5802.8 ± 2.4	2.5 ± 0.6
1828.2 ± 0.4	8.0 ± 0.9	5983.7 ± 2.6	17 ± 3
1982.9 ± 0.5	6.2 ± 0.7	6089.0 ± 2.6	10 ± 2
2065.7 ± 0.5	46 ± 4	6259.0 ± 2.7	2.9 ± 0.6
a 2100.4 ± 2.7	0.24 ± 0.09	6343.9 ± 2.8	6.8 ± 1.3
a 2353.0 ± 0.6	0.25 ± 0.06	6468.9 ± 2.9	2.5 ± 0.5
2376.0 ± 0.6	43 ± 4	6487.4 ± 2.9	2.2 ± 0.5
a 2410.6 ± 0.7	0.69 ± 0.12	6601.9 ± 3.0	4.8 ± 1.0
2476.8 ± 0.7	1.2 ± 0.2	6913.7 ± 3.2	12 ± 2
a 2492.0 ± 0.7	0.98 ± 0.16	6937.3 ± 3.2	2.9 ± 1.0
a 2723.2 ± 4.0	0.31 ± 0.12	7083.6 ± 3.3	3.9 ± 0.8
a 2739.7 ± 0.8	1.1 ± 0.2	7276.8 ± 3.4	3.3 ± 0.9
3042.2 ± 0.9	27 ± 4	7669.4 ± 3.7	13 ± 2
3105.7 ± 1.0	2.4 ± 0.4	8718.8 ± 4.5	14 ± 2
a 3153.1 ± 1.0	1.3 ± 0.3	8822.2 ± 4.6	30 ± 5
a 3183.7 ± 5.0	0.39 ± 0.14	8934.5 ± 4.7	5.6 ± 1.3
3375.1 ± 1.1	3.8 ± 0.7	9025.4 ± 4.7	5.4 ± 1.6
3496.4 ± 1.1	1.6 ± 0.3	9146.3 ± 4.8	10 ± 2
3636.4 ± 1.2	1.8 ± 0.3	9221.0 ± 4.9	9.6 ± 2.6
a 3667.8 ± 2.4	1.1 ± 0.2	9250.2 ± 4.9	10 ± 3
3702.1 ± 1.2	1.7 ± 0.3	9269.7 ± 4.9	18 ± 5
3744.4 ± 1.3	8.2 ± 1.1	9860.0 ± 5.4	23 ± 6
a 3804.4 ± 6.6	0.41 ± 0.16		

TABLE II. Resonance parameters for ^{112}Cd. See the caption of Table I concerning p levels.

	E_0 (eV)	Γ_n^0 (meV)		E_0 (eV)	Γ_n^0 (meV)
	66.77 ± 0.09	0.91 ± 0.07		4558.3 ± 1.7	25 ± 3
a	82.57 ± 0.09	0.009 ± 0.002		4798.3 ± 1.8	1.8 ± 0.4
a	83.24 ± 0.07	0.041 ± 0.003	a	4854.3 ± 1.9	1.3 ± 0.3
	226.46 ± 0.15	1.4 ± 0.1		4907.5 ± 1.9	13 ± 2
	442.97 ± 0.41	3.2 ± 0.2		5001.1 ± 2.0	3.4 ± 0.7
a	452.68 ± 0.27	0.16 ± 0.01		5136.5 ± 2.0	2.9 ± 0.6
a	565.76 ± 0.30	0.11 ± 0.01		5236.6 ± 2.1	2.2 ± 0.4
	737.28 ± 0.44	11.8 ± 1.1	a	5285.8 ± 2.1	1.3 ± 0.3
a	810.61 ± 0.65	0.11 ± 0.02	a	5552.2 ± 2.3	1.2 ± 0.2
a	884.47 ± 0.57	0.20 ± 0.03		5574.9 ± 2.3	10 ± 1
a	894.50 ± 0.30	0.22 ± 0.03		5686.0 ± 2.4	13 ± 2
	908.73 ± 0.30	8.3 ± 0.7	a	5734.4 ± 2.4	1.2 ± 0.3
a	1052.5 ± 0.4	0.40 ± 0.06		5948.3 ± 2.5	8.0 ± 1.6
a	1101.5 ± 0.4	0.18 ± 0.05		6085.1 ± 2.6	21 ± 4
	1115.4 ± 0.4	21 ± 2		6112.4 ± 2.6	2.0 ± 0.4
a	1207.3 ± 0.5	0.40 ± 0.09		6359.1 ± 2.8	6.5 ± 1.5
	1337.3 ± 0.5	0.98 ± 0.14		6433.4 ± 2.8	2.6 ± 0.6
	1423.3 ± 0.6	18 ± 2		6529.1 ± 2.9	4.0 ± 0.7
a	1640.1 ± 0.4	0.21 ± 0.04		6577.0 ± 2.9	30 ± 4
	1706.0 ± 0.4	7.0 ± 0.7	a	6874.5 ± 3.1	1.7 ± 0.5
a	1814.4 ± 0.4	0.45 ± 0.09		6920.0 ± 3.2	4.7 ± 1.2
	1942.5 ± 0.5	1.04 ± 0.14		6937.3 ± 3.2	2.6 ± 0.7
	2035.5 ± 0.5	33 ± 3		6975.5 ± 3.2	35 ± 5
	2226.2 ± 0.6	0.83 ± 0.13		7167.6 ± 3.3	4.7 ± 0.9
	2336.5 ± 0.6	15.5 ± 1.5		7640.0 ± 3.7	1.9 ± 0.6
a	2456.6 ± 0.7	0.34 ± 0.06		8007.5 ± 3.9	4.9 ± 1.3
	2573.7 ± 0.7	34 ± 3		8029.2 ± 3.9	5.4 ± 1.1
	2684.8 ± 0.8	31 ± 4		8248.3 ± 4.1	18 ± 3
a	2813.4 ± 0.8	0.36 ± 0.08		8377.0 ± 4.2	8.0 ± 1.7
	2817.5 ± 0.8	1.1 ± 0.2		8519.4 ± 4.3	5.9 ± 1.6
	2951.5 ± 0.9	4.2 ± 0.6		8536.7 ± 4.3	8.2 ± 2.2
a	3005.8 ± 1.8	0.38 ± 0.11		8665.5 ± 4.4	3.8 ± 1.0
	3103.8 ± 1.0	8.8 ± 1.1		9041.9 ± 4.7	4.2 ± 1.1
	3153.6 ± 1.0	1.9 ± 0.3		9153.5 ± 4.8	21 ± 4
a	3224.7 ± 1.0	0.83 ± 0.14		9225.9 ± 4.9	5.7 ± 1.5
a	3289.8 ± 1.0	0.80 ± 0.16		9570.1 ± 5.1	11 ± 2
	3306.9 ± 1.0	2.7 ± 0.4		9738.9 ± 5.3	3.7 ± 1.0
	3320.5 ± 1.1	2.4 ± 0.3		10 043 ± 6	6.3 ± 1.8
a	3404.3 ± 1.1	0.43 ± 0.10		10 262 ± 6	9.3 ± 3.0
	3491.9 ± 1.2	2.4 ± 0.3		10 464 ± 6	3.8 ± 1.2
a	3710.1 ± 1.2	0.77 ± 0.20		10 582 ± 6	5.9 ± 1.9
	3776.6 ± 1.3	3.6 ± 0.5		10 678 ± 6	32 ± 9
a	3861.9 ± 1.3	0.89 ± 0.16		10 899 ± 6	24 ± 6
a	3885.7 ± 1.3	1.1 ± 0.2		11 018 ± 7	4.9 ± 1.9
	4106.9 ± 1.4	3.3 ± 0.5		11 153 ± 7	8.1 ± 2.4
a	4152.7 ± 1.5	0.53 ± 0.12		11 280 ± 7	7.5 ± 2.4
	4200.0 ± 1.5	1.54 ± 0.31		11 322 ± 7	5.0 ± 1.9
a	4263.3 ± 1.5	0.70 ± 0.18		11 455 ± 7	20 ± 6
a	4392.7 ± 1.6	1.4 ± 0.3			
a	4486.7 ± 1.7	1.4 ± 0.4			

