Calculation of ^{3}H and ^{3}He electromagnetic form factors with hard-core two-nucleon potentials using the unitary pole approximation

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The triton wave function for a two body spin-dependent hard-core potential of exponential shape is obtained by solving the Faddeev three-body equations in the unitary pole approximation. It is then used to calculate the ^{3}H, ^{3}He electromagnetic form factors. As anticipated by Gupta *et al.*, the hard core is found to partially account for the difference between the rms charge radii of ^{3}He and ^{3}H which remains unexplained even after considering the S' state admixture as well as the neutron charge form factor.

[NUCLEAR STRUCTURE ^{3}H, ^{3}He calculated electromagnetic form factors, hard-core exponential potential, UPA calculations.]

While presenting the results of a calculation of the electromagnetic form factors of ^{3}H and ^{3}He using a nonvariational wave function obtained by exact solution of three-body equations for separable potentials, Gupta, Bhakar, and Mitra[1] observed that the experimental difference between the mean square charge radii of ^{3}H and ^{3}He could not be completely explained by considering the positive slope of the neutron charge form factor and the S' state admixture. They conjectured that the rest of this difference might be accounted for by a hard core in the two-nucleon interaction, $T=\frac{3}{2}$ state admixture, etc., which they had not considered. In the present note we attempt to give a quantitative idea of the role of hard core in explaining the aforesaid difference.[2] We use the triton wave function for a two-body spin-dependent potential with hard-core and exponential shape.[3] The former was obtained by solving the Faddeev three-body equations in the unitary pole approximation (UPA). This potential gives,[4] in UPA, a triton binding energy of 10.36 MeV for a zero hard-core radius and 9.33 MeV for a hard-core radius of 0.4 fm. For the "spectator functions" $F(q)$ and $G(q)$, thus obtained in both these cases,[4] we have obtained a least square fit to the expression:

$$F(q), \quad G(q)=\frac{A}{(q^2+\beta)^{3/2}}$$

so that these functions can be conveniently used in the calculation of electromagnetic form factors of ^{3}H and ^{3}He. The values of the best-fit parameters A and β are given in Table I. The S'-state probability was found to be 1.4% for hard-core radius $a=0.0$ fm and 2.1% for $a=0.4$ fm.

As is well known[1,5] the charge and magnetic form factors of ^{3}H and ^{3}He are expressible in terms of the charge and magnetic form factors of the neutron and proton, the body form factors F_L^c, F_0^c, F_L^m, F_0^m of the trinucleons and the isovector and isoscalar exchange form factors F_{XV} and G_{XS}. The latter arise due to the meson exchange contribution to the static magnetic moments and normalize the magnetic form factors to unity. We have adapted Gupta, Bhakar, and Mitra[1] expressions for the body form factors F_L^c, F_0^c, F_L^m, and F_0^m to the UPA formalism. These body form factors can be expressed as a sum of several integrals involving the spectator function $F(q)$ and $G(q)$ and the "UPA form factors"[4] $\Psi_{T,S}(k, -B_{T,S})$.[6] To cut the computing time of the above mentioned integrals, which involve integration over angles also, we have averaged over the angles according to Mitra's prescription.[7] The values of $F_{ch}^{p,n}$ and $F_{mag}^{p,n}$ used in the calculations have been taken from de Vries *et al.*[8] The values of F_{XV}, the isovector exchange form factor, are taken from the visual best-fit curve of Janssens *et al.*[9] resulting from the analysis of Levinger and Srivastava.[5] G_{XS}, being very small, is neglected.

The calculated charge and magnetic form factors of ^{3}H and ^{3}He are in good agreement with the experimental data[10] up to $q^2=8$ fm^{-2}. The agreement is better when the hard core is present in the potential. Though we do not get the dip in the charge

TABLE I. Best-fit parameters of the spectator functions.

Hard-core radius (fm)	$F(q)$ A	$F(q)$ β	$G(q)$ A	$G(q)$ β
$a=0.0$	0.229	0.374	0.210	0.549
$a=0.4$	0.163	0.298	0.142	0.467

form factor of ^{3}He, we do find that with hard core the ^{3}He charge form factor decreases more steeply with increasing q^2 than without hard core. The absence of the "dip" in our results could be a consequence of the simplicity of the potential used by us. The other important experimental quantity, the difference of the mean square charge radii of ^{3}He and ^{3}H, is found to be 0.367 fm^2 for no hard core and 0.424 fm^2 for the hard-core radius equal to 0.4 fm as against the experimental value 0.607 fm^2. This is in accordance with the anticipation of Gupta *et al.*[1] and it clearly shows that the hard core does play a role in bridging the gap between mean square charge radii of ^{3}He and ^{3}H. The reason why we have not been able to reproduce exactly the experimental difference in ${a_{^3\mathrm{He}}}^2$ and ${a_{^3\mathrm{H}}}^2$ with our wave function is that we have not considered the tensor component in the two-body potential, which, as Gupta *et al.* observe,[1] itself plays a role in bridging the above mentioned gap. UPA calculations on these lines with Hamada-Johnston and Yale potentials, which include tensor forces as well, are in progress.

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