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Deuteron electromagnetic structure at large momentum transfer*

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Pursuing the idea that exchange currents should dominate the very large momentum transfer behavior of the deuteron electromagnetic form factors, we evaluate the $\pi\rho$ and $\sigma\omega$ exchange contributions. Taking the meson-nucleon coupling constants from one-boson exchange calculations of the *NN* force and the photon-meson coupling strengths from a relativistic quark model, we find that the form factors begin to "flatten out" at momentum transfers $q^2 \gtrsim 50\ \text{fm}^{-2}$ and are an order of magnitude greater than the usual impulse prediction at $q^2 \approx 100\ \text{fm}^{-2}$.

[NUCLEAR STRUCTURE ^{2}H; calculated exchange current effects in large momentum transfer electromagnetic form factors.]

I. INTRODUCTION, RESULTS, AND CONCLUSIONS

The electromagnetic form factors of the deuteron represent one of the most sensitive testing grounds for models of the nucleon-nucleon interaction. Potentials derived from fits to the scattering phase shifts or from dynamical models must reproduce the two-body bound state properties and, while there are several "good" potentials, none of these is by itself adequate for describing the magnetic form factor. First, potentials fitted to the scattering and to the deuteron quadrupole moment seem to require a D-state probability $P_D \approx 6 \pm 1\%$. It is by now an old story that the calculated magnetic moment of the deuteron is then smaller than the observed value, requiring a 1 or 2% contribution from exchange currents and/or relativistic corrections. There have been several calculations including explicit mesonic degrees of freedom in the electromagnetic current operator[1,2] or isobar admixtures in the deuteron wave function.[3,4] All of these calculations predict corrections of approximately the correct magnitude, even though they are based upon different mechanisms, so there is not yet a consensus about the origin of the exchange moment. In fact, the relativistic wave function correction of Gross[5] gives a magnetic moment correction of similar magnitude but opposite sign, and a proper accounting for the magnetic moment discrepancy appears to be a very complicated affair.

There has also been a long-standing disagreement in the large momentum transfer ($q^2 \approx 15\ \text{fm}^{-2}$) magnetic form factor, the data lying somewhat above the prediction of most models. Blankenbecler and Gunion[6] (BG) have attempted to understand this within a framework based upon the ideas of simple vector meson dominance and double scattering. This model is represented schematically in Fig. 1: the initial (isoscalar) photon "converts" into an ω meson, which is then scattered or transformed into a ρ by one nucleon before being absorbed by the other. This is analogous to the double scattering term in high energy hadron-deuteron scattering which is known to dominate the impulse term in large momentum transfer elastic scattering. The reason is just that in double scattering the momentum transfer to the deuteron can be directly shared by the two nucleons, giving them little relative momentum and consequently a high probability of remaining bound. In contrast, the impulse form factor falls off very rapidly with momentum transfer and eventually dies off compared to the two-body term. BG find that this model is able to give a magnetic form factor which satisfactorily fits the data and which is completely dominated by the two-body term for $q^2 \gtrsim 25\ \text{fm}^{-2}$. However, we find it difficult to judge the quantitative significance of these results. Their calculation is certainly quite different from the more "conventional" exchange current approaches and relies heavily upon very

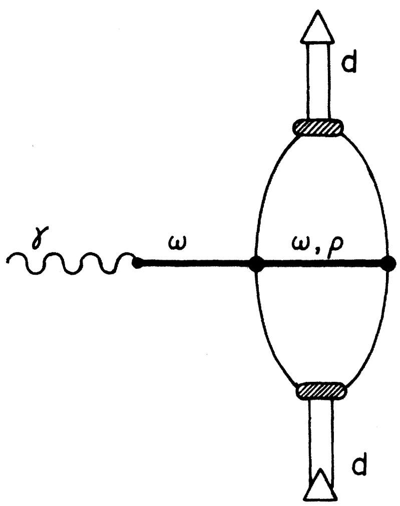

FIG. 1. Blankenbecler-Gunion model of the form factor correction.

simple assumptions concerning the vector meson-nucleon vertex.[7] Also, they set the scale of the exchange contribution by the magnetic moment discrepancy and, since completely different mechanisms seem capable of accounting for this discrepancy, this could be quite misleading.

Recently, Rand *et al.*[8] have recomputed the deuteron magnetic form factor using the most recent dipole fit for the proton and neutron form factors, and they now conclude that most deuteron models are consistent with the electron scattering data. While these results are very interesting, we do not feel that the question of exchange contributions to the magnetic moment distribution is a dead issue. It is clear that there must be some contribution to the magnetic form factor not only at the static value $q^2=0$ but also at larger momentum transfer and, lacking a theory of the *NN* interaction which is both fundamental and accurate in reproducing the phase shifts, we cannot answer the question of how large this contribution is simply by comparing two-nucleon wave functions with the electromagnetic form factors. The results of Ref. 8 indicate only modest (if any) exchange contributions for $q^2 \lesssim 15\ \mathrm{fm}^{-2}$, but we stress that this can be settled only by identifying the important components of the two-body current and evaluating them. Unfortunately, this is not a simple task (as already seen for the static magnetic moment) and the quantitative results can be extremely model dependent. We feel that one can more profitably search for a more *qualitative* exchange current modification of the form factors.