TABLE III. Resonance parameters for ^{114}Cd. See the caption of Table I concerning p levels.

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
a 56.40 ± 0.05	0.010 ± 0.002	3178.0 ± 1.0	60 ± 9
120.10 ± 0.15	4.3 ± 0.3	a 3305.4 ± 2.1	1.5 ± 0.4
227.07 ± 0.15	0.21 ± 0.03	3333.6 ± 1.1	5.7 ± 0.9
392.24 ± 0.34	42 ± 3	3698.4 ± 6.3	0.87 ± 0.38
a 567.78 ± 0.38	0.088 ± 0.013	3819.5 ± 1.3	16 ± 2
670.68 ± 0.38	14 ± 1	4258.0 ± 1.5	32 ± 5
752.19 ± 0.45	13.5 ± 1.1	4418.4 ± 3.2	1.8 ± 0.5
a 962.13 ± 0.33	0.55 ± 0.10	4582.8 ± 1.7	3.8 ± 0.9
a 1037.9 ± 0.9	0.15 ± 0.05	4645.5 ± 1.7	2.8 ± 0.7
1099.7 ± 0.4	7.5 ± 0.6	4691.8 ± 1.8	4.7 ± 0.7
1326.5 ± 0.3	0.85 ± 0.08	5357.0 ± 2.2	5.9 ± 1.2
1425.7 ± 0.6	48 ± 5	5515.0 ± 2.3	9.2 ± 1.3
1475.3 ± 0.3	1.1 ± 0.1	6042.4 ± 2.6	7.5 ± 1.2
a 1485.3 ± 0.6	0.31 ± 0.08	6406.6 ± 2.8	9.6 ± 1.9
a 1604.3 ± 0.4	0.7 ± 0.1	7202.6 ± 3.4	14 ± 2
a 1637.3 ± 1.9	0.49 ± 0.17	7393.8 ± 3.5	4.9 ± 0.8
a 1690.0 ± 0.7	0.36 ± 0.07	7495.8 ± 3.6	14 ± 3
1921.1 ± 0.5	11.2 ± 1.4	7662.0 ± 3.7	5.6 ± 1.4
a 1964.9 ± 0.5	0.83 ± 0.16	7837.6 ± 3.8	18 ± 5
2132.6 ± 0.6	6.7 ± 0.9	8368.6 ± 4.2	15 ± 4
a 2267.3 ± 0.6	0.46 ± 0.08	8946.0 ± 4.6	9.3 ± 2.6
2284.2 ± 0.6	1.4 ± 0.2	9262.4 ± 4.9	81 ± 15
a 2512.9 ± 0.7	1.12 ± 0.16	9958.3 ± 5.5	9.5 ± 1.7
a 2589.3 ± 3.7	0.88 ± 0.33	10 088 ± 6	13 ± 3
2635.9 ± 0.8	1.4 ± 0.3		
2678.3 ± 0.8	29 ± 4		
2804.0 ± 0.8	6.6 ± 0.9		
a 2849.3 ± 0.9	0.75 ± 0.15		
a 2955.6 ± 4.5	1.12 ± 0.44		
a 3027.1 ± 0.9	0.89 ± 0.18		

served transmission dip, or self-indication peak for each resonance for each sample thickness for which the level is well established. The intersection of these curves, defines a best $g\Gamma_n^0$, Γ, which should be consistent with $\Gamma \approx \Gamma_n + \langle \Gamma_\gamma \rangle$, since individual resonance Γ_γ values are expected to be nearly equal to $\langle \Gamma_\gamma \rangle$ for the population. Our area analysis programs for the self-indication measurements for the natural Cd isotope samples were rewritten to be more precise by including the effect of potential scattering due to all sample atoms for the case where the responsible isotope is of relatively low abundance. This permitted us to determine Γ_γ and favored J values for many

TABLE IV. Resonance parameters for ^{116}Cd.

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
28.97 ± 0.05	0.010 ± 0.001	2541.2 ± 0.7	0.48 ± 0.14
676.41 ± 0.39	0.85 ± 0.08	2651.6 ± 0.8	4.5 ± 1.0
888.96 ± 0.29	1.64 ± 0.17	3358.0 ± 1.1	4.1 ± 0.7
1048.3 ± 0.4	13.3 ± 1.2	3652.6 ± 1.2	11.1 ± 1.7
1122.4 ± 1.1	0.57 ± 0.18	4206.7 ± 1.5	2.2 ± 0.6
1384.1 ± 0.3	3.2 ± 0.3	4615.3 ± 1.7	2.5 ± 0.7
1566.5 ± 0.4	2.4 ± 0.4	4873.8 ± 1.9	10.7 ± 2.6
1857.7 ± 0.5	1.8 ± 0.4	5072.5 ± 2.0	9.0 ± 1.7
1968.5 ± 0.5	5.2 ± 0.9	5300.5 ± 2.2	22 ± 5
2361.5 ± 0.6	29 ± 4	7347.1 ± 3.5	17.5 ± 4.7
		8822.2 ± 4.6	38 ± 9

TABLE V. Resonance parameters for ^{111}Cd. See the caption of Table I concerning p levels. (Also see Table VII.)

