Influence of the particle-number fluctuation on the $E1$ $\Delta K = \pm 1$ transitions in deformed odd-mass nuclei

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Dipole electric transitions with $\Delta K = \pm 1$ are studied theoretically for odd-mass nuclei in the rare-earth region (i) in the frame work of an elaborated Nilsson model with rotation-particle coupling and with isospin-dependent parameters, (ii) with the usual BCS wave function, and (iii) with sharp particle-number-projected BCS wave functions. These projected wave functions are the limit of very speedily converging sequences of functions. General formulas for the pairing reduction factors with strict nucleon number conservation are established. The unphysical effects due to particle fluctuation in the usual BCS wave functions are found to be very important in the theoretical BCS evaluation of reduction factors and half-lives. In most of the analyzed $E1$ transitions, the hindrance factors are improved by the elimination of the unphysical effects due to particle fluctuation. The blocking effect is systematically taken into account. The importance and validity of the physical and mathematical approximations used is discussed.

[RADIOACTIVITY ^{153,155}Eu; ^{159,161}Tb; ^{175,177}Lu; ^{179,181}Ta; ^{155,157}Gd; ^{161}Dy; ^{169,171,173,177}Yb; ^{175,177,179}Hf; $E1$ $\Delta K = \pm 1$ particle-number-projected pairing reduction factor and $T_{1/2}$.]

I. INTRODUCTION : MODEL SURVEY

If the theoretical study of the phenomena of electromagnetic transitions in atomic nuclei was at its height some 15 years ago, very few new methods or techniques have been proposed recently, and the state of the theory has not progressed much since the calculations of Weisskopf,[1] Moszkowski,[2] Bohr and Mottelson,[3] and Nilsson.[4,5] A lack of precise experimental information concerning single-particle energies or collective vibrations, does not seem to be the only reason for the present lack of interest in the field of β- and γ-ray spectroscopy, because numerous experimental works concerning more especially the nuclei of the rare-earth region and the actinides have appeared recently.[6-10]

The different theoretical approaches may be summarized without any claim to completeness as follows:

(i) The usual shell model[11] and its more or less sophisticated[12] variants taking into account the different types of core polarization have not always produced results commensurate with the effort invested. They have only occasionally been used for other than light nuclei, and can only be applied to systematic multipolar transition analyses at the cost of a considerable amount of numerical calculations.[13]

(ii) The influence of the interaction between quasiparticles has been studied within the scope of Migdal's finite-Fermi-system theory.[14] This approach may explain[15,16] several single-particle $E1$ transitions with $|\Delta K| = 1$; for example in Ref. 16 an improvement of the hindrance factor, as compared with Nilsson's, was found in 75% of the cases. These results can be modified by the Coriolis effect which was completely ignored, and, moreover, the approximation is not free from phenomenology.

(iii) Various multipole sum rules have often been used in the studies of photonuclear reactions.[17-19] As it requires no knowledge of wave functions of the excited states of the nuclei, this approach is a redoubtable trial ground of two-body interactions and seems particularly suitable for the study of the nonlocality effect of realistic nuclear forces[18] on the integrated photoabsorption cross sections.

Calculations[18,19] performed in the electric dipole approximation and with a Fermi-gas nuclear model indicate fair agreement with experimental data in the heavy-nuclear region.

(iv) A Hartree-Fock (HF) calculation based on a detailed knowledge of nuclear forces may be more attractive than the method briefly indicated in the preceding paragraph. However, and especially in the case of nonlight nuclei, the excessively high number of matrix elements of two-body interactions to be calculated for the large number of particles involved, makes such a calculation impractical with computers presently available. The interesting and powerful generalized HF approximation of Kerman and Klein elaborated and analyzed by Klein and his co-workers[20] has been successfully applied in some numerical calculations.[21] However, the question of the strict conservation

of the number of particles remains open. Recently, a self-consistent core-particle coupling method derived from the generalized HF approximation was applied to the nickel isotopes.[22] The relatively wide discrepancies between the ensuing results and experimental data on ^{65}Ni are attributed by the author to the nonconservation of the number of particles. The problem of the nonstrict conservation of the number of nucleons has been reviewed and discussed recently, more particularly in relation to the rotation energies, and it has been found that nonstrict conservation of the nucleon number could alter the energies and the wave functions appreciably.[23] It is undeniable that in modifying, even slightly, the wave functions, the considered effect may, in consequence, influence phenomena which, as in the case of electromagnetic transitions, are sensitive to slight modifications of the wave functions. A similar conclusion has been obtained recently for several light nuclei by a Hartree-Bogoliubov calculation.[24]

(v) The BCS wave functions seem particularly suitable for a later projection in the occupation-number space (see Refs. 23 and 25; the strict particle-number conservation method of these references may be called the SBCS method). In the place of a HF potential which could only be known numerically, we employ a phenomenological one-body potential whose eigenfunctions are well known. Our aim is then to procede to calculate different transitions of the type $E1$, $|\Delta K| = 1$, with the BCS and SBCS functions so as to judge the extent of the nonphysical effects introduced by the fluctuation of the number of particles.