We go back to the point stressed by BG, namely that the two-body current effects should dominate the deuteron electromagnetic form factors at sufficiently large momentum transfer, and emphasize that the validity of this statement does not rest upon the detailed picture employed by BG but only upon the fact that both nucleons take up to the momentum transfer. We reexamine this problem of the high-$|\vec{q}|$ behavior of the electromagnetic form factors in a more "conventional" exchange current framework.[9] We find this approach more appealing in view of its considerable success for various low momentum transfer properties (slow neutron capture,[10] electrodisintegration of the deuteron,[11] etc.) and would like to push this program as far as possible. We argue that the $\pi\rho$ and $\sigma\omega$ [Fig. 2(b)] exchange currents should provide the largest contributions at very large momentum transfers; these arguments, together with a detailed description of the model and of the calculation, will be presented in the following section. Effectively, we evaluate the BG diagrams (Fig. 1) with an explicit model for the vector meson photoproduction vertices: ρ production is assumed to proceed via π exchange, and ω production via σ exchange. These are the lightest mass (longest range) exchanges possible for an isoscalar photon. The π, ρ, ω, and σ mesons are those which have been used with some success to compute the intermediate and long-range parts of the nucleon-nucleon potential in the one-boson exchange approach (OBEP), and it seems natural

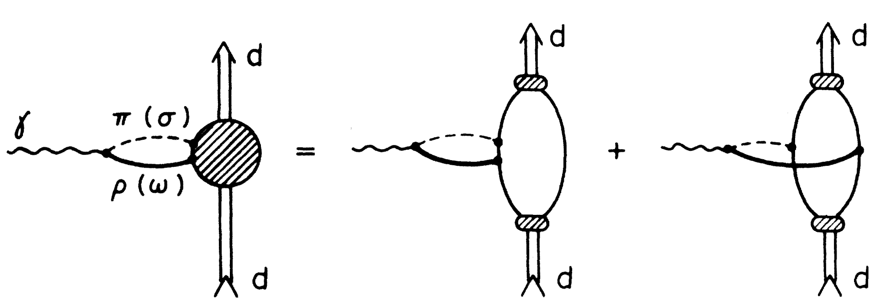

FIG. 2. Interaction of the $\pi\rho$ (or $\sigma\omega$) component of the electromagnetic current with the deuteron. (a) Contribution to the nucleon form factor in the usual impulse approximation. (b) Exchange current contribution.

to include these in our consideration of electromagnetic current corrections. The meson-nucleon coupling constants are taken from the OBEP calculations. For the $\gamma\pi\rho$ and $\gamma\sigma\omega$ vertices, we assume gauge-invariant interactions and calculate the coupling constants in the relativistic quark model of Feynman, Kislinger, and Ravndal[12]; these results are consistent with the experimental upper bounds on the relevant decay widths. In as much as the coupling constants are determined, there are no variable parameters in the calculation. We do not attempt to fit data but rather to get an idea about the expected "flattening out" of the form factors (i.e., at what momentum transfer is this likely to occur?). Experimental verification of this qualitative feature would in turn reflect upon the range of validity of the impulse approximation. We point out that the $\pi\rho$ exchange current is exactly that considered many years ago by Adler,[1] and our treatment is similar to his.[13] However, in addition to using more recent values for the various coupling constants, we have retained all the complicated spinology (e.g., terms involving products of d-state wave functions, which may be important at very large momentum transfer) and extend the calculations out to much larger momentum transfer. Since many readers may not be interested in following the details of the calculation, we first present the results.

The deuteron charge (F_C), quadrupole (F_Q), and magnetic (F_M) form factors contain all the information about the electromagnetic properties of the deuteron. With our conventions, the static limits of these form factors are the deuteron total charge, quadrupole moment, and magnetic moment (in nuclear magnetons), respectively. If beam and target are unpolarized, there are only two independent form factors, the magnetic and electric form factors, with the latter defined as

$$F_E(q) = \left[F_C{}^2(q) + \frac{q^4}{18} F_Q{}^2(q)\right]^{1/2}. \qquad (1)$$

We express the exchange current effects as modifications of the form factors and display the results in Figs. 3–6. The main result is clearly the enhancement of the form factors at large momentum transfer. Roughly speaking, the exchange contribution to all the form factors is an order of magnitude larger than the impulse value at $q^2 \approx 100\ \text{fm}^{-2}$ ($|\vec{q}| \approx 2\ \text{GeV}/c$), corresponding to a factor of a hundred in the cross section. The contribution to F_M and F_Q comes almost entirely from the $\pi\rho$ term, while the $\pi\rho$ and $\sigma\omega$ contributions are comparable in F_C. The zero in the charge form factor is affected very little in our model because the $\sigma\omega$ and $\pi\rho$ contributions add to zero very near to the zero in the impulse approximation. Note that the flattening of the form factors begins at $q^2 \gtrsim 50\ \text{fm}^{-2}$ and that the zero in the impulse magnetic form factor is removed. All calculations have been performed with the Reid[14] soft-core wave function, but we stress that, for any reasonable choice of the deuteron model, the exchange current contribution is far less sensitive to the short-range behavior of the wave function than is the impulse result. As noted before, this simply reflects the fact that the nucleons