E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)
27.53 ± 0.03	0.65 ± 0.04	1042.0 ± 0.4	1.08 ± 0.12
69.43 ± 0.10	0.013 ± 0.002	a 1056.9 ± 0.8	0.086 ± 0.043
86.11 ± 0.07	0.20 ± 0.02	1067.7 ± 0.4	0.44 ± 0.09
99.41 ± 0.09	0.98 ± 0.06	a 1082.1 ± 1.0	0.061 ± 0.030
102.93 ± 0.09	0.079 ± 0.007	1138.9 ± 0.8	0.16 ± 0.08
a 114.75 ± 0.11	0.013 ± 0.005	1148.8 ± 0.4	0.26 ± 0.09
138.11 ± 0.14	0.66 ± 0.04	a 1157.1 ± 0.9	0.073 ± 0.038
a 140.78 ± 0.15	0.016 ± 0.003	1174.4 ± 0.5	0.16 ± 0.06
164.11 ± 0.18	3.8 ± 0.2	1202.2 ± 0.5	0.15 ± 0.06
a 203.52 ± 0.16	0.016 ± 0.006	1216.8 ± 0.5	3.2 ± 0.4
a 208.57 ± 0.17	0.018 ± 0.006	1221.0 ± 0.5	0.13 ± 0.07
225.05 ± 0.15	1.5 ± 0.2	1237.2 ± 0.5	0.26 ± 0.09
233.41 ± 0.16	3.7 ± 0.3	1252.1 ± 0.5	4.4 ± 0.4
275.56 ± 0.21	0.87 ± 0.09	1261.6 ± 0.5	0.62 ± 0.08
a 286.47 ± 0.27	0.028 ± 0.008	1289.4 ± 0.5	0.28 ± 0.07
311.38 ± 0.24	2.6 ± 0.2	1307.6 ± 0.5	0.30 ± 0.11
331.97 ± 0.27	0.26 ± 0.02	1371.2 ± 0.6	0.46 ± 0.11
336.66 ± 0.27	0.06 ± 0.01	1399.1 ± 0.3	1.2 ± 0.2
355.99 ± 0.29	1.75 ± 0.16	1403.2 ± 0.3	1.8 ± 0.3
389.02 ± 0.33	1.0 ± 0.3	1448.6 ± 0.6	0.47 ± 0.16
410.01 ± 0.36	0.084 ± 0.025	1467.7 ± 0.6	2.9 ± 0.5
422.66 ± 0.38	0.083 ± 0.024	a 1511.1 ± 0.7	0.12 ± 0.08
438.31 ± 0.40	0.32 ± 0.03	1522.3 ± 0.3	0.67 ± 0.10
a 465.37 ± 0.56		1566.5 ± 0.6	1.2 ± 0.4
478.01 ± 0.45	0.13 ± 0.03	1581.7 ± 0.4	3.8 ± 0.5
484.01 ± 0.47	0.12 ± 0.03	1596.9 ± 0.4	2.0 ± 0.3
a 517.90 ± 0.33	0.034 ± 0.013	1617.4 ± 0.4	3.5 ± 0.6
a 530.64 ± 0.27	0.042 ± 0.017	1630.9 ± 0.4	4.2 ± 1.0
540.27 ± 0.28	0.77 ± 0.09	1654.2 ± 0.4	2.0 ± 0.2
a 543.40 ± 0.40	0.041 ± 0.013	a 1698.5 ± 0.7	0.10 ± 0.05
548.20 ± 0.28	0.12 ± 0.28	1742.3 ± 0.4	6.7 ± 1.0
575.92 ± 0.30	1.54 ± 0.13	1766.5 ± 0.8	0.26 ± 0.07
a 598.67 ± 0.32	0.069 ± 0.016	1787.0 ± 0.4	4.0 ± 0.7
603.52 ± 0.33	1.1 ± 0.1	a 1791.1 ± 0.8	0.18 ± 0.09
622.63 ± 0.34	2.4 ± 0.2	1820.5 ± 0.4	2.5 ± 0.5
688.53 ± 0.79	0.10 ± 0.03	1825.6 ± 0.9	0.87 ± 0.42
706.28 ± 0.82	0.20 ± 0.06	1883.6 ± 0.5	2.3 ± 0.3
764.32 ± 0.47	0.43 ± 0.07	1934.1 ± 0.5	2.5 ± 0.5
a 782.69 ± 0.47	0.075 ± 0.029	2022.7 ± 0.5	1.3 ± 0.2
790.43 ± 0.49	2.0 ± 0.2	2082.8 ± 1.1	0.53 ± 0.35
809.26 ± 0.51	1.8 ± 0.2	2110.7 ± 1.1	0.28 ± 0.15
860.48 ± 0.55	0.65 ± 0.10	2118.6 ± 1.1	0.13 ± 0.07
878.17 ± 0.57	1.1 ± 0.1	2138.3 ± 1.1	0.78 ± 0.39
903.65 ± 0.30	0.30 ± 0.07	2144.0 ± 1.1	1.9 ± 0.4
a 924.69 ± 0.62	0.092 ± 0.036	2158.2 ± 1.1	0.26 ± 0.15
a 928.10 ± 0.62	0.079 ± 0.036	2172.0 ± 1.1	0.24 ± 0.13
965.24 ± 0.33	1.9 ± 0.3	2236.6 ± 0.6	0.87 ± 0.25
a 1003.1 ± 0.4	0.11 ± 0.04	2294.4 ± 1.2	0.50 ± 0.25
1018.2 ± 0.7	0.17 ± 0.06		
a 1023.1 ± 1.1	0.075 ± 0.034		

TABLE VI. Resonance parameters for ^{113}Cd. (Also, see Table VII.)

E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)
0.178 ± 0.002	1.16 ± 0.05	851.43 ± 0.55	12 ± 1
18.40 ± 0.03	0.030 ± 0.002	1088.6 ± 0.4	1.4 ± 0.2
63.68 ± 0.09	0.33 ± 0.03	1120.0 ± 0.4	3.0 ± 0.7
84.82 ± 0.07	2.5 ± 0.2	1267.3 ± 0.5	4.2 ± 0.6
108.30 ± 0.10	0.77 ± 0.06	1310.2 ± 0.5	0.8 ± 0.2
143.08 ± 0.15	0.18 ± 0.02	1381.1 ± 0.5	2.1 ± 0.7
158.72 ± 0.17	0.53 ± 0.04	1364.0 ± 0.6	3.5 ± 0.5
192.82 ± 0.14	3.1 ± 0.2	1628.0 ± 0.4	1.4 ± 0.4
215.16 ± 0.28	1.50 ± 0.14	1645.6 ± 0.4	0.81 ± 0.17
261.03 ± 0.19	1.61 ± 0.12	1659.7 ± 0.4	1.1 ± 0.2
269.26 ± 0.20	1.16 ± 0.06	1866.9 ± 0.5	0.65 ± 0.14
291.56 ± 0.22	0.26 ± 0.02	1908.2 ± 0.5	1.8 ± 0.3
414.03 ± 0.37	4.5 ± 0.3	1963.3 ± 1.0	1.4 ± 0.5
431.94 ± 0.39	1.1 ± 0.1	2053.2 ± 0.5	1.7 ± 0.4
500.89 ± 0.25	1.8 ± 0.1	2114.6 ± 0.6	0.76 ± 0.26
524.66 ± 0.26	1.2 ± 0.1	2200.0 ± 0.6	1.3 ± 0.4
551.59 ± 0.29	3.2 ± 0.3	2241.2 ± 0.6	0.95 ± 0.30
634.89 ± 0.35	3.5 ± 0.3		
723.45 ± 0.43	0.63 ± 0.07		
841.45 ± 0.54	1.9 ± 0.2		