The present renewed experimental interest in $E1$ transitions in the rare-earth region justifies a closer study of this question. In fact a comparison between experimental and theoretical results could be falsified by spurious number-fluctuation effects.[25] A theoretical study of $E1$ transitions could give vital information on nuclear structure, deformation of nuclei in different excited states, the quality of wave functions, β transitions, etc. A method of getting information on, for example, the β decay by studying $E1$ γ transitions from the isobaric analog state, was suggested by Fujita,[26] and extensively used in the study of the first-forbidden β decay in neutron magic nuclei.[27] In all these cases care must be taken that the results are not over-affected by the nonphysical effect under consideration.

As yet, very few particle-conserving calculations have been made. In Ref. 24 a few quadrupole moments and $B(E2)$ values were calculated for light nuclei with approximate particle-number-projected wave functions. But the approximate character of the number-projection operators makes its use tricky and open to error for the non-light nuclei which are our present concern.

Miranda and Preston[28] used the well-known generating function of Bayman.[29] This method excludes large fluctuations by using the saddle-point approximation. However, in its simplest form this method is the usual BCS approximation. The calculated β-decay and single-particle $M4$ transition reduction factors are unequally affected by the extraction of the unphysical components, but always in the right direction.

To our knowledge the only attempt made to study the influence on $E1$ transitions by number-projected BCS wave functions is the calculation of Monsonego and Piepenbring,[30] using once again, Bayman's generating function. As discussed in Ref. 23, this technique may not be the most powerful and is certainly not the easiest to handle. On the other hand, the general lack of agreement obtained with the experiment might be partly explained by the rotation-particle coupling (RPC) which was completely ignored in the calculation. Moreover, a large number of new transitions have since been brought to light experimentally, particularly in the rare-earth region,[6-10,31,32] thus justifying a fresh study of this problem. Finally, the description of the one-body central potential has in the mean time been considerably improved[33-38] allowing, for example, an isospin dependence of the neutron or proton Nilsson oscillator frequency.

It seems to us that for all these reasons a fresh analysis of the problem has become indispensable. We propose to study in this work the influence of the nonstrict conservation of the number of nucleons in the usual BCS theory on the probabilities of electromagnetic transitions, and we shall establish, in particular, on the basis of the projected BCS functions, the strict particle-conserving pairing reduction factor R_{SBCS} for any multipole transition.

II. NUCLEAR FIELD

According to the foregoing, the BCS theory would appear to be an appropriate starting point for establishing exact particle-number-conserving wave functions, and the Hamiltonian of a distorted harmonic oscillator with axial symmetry may advantageously replace a more realistic nonlocal self-consistent field. At first sight a Woods-Saxon potential with constant surface diffuseness seems to more in agreement with the experimental knowledge possessed on the nuclear single-particle potential, but quite apart from the computational difficulties associated with a Woods-Saxon potential, many numerical calculations have shown that as long as one is not dealing with superheavy nuclei

a deformed oscillator potential in the shape of a spheroid will do as well as the Woods-Saxon potential from the point of view of both the energy levels and their angular momentum configuration.[35] The use of a nonlocal phenomenological potential[36] as, for example, the velocity-dependent Lemmer-Green potential,[36] does not make any fundamental difference.

Thus, the single-particle Hamiltonian will be written (in a coordinate system fixed in the nucleus):

$$H^{\pi} = \tfrac{1}{2}\hbar\omega_0^{\pi}(\delta)(-\nabla^2 + r^2) - \frac{4}{3}\left(\frac{\pi}{5}\right)^{1/2}\delta\hbar\omega_0^{\pi}(\delta)r^2Y_2^0 - 2\chi^{\pi}\hbar\omega_0^{0\pi}\vec{l}\cdot\vec{s} - \mu^{\pi}\chi^{\pi}\hbar\omega_0^{0\pi}(\vec{l}^2 - \langle l^2\rangle_{\text{shell}}). \quad (1)$$

The radial distance r is given in oscillator units $(\hbar/m\omega_0^0)^{1/2}$ and π indicates the nature of the particle: neutron (n) or proton (p), all constants being isospin-dependent. We note the eigenkets of Eq. (1) by $|N\Omega\nu\rangle$ and the corresponding eigenvalues by:

$$E_{\nu}^{\pi N\Omega} = (N + \tfrac{3}{2})\hbar\omega_0^{\pi}(\delta) + \chi^{\pi}\hbar\omega_0^{0\pi}r_{\nu}^{N\Omega}. \quad (2)$$

All energies within the same shell have been renormalized by $\chi^{\pi}\mu^{\pi}\hbar\omega_0^{0\pi}\langle l^2\rangle_{\text{shell}}$ $[=\frac{1}{2}\chi^{\pi}\mu^{\pi}\hbar\omega_0^{0\pi}N(N+3)]$ which takes into account the conservation of the distance between centers of gravity of the successive N shells. The parameters χ^{π} and μ^{π} depend linearly on the atomic mass of the nuclei (see Table I).