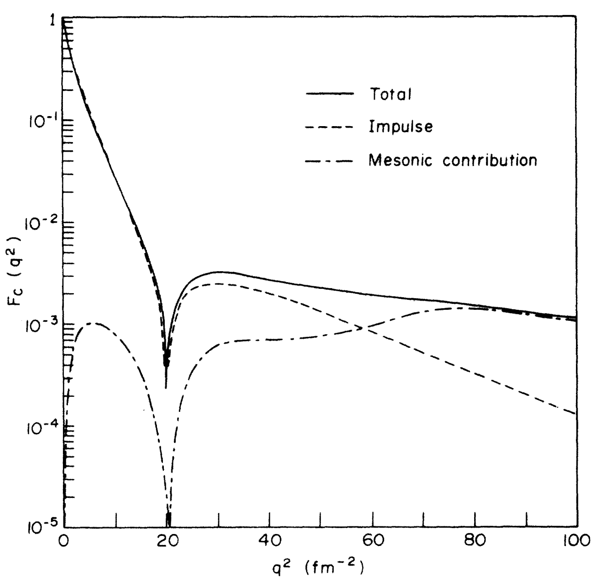

FIG. 3. Deuteron charge form factor, with and without the exchange current contributions. All calculations use Reid soft-core wave functions.

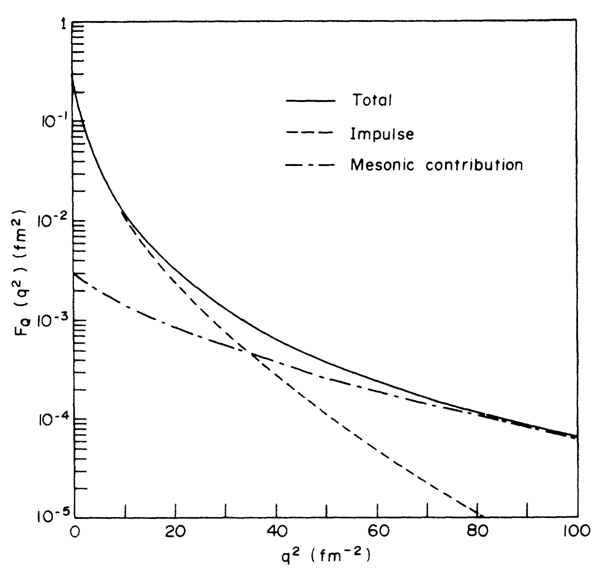

FIG. 4. Deuteron quadrupole form factor.

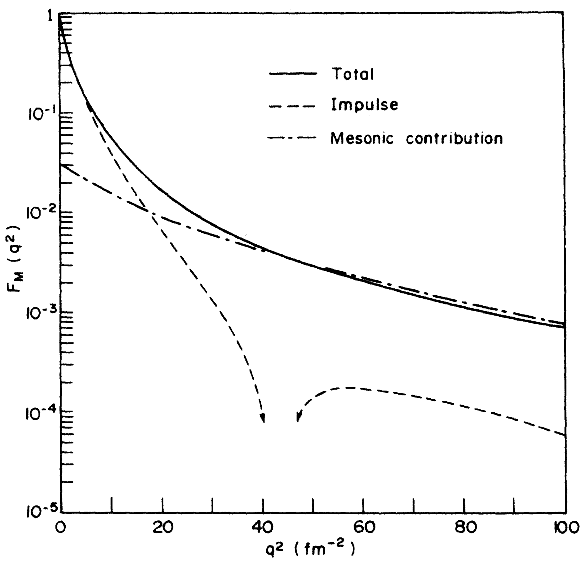

FIG. 5. Deuteron magnetic form factor.

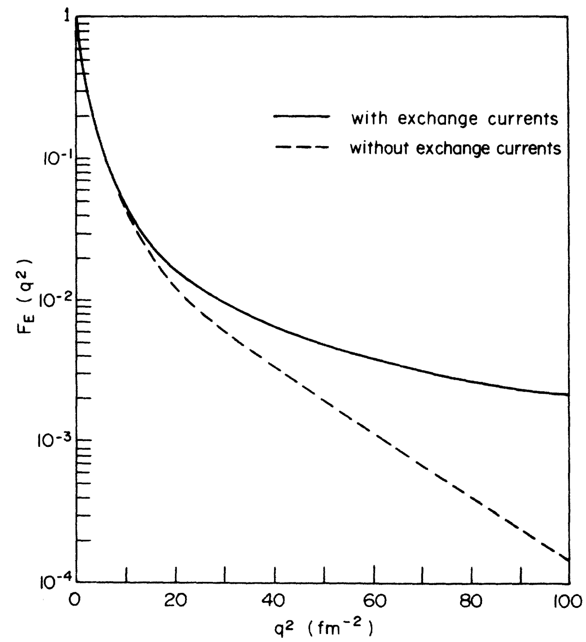

FIG. 6. Deuteron electric form factor.

share the momentum transfer and so have low relative momenta.