resonances of ^{111}Cd and ^{113}Cd. The results for the level energies and $g\Gamma_n^0$ values are given in Tables I to VI. Table VII lists the cases for ^{111}Cd and ^{113}Cd where Γ_γ for the resonance was obtained, and those cases where favored resonance J were obtained. Table VIII summarizes the final choices for $\langle\Gamma_\gamma\rangle$, $\langle D_0\rangle$ (the average s level spacing), $10^4 S_0$ (the s strength function), and $10^4 S_1$ (the p strength function). We obtain values of Γ_γ for six levels in ^{110}Cd, five levels in ^{112}Cd, four levels in ^{114}Cd, and one level in ^{116}Cd. The results are consistent with nearly equal values for Γ_γ for the different resonances of a given isotope. As in the cases of Yb[4] and W,[5] we also obtain the behavior of the natural element total cross section between resonances in the lower energy region. Figure 1 shows σ_t vs E for natural Cd to 200 eV, based on the thickest sample ($1/n = 5.36$ b/atom) transmission data. Many channel averages were used to emphasize the between level cross sections (see the caption). A potential scattering cross section of about 5 b applies, with some flucutuation in the between level σ_t values due to fluctuations in the net contribution of the wing effects of neighboring

TABLE VII. Cases where Γ_γ, and in some cases, favored resonance J values were obtained for the odd Cd isotopes.

E_0 (eV)	Γ_γ (meV)	J	E_0 (eV)	Γ_γ (meV)	J
^{111}Cd			^{113}Cd		
27.53 ± 0.03	96 ± 15		18.40 ± 0.03	95 ± 20	
99.41 ± 0.09	92 ± 14	1	63.68 ± 0.09	85 ± 15	
138.11 ± 0.14	96 ± 12	0	84.82 ± 0.07	105 ± 15	1
164.11 ± 0.18	115 ± 12	1	108.30 ± 0.10	93 ± 12	1
233.41 ± 0.16	120 ± 30	1	158.72 ± 0.17	90 ± 20	
275.56 ± 0.21	106 ± 15		192.82 ± 0.14	110 ± 15	0
355.99 ± 0.29	96 ± 15	1	215.16 ± 0.28	110 ± 16	1
540.27 ± 0.28	120 ± 25		261.03 ± 0.19	110 ± 15	1
575.92 ± 0.30	105 ± 16	1	269.26 ± 0.20	96 ± 20	0
603.52 ± 0.33	104 ± 18	1	414.03 ± 0.37	100 ± 16	1
622.63 ± 0.34	115 ± 15	1	431.94 ± 0.39	100 ± 20	
790.43 ± 0.49	124 ± 18	1	500.89 ± 0.25	100 ± 20	1
809.26 ± 0.51	105 ± 30		524.66 ± 0.26	115 ± 25	1
1252.1 ± 0.5	65 ± 25	1	551.59 ± 0.29	115 ± 18	1
1581.7 ± 0.4	75 ± 30	0	634.89 ± 0.35	92 ± 30	0
			723.45 ± 0.43	86 ± 20	
			851.43 ± 0.55	125 ± 30	1
			1267.3 ± 0.5	95 ± 25	1
			1364.0 ± 0.6	90 ± 30	1

TABLE VIII. Final choices of average capture widths, $l = 0$ average nearest neighbor spacings, s-wave strength functions, and p-wave strength functions for the Cd isotopes.

Isotope	$\langle\Gamma_\gamma\rangle$ (meV)	$\langle D_0\rangle$ (eV)	$10^4 S_0$	$10^4 S_1$
^{110}Cd	101	174 ± 18	0.50 ± 0.10	2.8 ± 0.6
^{111}Cd	102	24.0 ± 1.5	0.38 ± 0.06	3.5 ± 0.7
^{112}Cd	102	137 ± 8	0.53 ± 0.09	2.5 ± 0.5
^{113}Cd	101	22.1 ± 3.8	0.43 ± 0.07	
^{114}Cd	110	183 ± 29	0.70 ± 0.19	3.2 ± 1.0
^{116}Cd		264 ± 38	0.20 ± 0.06	

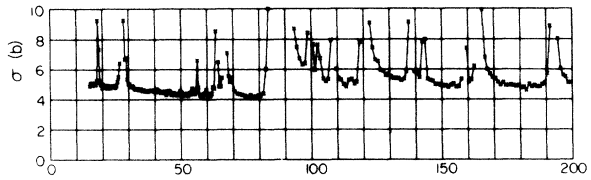

FIG. 1. The between level total cross section vs energy for natural Cd to 200 eV with sample thickness $1/n = 5.36$ b/atom. Each plotted point represents a many channel average satisfying the requirements that $(T_{max} - T_{min}) < 0.1$ within the average group, and that $\sigma < 10$ b where $\sigma = -(1/n)\ln\langle T\rangle$ (or else the average is not used). A line connecting two neighboring points means that no excluded set of channels comes between them.

levels.

Figure 2 shows a detailed shape fit to the transmission curve for the $(1/n) = 166$ b/atom sample of ^{110}Cd in the vicinity of the strong level at 89.52 eV. The best fitting parameters are given in the caption. The fit curve uses a single level Breit Wigner formula, including the effects of Doppler broadening. The $E^{1/2}$ energy dependence of Γ_n is included. The experimental points are many channel averages. The value $R' = 6.0$ fm corresponds to $4\pi R'^2 = 4.5$ b, which is in reasonable agreement with Fig. 1 for natural Cd.

IV. SYSTEMATICS OF THE RESULTS

Figures 3(a)–3(f) give plots of the cumulative number of resonances observed for the Cd isotopes 110, 112, 114, 116, 111, and 113, respectively. In the case where two sets of N vs E are given, the upper plot is for all observed levels, while the lower is for levels considered to be $l = 0$. The indicated slopes, $\langle D\rangle$, are visually fitted values, and do not represent our final choices, since other considerations must also be included to establish best choice $\langle D\rangle$ values for s levels. The extra levels are due to our observing part of the p level population. The self-indication capture data was most sensitive for detecting weak resonances, but it was of lower energy resolution and was not used for the last few keV regions for the even A isotopes. Only the stronger levels, seen in transmission, are included in the upper energy part of ranges shown in Figs. 3(a)–3(d) where many s levels, and probably all p levels are missed.

Figures 4(a)–4(f) give plots of $\sum \Gamma_n^0$ or $\sum g\Gamma_n^0$ vs energy. For ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd, the small contribution of levels regarded as being $l = 1$ have been omitted. The stronger levels dominate in such a plot, so the effect of missed weak s levels in the upper parts of the energy regions is relatively unimportant. The decreased slope for ^{113}Cd above 1400 eV may be due to our missing many levels. The indicated slopes of the straight lines in all cases show our final choices for the s strength functions S_0. For ^{111}Cd, we can compare our value of $10^4 S_0 = 0.38 \pm 0.06$ to the much smaller value (0.15 ± 0.02) of Wasson and Allen[10] and the value $(0.46^{+0.29}_{-0.15})$ of Ref. 9 (to 625 eV). We believe that the $g\Gamma_n^0$ values of Wasson and Allen are systematically too low. Our results are in rough agreement with the Shepkin *et al.*[9] values for ^{111}Cd, ^{112}Cd, ^{113}Cd, and ^{114}Cd, although their results are based on relatively few levels.

The ^{111}Cd results of Wasson and Allen[10] are based on measurements using a single ^{111}Cd sample, with main emphasis on weak p levels. The quantity evaluated (by them) is $g\Gamma_n\Gamma_\gamma/\Gamma$ and they assume that $\Gamma_n \ll \Gamma_\gamma$ so this is the same as $g\Gamma_n$ in all cases. Our results for $g\Gamma_n^0$ are based on data from several sample thicknesses which tend to be self-consistent, but generally larger than those implied from the $g\Gamma_n\Gamma_\gamma/\Gamma$ values of Ref. 10. In addition, we find that the assumption $g\Gamma_n\Gamma_\gamma/\Gamma = g\Gamma_n$ introduces a significant underestimate of $g\Gamma_n$ which is greatest for the strongest $l = 0$ levels which are most important in evaluating S_0. We agree much better with the individual level $g\Gamma_n^0$ values of Shepkin *et al.*[9] for ^{111}Cd. The level assignments of Ref. 10 for ^{111}Cd were essential to our ^{111}Cd evaluation.

In Figs. 5(a)–5(f), we show the distributions of observed $(g\Gamma_n^0)^{1/2}$ values for the Cd isotopes to some upper energy limits where most s levels are believed to be observed. The sets of observed levels tend to include many p levels, leading to

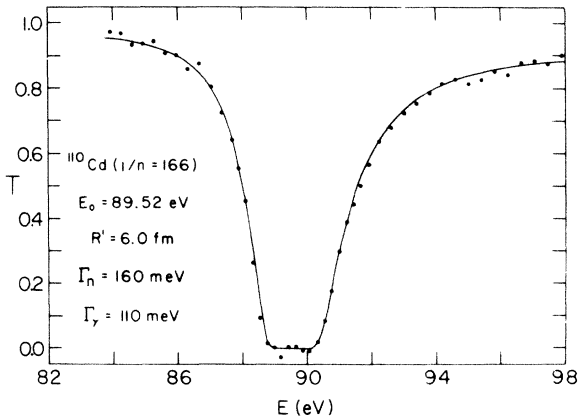

FIG. 2. Result of a shape fit to the transmission data of the ^{110}Cd separated isotope $(1/n = 166$ b/atom) for the strong level at 89.52 eV with the indicated parameters. The analysis uses a Breit-Wigner single level formula, including the Doppler effect and the $(E)^{1/2}$ variation of Γ_n. The interference asymmetry, permits an evaluation of R', giving $R' = 6.0$ fm for this level.

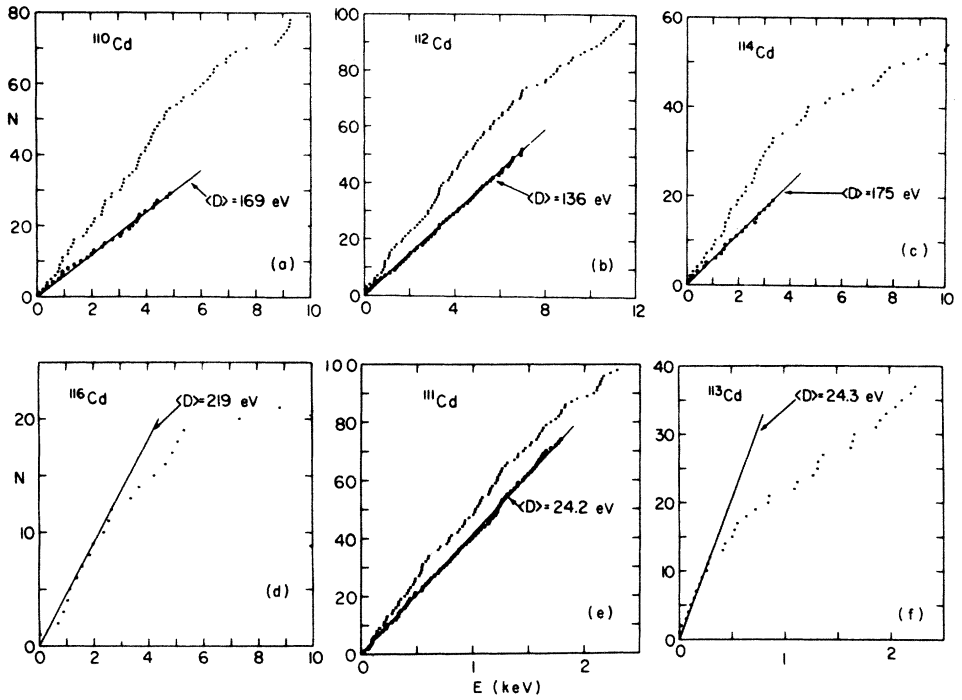

FIG. 3. Plots of the cumulative level counts vs energy for: (a) ^{110}Cd, (b) ^{112}Cd, (c) ^{114}Cd, (d) ^{116}Cd, (e) ^{111}Cd, (f) ^{113}Cd. The plots in (d) and (f), and the upper plots in (a), (b), (c), and (e) represent the full observed level population for each isotope, while the lower plots in (a), (b), (c), and (e) represent the final choices of s level sets as described in text. The values of $\langle D \rangle$ shown in the plots express only the slopes of visually fitted straight lines. Our final choices for $l=0$ $\langle D \rangle$ values are given in Table VIII.