Moravcsik and Ghosh[15] have commented that the tensor polarization in elastic electron-deuteron scattering is very sensitive in the region $q=5-10$ fm^{-1} to the deuteron wave function at short distance. The tensor polarization is given by[15]

$$P=\frac{2F_C G_Q + G_Q^2/\sqrt{2}}{F_C^2+G_Q^2}, \quad G_Q \equiv \frac{q^2}{\sqrt{18}} F_Q \tag{2}$$

and is shown with and without exchange currents in Fig. 7. Comparing this with Fig. 4 of Ref. 15, we see that the exchange current effect is large compared to the differences predicted by various "good" wave functions and that the polarization behaves quite differently in our model from that predicted in impulse approximation by any of the deuteron models. In other words, the suggested measurement of the tensor polarization yields information not about the short distance behavior of the two nucleon wave functions but rather about the two-body (isoscalar) electromagnetic current.

Experimental verification of these predictions would be quite significant, for one would be seeing directly the effects of meson degrees of freedom in the deuteron electromagnetic current. In turn this could lead to increased confidence in the rapidly developing phenomenology of nuclear exchange currents. More specifically, experimental support for this model would allow us to infer the contribution of the $\pi\rho$ (and to some extent the $\sigma\omega$) currents at lower q^2, our results being a correction to the static magnetic moment of $+0.032\,\mu_N$ (roughly twice the discrepancy for the Reid soft core wave function), a correction to the quadrupole moment of $+0.003$ fm^2 (much less than the present error bars on the measurement of the quadrupole moment), and appreciable contributions to F_E ($\approx$20%) and F_M (a factor of 2) already at $q^2 \approx 15$ fm^{-2} (requiring a "softer" deuteron wave function). At this time, these results can be taken only as very speculative, requiring for example a $\gamma\pi\rho$ coupling constant close to the present experimental upper bound. In any case, we feel that the problem is an important one and one which can be clarified only by good large momentum transfer data.[16]

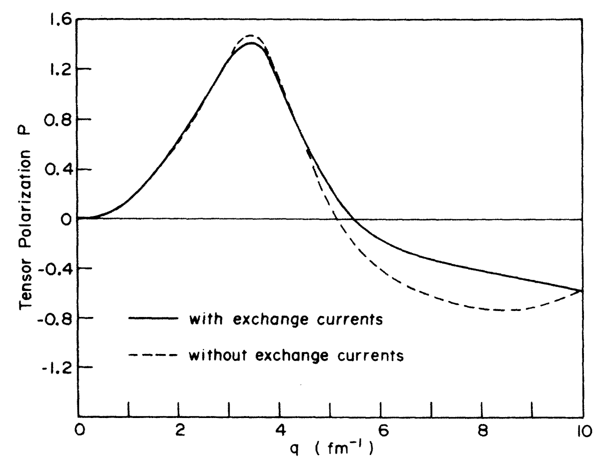

FIG. 7. Tensor polarization, with and without exchange current.

II. THEORY

This section supplies the theoretical background and calculational details. The model used and the approximations made will be described here. We have tried to make the first section of the paper reasonably self-contained (Figs. 3–6 summarize our results), and those not interested in the technical details of the calculation need not be overly concerned with this section.

A. Model

We evaluate corrections to the deuteron electromagnetic form factors arising from interaction of the virtual photon with the meson cloud in the deuteron. Our considerations are limited to effects associated with the modification of the electromagnetic current and not with wave function effects such as the role of nucleon resonances in the deuteron wave function.[3,4] While the distinction between wave function and electromagnetic current effects is not always clear, we argue that two-body current diagrams [such as those in Fig. 2(b)] which can be viewed as the direct interaction of components of the photon with both target nucleons will dominate at large momentum transfer. This is because there is never a large relative momentum between the baryon lines, and available calculations support this assertion: for example, inclusion of $\Delta\Delta$ components in the wave function[3] adds corrections to the form factors which fall off as rapidly with momentum transfer as do the impulse form factors, and hence considerably faster than the form factors computed in our model. Our specific selection of the $\pi\rho$ and $\sigma\omega$ exchange currents is dictated by G parity, which requires that the isoscalar photon couples to an odd number of pions or alternatively to mesons (in the sense of independent particles) of an over-all odd G parity, and by the basic assumption that the low-mass multipion continuum can be approximated by π, σ, ρ, and ω mesons.

Besides wave function corrections, we also ignore the effects generally classified as relativistic corrections.[5,17] This might seem *a priori* to be a dangerous procedure, considering that we evaluate the form factors out to $q^2 = 100\ \text{fm}^{-2}$. However, the available calculations indicate that up to moderate momentum transfers, the net contribution of the whole class of relativistic corrections amounts at most to a few tens of percent of the impulse terms[5,17] and we do not expect their importance to increase drastically at higher momentum transfer. We are looking for much larger effects than these. Furthermore, we can expect the exchange current terms to be even less affected by relativistic corrections, since the nucleons do not acquire large relative four-momentum.

Next we discuss the elementary couplings. There are two sorts involved in the model: the meson coupling to nucleons and the (virtual) photon coupling to mesons. The strengths of the coupling of π, ρ, and ω to nucleons are rather well established and we quote the accepted values in Table I.[18] The σ presents a problem, however, since the $T=J=0$ $\pi\pi$ resonance is so broad that it is difficult to interpret it as a meson with a definite mass. As a way out, we assume that the σ we are considering can be identified with that introduced in the one-boson-exchange model of $N-N$ potentials to account for the intermediate range attraction.[18] Even so, the problem persists since there is an arbitrariness in assigning a mass to it and, as we shall see, the $\gamma\sigma\omega$ coupling is quite sensitive to the mass of the σ. In this paper, we take the generally favored value $M_\sigma = 550$ MeV. The coupling constant $g_{\sigma NN}$ given in Table I corresponds to this mass.