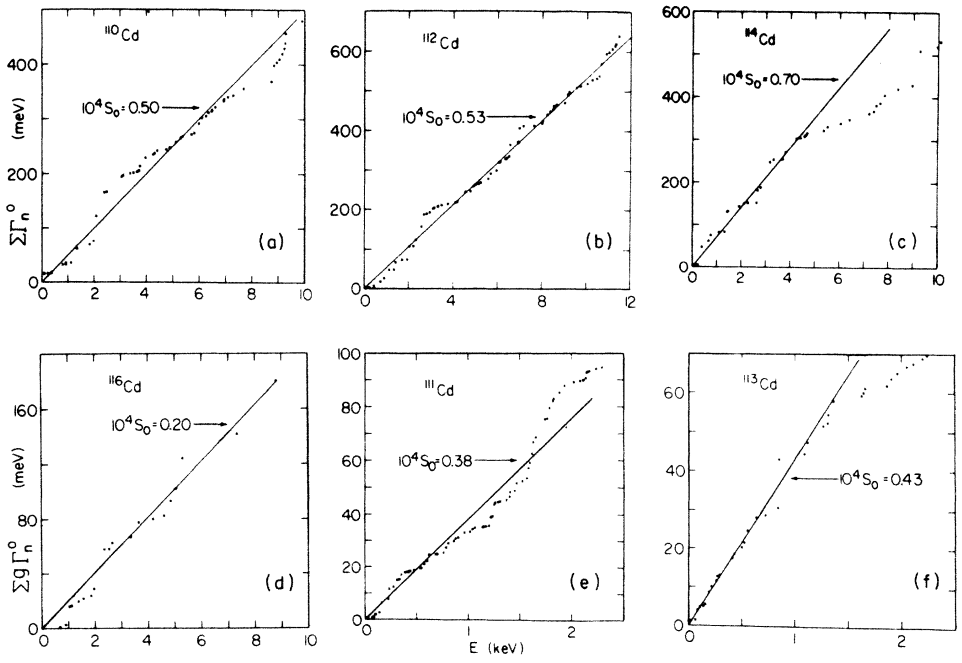

FIG. 4. Plots of $\sum \Gamma_n^0$ or $\sum g\Gamma_n^0$ vs energy for: (a) ^{110}Cd, (b) ^{112}Cd, (c) ^{114}Cd, (d) ^{116}Cd, (e) ^{111}Cd, (f) ^{113}Cd. The slopes of the fitted straight lines give the s strength functions. In (a), (b), (c), and (e) the levels likely to be p levels from a Bayes's theorem analysis as indicated by an a in Tables II, III, IV, and VI are not included.

relatively more weak levels than would be expected from the Porter-Thomas (PT) theory for a pure, complete s population. The parts of the histogram for the stronger levels should follow the PT distribution within statistical uncertainties. This provides one method of estimating the number of p levels included for each isotope in the indicated energy region. For ^{111}Cd and ^{113}Cd, we assume that $\langle g\Gamma_n^0 \rangle$ is nearly the same for the $J=0$ and $J=1$ level populations ($l=0$).

Consider Fig. 5(a) for ^{110}Cd, as a typical example. The "extra" p levels are believed to be essentially all contained in the first two histogram boxes. On the basis of the third and higher boxes, an s population of about 29 levels is implied. The more detailed analysis proceeded on the basis of two different approaches. The first approach involved an estimate of a threshold level strength vs energy for detection, and a calculation of the mean number of s levels missed for each 1 keV energy interval. The calculation indicated that about three s levels would be expected to be missed to 5 keV. For each of a series of choices of the p strength function, $10^4 S_1$, a similar calculation was made of the mean number of p levels we should detect in each 1 keV energy interval. The choice $10^4 S_1 = 2.8$ corresponds to a mean of 27 p levels expected to be detected to 5 keV. The difference $(53-29)=24=(27-3)$ is the mean expected number of p levels seen, minus the mean number of s levels missed for this choice of S_1.

The second method uses Bayes's theorem.[12] For each of a number of trial choices for $10^4 S_1$, a relative probability is calculated that each observed level is a p level on the basis of its observed $g\Gamma_n$ value. For the choice $10^4 S_1 = 2.8$, we find $\sum p_{j1}$ to be 28, where p_{j1} is the probability of the jth level to be a p level. The 27 levels having p_{j1}

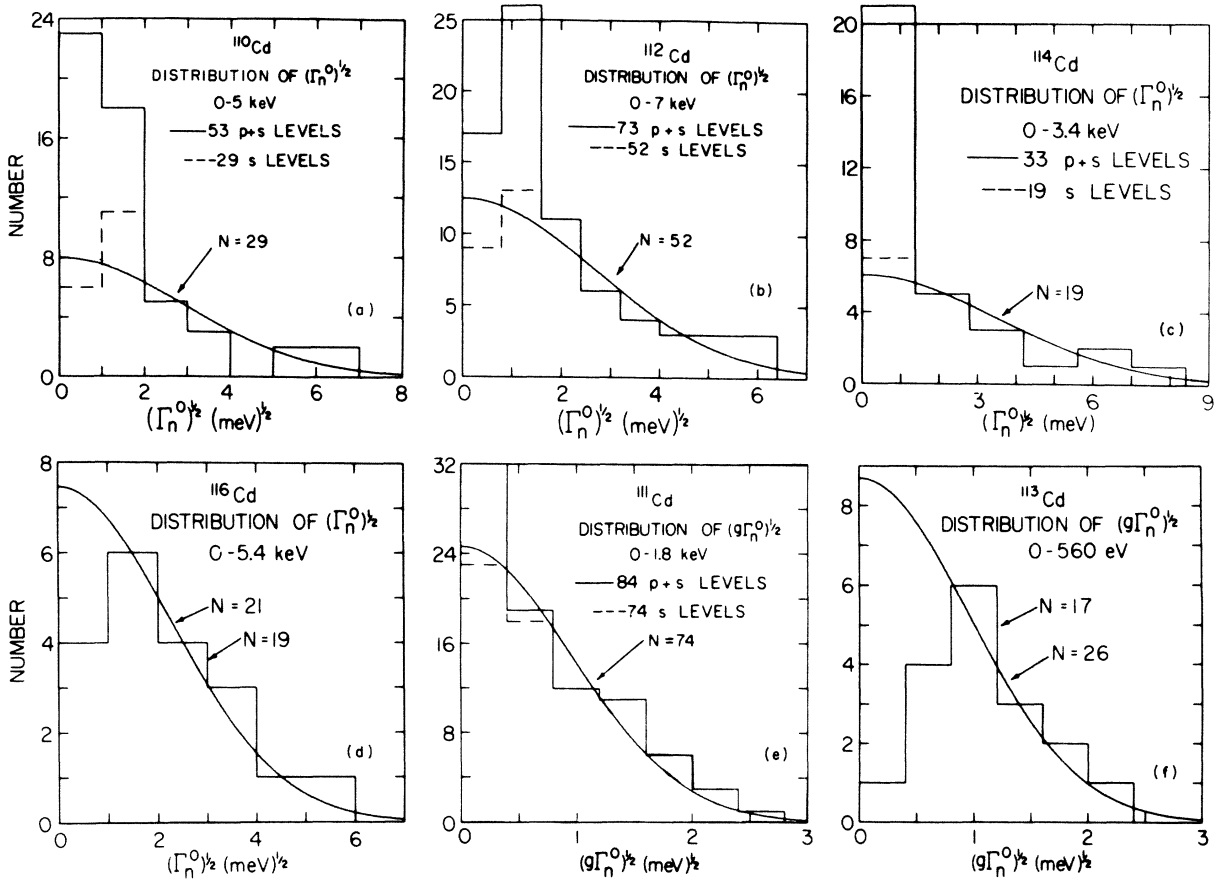

FIG. 5. Histograms of $(\Gamma_n^0)^{1/2}$ or $(g\Gamma_n^0)^{1/2}$ values for: (a) ^{110}Cd, (b) ^{112}Cd, (c) ^{114}Cd, (d) ^{116}Cd, (e) ^{111}Cd, (f) ^{113}Cd. In each isotope case, the solid histogram represents the full observed level population for a chosen energy interval, and the part excluding the first (two) histogram box(es) is (are) fitted with a Porter-Thomas single channel curve which is normalized to the experimental S_0 value. In (a), (b), (c), and (e) the dashed histogram represents our selected s level set for the same energy interval. It shows that the fits of their lower parts with the same Porter-Thomas curves are greatly improved.

> 0.66 are considered to be p levels. Three extra s levels were added to account for the expectation that we would miss three s levels in the first 5 keV energy interval. These were added at the midpoints of the three largest nearest neighbor level spacings for the above indicated s population, at 585, 1588, and 2760 eV. These "virtual" levels were added to the first histogram box in Fig. 5(a). The resulting s level set was then tested using the Dyson-Mehta Δ statistic[13] based on orthogonal ensemble theory, which agreed so well with our experimental results[1-5] for ^{166}Er, ^{152}Sm, ^{172}Yb, ^{182}W, and other nuclei. The value of Δ is the mean square deviation of the staircase plot of N vs E from a best fit straight line. This fit is expected to be very good for a pure, complete level population. We also calculate the observed value of the correlation coefficient ρ_{exp} for adjacent nearest neighbor level spacings. A detailed discussion of these tests is given in our paper[1] on the Er isotopes. The results of these statistical tests are given in Table IX, for our final s level choices, for ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd. The results are in general agreement with the predictions for the statistical orthogonal ensemble (OE) for the final s level choices, but not for the total observed level sets (over the indicated energy intervals). The agreement was also generally poor before the extra "missed" s levels were added by procedures such as that described above for ^{110}Cd. The added "virtual" levels were at 335, 590, 1565, 1824, 3634, 3942, 4379, 4678, and 5405 eV for ^{112}Cd to 7 keV; at 1698 and 2991 eV for ^{114}Cd to 3400 eV; and at 195, 656, 735, 835, 935, 992, 1103, 1339, 1495, 1544, and 1698 eV for ^{111}Cd to 1800 eV. The levels which are treated as p levels from the Bayes's theorem analysis are indicated by the letter "a" before the level energy in Tables I, II, III, and V.

In the analysis for ^{112}Cd, the expected number of missed s levels was nine. Those observed levels having $p_{j1} > 0.64$ for $10^4 S_1 = 2.5$ were treated as p levels. Seven of the added "virtual" levels were placed in the center of the seven largest "observed" nearest neighbor s level spacings. For a best simultaneous fit to the theoretical values of Δ, ρ, and the Wigner distribution for nearest neighbor level spacings, the last two "virtual" levels were placed at the center of those nearest neighbor spacing intervals which were tenth and twelfth, rather than eighth and ninth in order of decreasing spacing size.

For ^{114}Cd, the two virtual levels were placed at the center of the two largest nearest neighbor spacing intervals. The levels having $p_{j1} > 0.69$ for $10^4 S_1 = 3.2$ were treated as being p levels.

For ^{111}Cd, an average of 11 s levels were expected to be missed. Levels having $p_{j1} > 0.61$ for $10^4 S_1 = 3.5$ were treated as being p levels. The 11 inserted "virtual" s levels were all placed at the middle of 1 of the 13 largest observed nearest neighbor spacings, skipping the eighth and twelfth largest, to obtain a simultaneous best fit to the above several statistical tests.

The comparisons of the nearest neighbor level spacings for ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd with the Wigner formula are shown in Figs. 6(a)–6(d). It is made both for all observed levels in the indicated energy intervals, and for the final adjusted

TABLE IX. Summary of the results of statistical OE tests (Δ and ρ) for the final s level selections for ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd.

Isotope	E_{max} (keV)	N	Δ_{exp}	Δ_{theo}	ρ_{exp}	ρ_{theo}
^{110}Cd	5.0	29	0.41	0.33 ± 0.11	−0.18	-0.27 ± 0.17
^{112}Cd	7.0	52	0.30	0.39 ± 0.11	−0.32	-0.27 ± 0.13
^{114}Cd	3.4	19	0.22	0.29 ± 0.11	−0.28	-0.27 ± 0.22
^{111}Cd	1.8	74	0.63	0.69 ± 0.22	−0.14	-0.21 ± 0.10

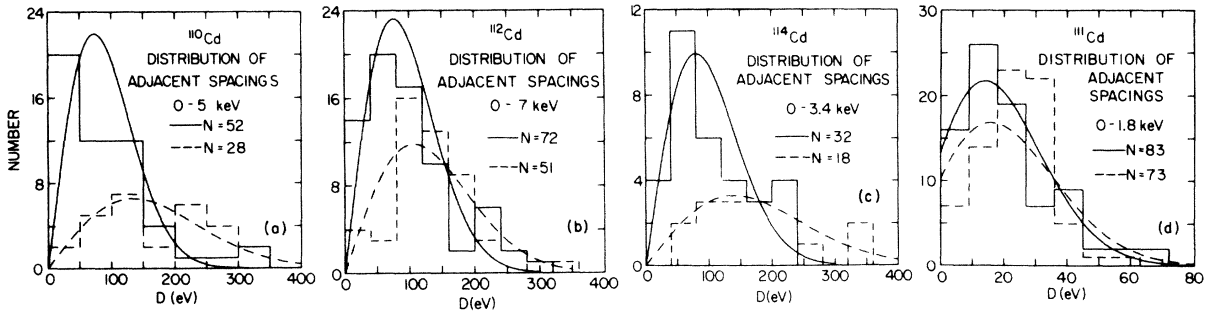

FIG. 6. Plots of the nearest level spacing histograms and the comparison Wigner distributions (normalizing to the same number of spacings as the histogram) for: (a) ^{110}Cd, (b) ^{112}Cd, (c) ^{114}Cd, (d) ^{111}Cd. In each plot, the solid histogram and curve represent the full observed level set, and the dashed histogram and curve represent the selected s level set. For ^{111}Cd, the theoretical curves are for a merged two-population distribution with the relative density in the ratio of $(2J+1)$ values.