Consider now the $\rho\pi\gamma$ and $\omega\sigma\gamma$ couplings. We follow Adler[1] and write the Lagrangians in the simplest gauge invariant form:

$$\mathcal{L}_{\gamma\pi\rho} = -\frac{e g_{\rho\pi\gamma}}{2m_\rho}\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}\vec{\rho}_\gamma\cdot\partial_\delta\vec{\pi}\,, \tag{3}$$

$$\mathcal{L}_{\gamma\sigma\omega} = -\frac{e g_{\omega\sigma\gamma}}{m_\omega}F_{\alpha\beta}\omega_\alpha\partial_\beta\sigma\,,$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field tensor, and ρ, ω, π, and σ are the field operators of the corresponding mesons. The matrix elements of the isoscalar electromagnetic current J^s_λ are then given by

$$\langle\pi^n(k)|J^s_\lambda|\rho^m(Q)\rangle = -i\frac{e g_{\rho\pi\gamma}(q^2)}{m_\rho}\times\epsilon_{\lambda\nu\sigma\mu}q_\nu k_\sigma S_\mu\delta_{mn}\,, \tag{4}$$

$$\langle\sigma(k)|J^s_\lambda|\omega(Q)\rangle = \frac{e g_{\omega\sigma\gamma}(q^2)}{m_\omega}\times q\cdot k\left(\delta_{\lambda\mu} - \frac{q_\mu k_\lambda}{q\cdot k}\right)S_\mu\,,$$

where $q = Q - k$ is the photon momentum, S_μ the polarization vector of spin-1 objects, and n, m isospin indices. One has no information as to

TABLE I. Meson-nucleon coupling constants.

B	g_{BNN}
π	13.5
ρ	2.56
ω	7.68
σ ($M_\sigma = 550$ MeV)	7.77

how $g_{\rho\pi\gamma}(q^2)$ and $g_{\omega\sigma\gamma}(q^2)$ behave in q^2, so we are forced to make a guess on this. Strictly speaking, since the mesons are virtual, there is an ambiguity as to which variables are relevant for the form factor, and we assume that they are functions of q^2 only. Finally, we assume that the form factors scale in accordance with the vector dominance model of the isoscalar current:

$$\frac{g_{\rho\pi\gamma}(q^2)}{g_{\rho\pi\gamma}} = \frac{g_{\omega\sigma\gamma}(q^2)}{g_{\omega\sigma\gamma}} = (1 + q^2/m_\omega{}^2)^{-1} . \tag{5}$$

It is difficult to evaluate the reliability of this extrapolation, but recent parton model calculations do predict an asymptotic dependence proportional to $(q^2)^{-1}$ and we think it unlikely that the scaling relation in Eq. (5) could be off by an order of magnitude for $q^2 \lesssim 100\ \text{fm}^{-2} \approx (2.5M_\omega)^2$. What remains now is the determination of the coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\sigma\gamma}$ relevant for the real photon processes

$$\rho \to \pi + \gamma ,$$

$$\omega \to \sigma + \gamma .$$

We use two methods to do this: (i) we obtain upper bounds on the coupling constants using experimental data; (ii) we use the quark model to calculate them. The two procedures complement each other and provide a consistency check. It is relatively straightforward to extract $g_{\rho\pi\gamma}$ in both methods and we simply state those results. The experimental width for $\rho \to \pi\gamma$ is not known, but an upper limit is available[19]:

$$\Gamma(\rho \to \pi\gamma) < 0.25\ \text{MeV} . \tag{6}$$

This implies a bound on $g_{\rho\pi\gamma}$, defined by Eq. (3), of

$$|g_{\rho\pi\gamma}| < 1.02 . \tag{7}$$

In the relativistic quark model of Feynman, Kislinger, and Ravndal (FKR)[12,20] which will be used throughout, $g_{\rho\pi\gamma}$ is very simple

$$g_{\rho\pi\gamma} = \pm\frac{2}{3}\,\frac{2m_\rho}{m_\rho + m_\pi} \approx \pm 1 . \tag{8}$$

This can be derived from FKR's Eq. (38) with a slight modification to apply to mesons. This is essentially identical to the experimental upper bound and suggests that Eq. (8) may be somewhat too large. An indication of this comes from calculating with the same model the coupling constant $g_{\omega\pi\gamma}$ (with the Lagrangian defined analogous to $\mathcal{L}_{\gamma\pi\rho}$) for

$$\omega \to \pi + \gamma$$

for which the branching ratio has been measured. The FKR model predicts $g_{\omega\pi\gamma} = \pm 3\,|g_{\rho\pi\gamma}| = \pm 3.0$, whereas the experimental width $\Gamma(\omega \to \pi\gamma) = 0.89$ MeV[19] leads to the empirical value $g_{\omega\pi\gamma}^{\text{exp}} = \pm 2.1$. Since the current and wave functions are similar in the two cases, it is reasonable to suppose that the quark model may overestimate $g_{\rho\pi\gamma}$ by the same amount, i.e., about 50%.