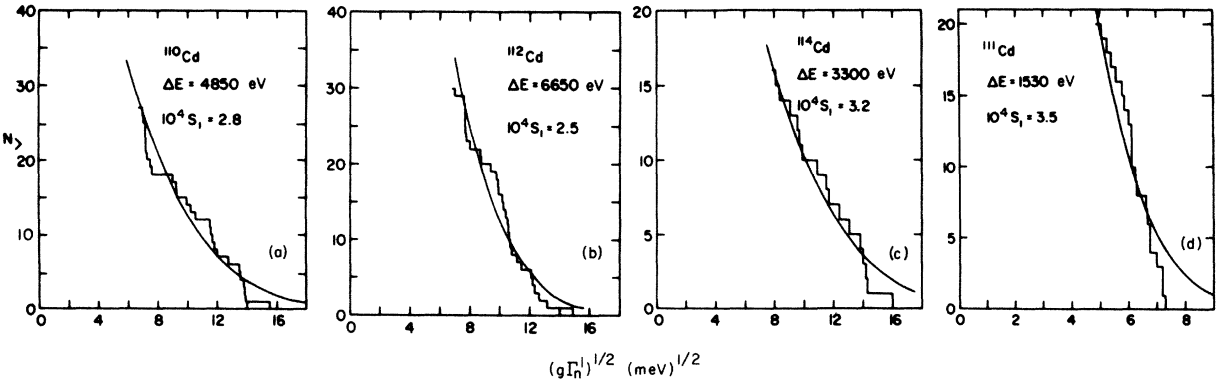

FIG. 7. Distributions of $(g\Gamma_n^1)^{1/2}$ values for the p levels chosen from a Bayes's theorem analysis for: (a) ^{110}Cd, (b) ^{112}Cd, (c) ^{114}Cd, (d) ^{111}Cd. In each case, an integral Porter-Thomas curve is plotted for comparison, which is normalized to the 10^4S_1 value determined from our level strength detection sensitivity. The average p level adjacent spacing is assumed to be $\frac{1}{3}$ of the average s level adjacent spacing for the even isotopes, but $\frac{4}{9}$ of it for the odd isotope, ^{111}Cd. Shown in each plot is also ΔE, the effective energy interval for observation of p levels, which is the observed interval minus the portions masked by the strong s levels.

set of s levels as described above. Our arbitrary method of inserting a "missed" s level at the center of a large spacing for observed s levels tends to artificially raise the center part of the histogram. This effect is large for ^{112}Cd and ^{111}Cd, but is relatively unimportant for ^{110}Cd and ^{114}Cd.

Figures 7(a)–7(d) give the integral $(g\Gamma_n^1)^{1/2}$ distributions for the levels in ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd, which were considered to be p levels from the Bayes's theorem analysis. The curves show the upper parts of the integral PT distributions of $(g\Gamma_n^1)^{1/2}$ values using the 10^4S_1 values obtained as described above. The level density for each s or p level population compound nucleus J value set is assumed to be proportional to $(2J+1)$. This gives 3 times as many p as s levels for the even isotopes, and $\frac{9}{4}$ times as many levels for the odd isotope ^{111}Cd ($I=\frac{1}{2}$). In each case, we use an effective energy interval for observing p levels equal to the observed interval minus the portions of these regions where observation of p levels would be blocked by the presence of the strong s levels. This latter correction is not large except for ^{111}Cd where it is 15%. A comparison of the theoretical curves with the experimental distributions shows reasonable agreement. The few largest $(g\Gamma_n^1)^{1/2}$ values predicted theoretically were excluded by the Bayes's theorem method used to select the p level population.

Another approach for establishing the p strength function was given in our paper[3] on ^{238}U and ^{232}Th. This method sets up a proper lower boundary C_1 for $g\Gamma_n^1$ values of all observed p levels chosen from the Bayes's theorem analysis in a selected effective energy interval, and assumes that the distribution of $g\Gamma_n^1$ follows the PT theory. Then there corresponds a unique value of 10^4S_1 which gives the same number of p levels as observed having $g\Gamma_n^1 \geq C_1$. For ^{110}Cd we have 27 levels considered to be p levels over the effective energy interval 4850 eV. For a choice of $C_1^{1/2} = 6.9$ $(\text{meV})^{1/2}$, the predicted number of p levels having $g\Gamma_n^1 \geq C_1$ are 22, 27, and 32 for $10^4S_1 = 2.18$, 2.80, and 3.58, respectively. These are taken as defining the uncertainty in our experimental choice for 10^4S_1 for ^{110}Cd. A similar method was used for ^{112}Cd, ^{114}Cd, and ^{111}Cd. The results are summarized in Table VIII.

The indicated values for $\langle D_0 \rangle$ (the mean s level

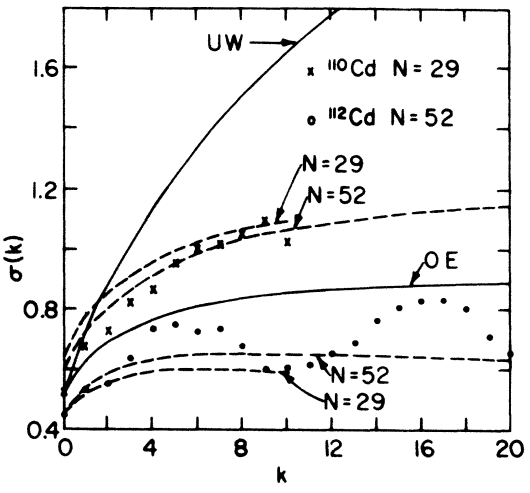

FIG. 8. Comparison of $\sigma(k)$ values for our selected s level sets for ^{110}Cd to 5 keV and ^{112}Cd to 7 keV with Monte Carlo results for OE and UW theories. $\sigma(k)$ is the standard deviation from the mean for the spacings of levels having k levels between (in units of $\langle D_0 \rangle$). The dashed curves give the 10 and 90% confidence limits for OE for the two isotopes.

spacing) in Table VIII were obtained from the Dyson-Mehta Δ statistic fit as described above for the final s level choices for ^{110}Cd, ^{112}Cd, ^{114}Cd, and ^{111}Cd. For ^{116}Cd and ^{113}Cd, the $\langle D_0 \rangle$ values were obtained from a study of Figs. 5(d)–5(f) for a best fit to the upper part of the observed $(g\Gamma_n^0)^{1/2}$ histogram, by a PT distribution.

Figure 8 shows, for our selected s populations, for ^{110}Cd and ^{112}Cd, the observed values of the statistic $\sigma(k)$ vs k, where $\sigma(k)$ is the standard deviation from its mean of the spacings of levels having k levels between.[14] The value of $\sigma(k)$ is normalized by expressing it in units of $\langle D_0 \rangle$. The observed $\sigma(k)$ are compared with the predictions for the OE and for a set of uncorrelated adjacent spacings generated from the Wigner distribution (UW). The 10 and 90% confidence limits for the OE are shown by the dashed curves for the cases appropriate to ^{110}Cd and ^{112}Cd. The observed values generally fall within these limits.

In our earlier papers,[1-5] we included an extra curve labeled TBRE (two body random matrix ensemble) which lies above the UW curve and is in greatest disagreement with the experimental $\sigma(k)$ curves. The TBRE curve was presented in Ref. 14. Since then it has been found[15] that that curve was in error and that the results for a two body random matrix ensemble should be the same as for the OE case so far as the level spacing ordering or fluctuation is concerned. This correction should be noted for our earlier papers.

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