Extracting $g_{\omega\sigma\gamma}$ is less straightforward. The reason is that besides there not being any experimental information on the decay $\omega \to \sigma\gamma$, the quark description of σ is not yet well established. To proceed, we do the following. For the "experimental bound," we suppose that

$$\Gamma(\omega \to \sigma\gamma) < 3\Gamma(\omega \to \pi^0\pi^0\gamma) < 3(0.098\ \text{MeV}) . \tag{9}$$

Then, using Eq. (3) and the result

$$\frac{e^2 g_{\omega\sigma\gamma}{}^2}{4\pi} = \frac{24\Gamma(\omega \to \sigma\gamma)}{m_\omega(1 - m_\sigma{}^2/m_\omega{}^2)^3} \tag{10}$$

and the mass $M_\sigma = 550$ MeV, we get

$$|g_{\omega\sigma\gamma}| < 3 . \tag{11}$$

In quark models, the σ may be described as an orbital (L)-excited state (i.e., $L = 1$)[21] with the SU(3) content of the wave function identical to that of $\omega(L = 0)$. The FKR model enables one to calculate transitions between different L states and yields for this case

$$g_{\omega\sigma\gamma} = \frac{2}{3\sqrt{3\Omega}}\,\frac{m_\omega}{m_\sigma{}^2 - m_\omega{}^2}\left(\Omega + \frac{4m_\omega Q^2}{m_\omega + m_\sigma}\right), \tag{12}$$

$$\Omega \approx 1\ \text{GeV}^2 ,$$

$$Q^2 = \left(\frac{m_\omega{}^2 - m_\sigma{}^2}{2m_\omega}\right)^2 ,$$

where the orbital quark current contribution to $g_{\omega\sigma\gamma}$ (the first term in the parentheses) dominates the spin current contribution. The choice $M_\sigma = 550$ MeV leads to

$$g_{\omega\sigma\gamma} \approx -1.1 . \tag{13}$$

In contrast to $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$, for which the phase is arbitrary, the sign of $g_{\omega\sigma\gamma}$ is determined by the model. This follows because the SU(3) wave function is taken to be the same for both σ and ω. The predicted value is consistent with the bound Eq. (11), but the important point to notice in Eqs. (10) and (12) is the sensitivity of the coupling constant to the chosen σ mass. As already remarked, we take the value, shown in Table I, from calculations of the nucleon-nucleon potential and find the $\omega\sigma$ current is far less important at large momentum transfer than the $\rho\pi$ current. Finally, we can simply summarize the results of this section with the values we have used in the calculations:

$$g_{\rho\pi\gamma} = -g_{\omega\sigma\gamma} = 1\,, \tag{14}$$

where the sign of $g_{\rho\pi\gamma}$, hitherto undetermined, is chosen to be consistent with the sign of the magnetic moment discrepancy.

B. Electromagnetic form factors

The deuteron electromagnetic form factors are defined by the matrix element of the isoscalar electromagnetic current J_λ^s taken between initial (momentum P_λ) and final (momentum P'_λ) deuteron states

$$\tau_\lambda(q) = \langle d(P')\,|J_\lambda^s\,|\,d(P)\rangle\,, \tag{15}$$

where $q_\lambda = (\vec{q}, q_0) = (P' - P)_\lambda$ is the photon momentum. Since we are working with nonrelativistic deuteron wave functions, we will consistently use nonrelativistic kinematics and identify the form factors as functions of $q^2 \equiv |\vec{q}\,|^2$. Then the charge, quadrupole, and magnetic form factors, denoted, respectively, as $F_C(q)$, $F_Q(q)$, and $F_M(q)$, are related to $\tau_\lambda(q)$ by

$$\begin{aligned} \tau_0(q) &= F_C(q) - \frac{q^2}{2\sqrt{6}}(\vec{\sigma}_p \times \vec{\sigma}_n)^{(2)} F_Q(q)\,,\\ \vec{\tau}(q) &= \frac{i}{2M}\,\frac{1}{2}\,(\vec{\sigma}_p + \vec{\sigma}_n)\times\vec{q}\,F_M(q)\,, \end{aligned} \tag{16}$$

where M is the nucleon mass, and $(\vec{\sigma}_p \times \vec{\sigma}_n)^{(2)}$ is a tensor of rank 2 in spin space. For bookkeeping, we will write

$$F = F^I + F^\rho + F^\omega$$

corresponding to the impulse, ρ-exchange, and ω-exchange currents. The kinematics for $F^{\rho,\omega}$ are given in Fig. 8. Calculation of the form factors is straightforward but quite tedious. We

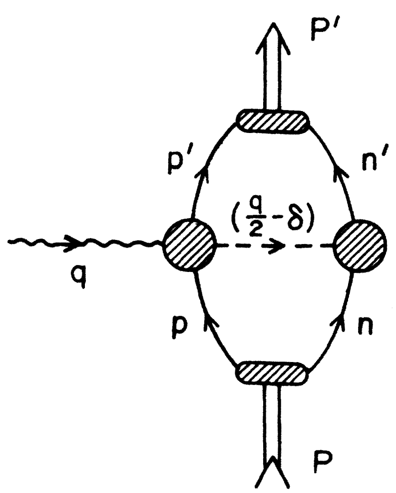

FIG. 8. Kinematics for the exchange current calculation. The dashed line represents the vector meson.

shall state the main assumptions and approximations and then give the final formulas.

1. *Approximations*

(a) The nucleons between which the mesons propagate are taken to be on the mass shell in accordance with the spirit of using nonrelativistic deuteron wave functions.
(b) The vector meson propagators are approximated by the unretarded ones:

$$\frac{1}{k_\mu^{\,2} + m_V^{\,2}} \to \frac{1}{\vec{k}^2 + m_V^{\,2}}\,.$$

we have estimated the retardation effects with an eikonal approach and find very small effects out to extremely large momentum transfers.
(c) The dependence on δ_0 (see Fig. 8) has been ignored, consistent with the nonrelativistic nature of the wave functions. The q_0 dependence has been kept but found to be numerically insignificant.
(d) Terms of third or higher order in $1/M$ are ignored at the meson-nucleon vertices.

2. *Results for the form factors*

We write the deuteron wave function in the conventional form

$$\begin{aligned} \psi_m(\vec{r}) &= \frac{1}{\sqrt{4\pi}\,r}\left(u(r) + \frac{S_{pn}(\hat{r})}{\sqrt{8}}\,w(r)\right)\chi_{1m}\,,\\ S_{pn}(\hat{r}) &= 3\vec{\sigma}_p\cdot\hat{r}\,\vec{\sigma}_n\cdot\hat{r} - \vec{\sigma}_p\cdot\vec{\sigma}_n \end{aligned} \tag{17}$$

and define the integrals

$$\begin{aligned} h_i(\delta) &= \int_0^\infty dr\, I_i(\delta r)\,, \quad i = 1, \ldots, 8\,,\\ I_1(\delta r) &= (u^2 - \tfrac{1}{2}w^2)\,j_0(\delta r)\,,\\ I_2(\delta r) &= \frac{1}{\sqrt{2}}\,w\left(u + \frac{w}{\sqrt{2}}\right) j_2(\delta r)\,,\\ I_3(\delta r) &= (u^2 + w^2)\,j_0(\delta r)\,,\\ I_4(\delta r) &= \frac{1}{\sqrt{2}}\,w\,(u - w/\sqrt{8}\,)\,j_2(\delta r)\,,\\ I_5(\delta r) &= w^2[\,j_0(\delta r) + j_2(\delta r)]\,,\\ I_6(\delta r) &= (u^2 + w^2/10)\,j_0(\delta r)\,,\\ I_7(\delta r) &= w^2[\,j_2(\delta r) + j_4(\delta r)]\,,\\ I_8(\delta r) &= \frac{1}{\sqrt{2}}\,w\left(u - r\,\frac{d}{dr}u\right)\frac{j_2(\delta r)}{\delta^2 r^2}\,,\\ F_l(\delta) &= \int_0^\infty dr\, u^2 j_l(\delta r)\,,\\ G_l(\delta) &= \frac{1}{\sqrt{2}}\int_0^\infty dr\, uw\, j_l(\delta r)\,,\\ H_l(\delta) &= \frac{1}{2}\int_0^\infty dr\, w^2 j_l(\delta r)\,. \end{aligned} \tag{18}$$

In terms of these quantities, the impulse form factors are

$$F_C^I(q) = G_{ES}(q) h_3(q/2)\,,$$
$$F_Q^I(q) = 12 q^{-2} G_{ES}(q) h_4(q/2)\,, \quad (19)$$
$$F_M^I(q) = G_{ES}\{\tfrac{3}{4} h_5(q/2) + (1+\kappa_s)[h_1(q/2) + h_2(q/2)]\}\,,$$

where κ_s (κ_v) is the isoscalar (isovector) nucleon anomalous magnetic moment and $G_{ES}(q)$ the nucleon isoscalar form factor, taken to be of the dipole form

$$G_{ES}(q) = (1 + q^2/0.71\ \text{GeV}^2)^{-2}\,.$$

The ω-exchange form factors are

$$F_C^\omega(q) = -\frac{2g_{\omega NN}}{1+q^2/m_\omega^2}\int \frac{d\vec{\delta}}{(2\pi)^3} R^\omega(\vec{q},\vec{\delta}) M(\vec{q},\vec{\delta}) \vec{q}\cdot\left(\frac{\vec{q}}{2}+\vec{\delta}\right) h_3(\delta)\,,$$

$$F_Q^\omega(q) = -\frac{12 g_{\omega NN}}{1+q^2/m_\omega^2}\int \frac{d\vec{\delta}}{(2\pi)^3} R^\omega(\vec{q},\vec{\delta}) M(\vec{q},\vec{\delta}) \vec{q}\cdot\left(\frac{\vec{q}}{2}+\vec{\delta}\right)(3x^2-1) h_4(\delta)\,,$$

$$F_M^\omega(q) = -\frac{2g_{\omega NN}}{1+q^2/m_\omega^2}\int \frac{d\vec{\delta}}{(2\pi)^3} R^\omega(\vec{q},\vec{\delta}) \vec{q}\cdot\left(\frac{\vec{q}}{2}+\vec{\delta}\right) \quad (20)$$
$$\times\left\{\frac{1+\kappa_s}{2}\left[[h_1(\delta)+h_2(\delta)]\left(1-2\frac{\delta}{q}x-\frac{\delta_\perp^2}{\frac{1}{2}q^2+q\delta x}\right)-\frac{3}{2}(1-x^2)h_2(\delta)\right]\right.$$
$$\left.+\frac{3}{2}\left(\frac{\delta x}{q}+\frac{1}{2}\frac{\delta_\perp^2}{\vec{q}\cdot(\vec{q}/2+\vec{\delta})}\right)h_5(\delta)\right\}\,,$$

where $x \equiv \hat{q}\cdot\hat{\delta}$ and

$$R^\omega(\vec{q},\vec{\delta}) = \frac{g_{\sigma NN}\, g_{\omega\sigma\gamma}/m_\omega}{[(\vec{q}/2-\vec{\delta})^2+m_\omega^2][(\vec{q}/2+\vec{\delta})^2+m_\sigma^2]}\,,$$
$$M(\vec{q},\vec{\delta}) = 1-\frac{q_0}{8M}\frac{\vec{q}\cdot(\vec{q}-4\vec{\delta})}{\vec{q}\cdot(\vec{q}/2+\vec{\delta})}-\frac{q_0^2}{2\vec{q}\cdot(\vec{q}/2+\vec{\delta})}\,, \quad (21)$$
$$q_0 = 2M[(1+q^2/4M^2)^{1/2}-1]\,.$$

Finally the ρ-exchange form factors are

$$F_C^\rho(q) = \frac{3g_{\rho NN}}{4M^2(1+q^2/m_\omega^2)} q^2 \int \frac{d\vec{\delta}}{(2\pi)^3} R^\rho(\vec{q},\vec{\delta})\delta_\perp^2 \left\{h_5(\delta)+\tfrac{2}{3}(1+\kappa_v)\left[h_3(\delta)-2\left(1+\frac{3}{2}\frac{qx}{\delta}\right)h_4(\delta)\right]\right\}\,,$$

$$F_Q^\rho(q) = \frac{9g_{\rho NN}}{M^2(1+q^2/m_\omega^2)}\int \frac{d\vec{\delta}}{(2\pi)^3} R^\rho(\vec{q},\vec{\delta})\delta_\perp^2$$
$$\times\left(24\frac{\delta x}{q}h_8(\delta)-\frac{2}{5}\left(1+3\frac{\delta x}{q}\right)h_5(\delta)+\frac{9}{35}\left(1-5x^2-\frac{16}{3}\frac{\delta x}{q}\right)h_7(\delta)\right.$$
$$+\tfrac{2}{3}(1+\kappa_v)\left\{-\left(1+6\frac{\delta x}{q}\right)h_6(\delta)-\frac{11}{7}H_2(\delta)\left[-1+\frac{27}{11}x^2+\frac{6}{11}x\left(\frac{q}{4\delta}+\frac{\delta}{q}\right)\right]\right. \quad (22)$$
$$\left.\left.+2G_2(\delta)\left[1-3x\left(\frac{q}{4\delta}+\frac{\delta}{q}\right)\right]+\tfrac{27}{35}H_4(\delta)\left[1+\frac{x}{2}\left(32\frac{\delta}{q}+15\frac{q}{\delta}\right)-5x^2-\frac{35}{2}\frac{qx^3}{\delta}\right]\right\}\right)\,,$$

$$F_M^\rho(q) = -\frac{3g_{\rho NN}}{1+q^2/m_\omega^2}\int \frac{d\vec{\delta}}{(2\pi)^3} R^\rho(\vec{q},\vec{\delta})\delta_\perp^2$$
$$\times\left(h_1(\delta)-\left(2+\frac{3}{2}\frac{qx}{\delta}\right)h_2(\delta)+\frac{q_0}{8m}\left[h_1(\delta)+h_2(\delta)+3h_2(\delta)\frac{qx/2+\delta}{\delta}\right]\right.$$
$$\left.-\frac{q_0}{4m}\left(\frac{1}{2}\delta q x-\delta^2\right)\left\{\left(1+2\frac{\delta x}{q}\right)[h_1(\delta)+h_2(\delta)]-\frac{3}{2}\frac{\delta_\perp^2}{\delta^2}h_2(\delta)\right\}\right)\,,$$

where

$$\delta_\perp^2 = \delta^2 - (\vec{\delta} \cdot \hat{q})^2$$

and

$$R^\rho(\vec{q}, \vec{\delta}) = \frac{g_{\pi NN} g_{\rho\pi\gamma}/m_\rho}{[(\vec{q}/2 - \vec{\delta})^2 + m_\rho^2][(\vec{q}/2 + \vec{\delta})^2 + m_\pi^2]} . \quad (23)$$

Except for the q_0 dependent terms and for the diagonal D-state terms, the ρ-exchange form factors are similar to those given by Adler.[1] We are quite confident in the correctness of these expressions and, despite their horrendous appearance, numerical evaluation is quite straightforward. The results have already been displayed in Figs. 3–6 and discussed in the first section. We repeat that the quantitative predictions must be viewed as somewhat speculative, depending as they do upon the unmeasured photon-meson couplings. Nevertheless, our calculation shows that the form factors do flatten out at sufficiently large momentum transfer within a "conventional" exchange current picture. The question of where this happens is important both for learning about the two-body currents and in its implications for the interpretation of large momentum transfer electron scattering data in terms of the nuclear current distribution, and our model predicts large exchange effects at momentum transfers easily accessible to existing high energy electron accelerators.

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