If one admits for neutron and protons a common value $\hbar\omega_0^0$ $(=41A^{-1/3}$ MeV) one obtains, in the spherical case, a root-mean-square radius lower for protons than for neutrons.[33] Thus, with Nilsson *et al.*,[33] we permit the one-body potential to be isospin-dependent. The energy quantum $\hbar\omega_0^{\pi}(\delta)$ depends on the relative neutron excess $(N-Z)/A$ and, for a nucleus with $A=N+Z$ particles, we have, for protons and neutrons, respectively:

$$\hbar\omega_0^{0p} = \hbar\omega_0^0\left(1 - \frac{N-Z}{3A}\right), \qquad \hbar\omega_0^{0n} = \hbar\omega_0^0\left(1 + \frac{N-Z}{3A}\right). \quad (3)$$

TABLE I. Values of the two isospin-dependent shell parameters of the single-particle Hamiltonion H^{π}. They are optimized (Ref. 33) to reproduce the experimental level schemes for the rare-earth nuclei and the actinides.

	Neutrons	Protons
χ	$0.0641 - 0.0026\,\frac{A}{1000}$	$0.0766 - 0.0779\,\frac{A}{1000}$
μ	$0.624 - 1.234\,\frac{A}{1000}$	$0.493 + 0.649\,\frac{A}{1000}$

We choose furthermore $\hbar\omega_0^0 = 41A^{-1/3}$ MeV for simplicity. We also suppose an isospin dependence for the pairing constant:

$$G^{\pi}A \simeq g_0 \pm g_1\frac{N-Z}{A} + \cdots,$$

where the plus sign holds for protons and the minus sign holds for neutrons.[33] The constants g_0 and g_1 giving values of Δ close to the experimental results for the rare-earth region are: $g_0 = 19.2$ MeV and $g_1 = 7.4$ MeV.

For the whole of the nuclei studied, the values of the parameter of deformation δ employed are those of Ref. 31 and the values of the parameter of inertia $\hbar^2/2\mathcal{I}$ are taken from Ref. 34.

The particle-rotation coupling term H' ignored in most theoretical studies of $E1$ transitions (for example in Refs. 16, 30, 31) is in fact rarely negligible[10,34,37-39] even for low-j values, and is especially large between deformed orbitals originating from the same spherical j level. The model Hamiltonian of a particle moving in a deformed potential coupled to a rotating core which has axial symmetry about the z axis is

$$H = H^c + H^{\text{int}} + H',$$

where the unperturbed collective part H^c describes the rotation of a symmetric top

$$H^c = \frac{\hbar^2}{2\mathcal{I}}(I^2 - I_3^2 - j_3^2). \quad (4)$$

As for the Coriolis interaction

$$H' = -\frac{\hbar^2}{2\mathcal{I}}(I_+j_- + I_-j_+), \quad (5)$$

it may be treated as a perturbation. As usual I and I_3 are the total angular momentum of the nucleus and its projection along the z axis, j is the angular momentum of the single particle, and $\mathcal{I}$ is the moment of inertia.

As our basic aim is the study of the influence of the particle-number fluctuations in electric dipole transitions, we ignore the $|\Delta N| = 2$ mixing between one-particle states. This effect may possibly be noticeable for the heavy rare-earth nuclei as well as for the actinide,[39,40] but cannot influence the conclusions of our study.[41] The same is true for octupole coupling. It is a well-known fact that the low-lying octupole vibrational band can influence the transition probabilities of $E1$, $\Delta K = 0$ transitions through mixing by particle-vibrational interaction.[42] The $E1$, $|\Delta K| = 1$ transition probabilities can, however, only be disturbed by the $K = \Omega \pm 1$ octupole vibrational bands, which lie much higher in energy and have smaller collective strength in comparison with the $K = \Omega$ octupole vibrational band. It is therefore generally believed that the

$E1$, $|\Delta K| = 1$ transitions are not noticeably contaminated by octupole components.[42]

The eigenkets and eigenenergies of the unperturbed $H^c + H^{\text{int}}$ Hamiltonian are written, respectively:

$$|I, KM; \nu\rangle = \left(\frac{2I+1}{16\pi^2}\right)^{1/2} (D^I_{MK}|\nu K\rangle + \text{phase} D^I_{M-K}|\nu -K\rangle) \tag{6a}$$

and

$$E^{\pi NK}_{I\nu} = E^{\pi NK}_{\nu} + [I(I+1) - 2K^2]\frac{\hbar^2}{2\mathcal{J}}. \tag{6b}$$

The ket $|\nu K\rangle$ is an intrinsic state ket and may be, for example, an eigenket of H^π [Eq. (1)] or $|\text{BCS}\rangle$ or $|\text{SBCS}\rangle$.

If the Coriolis interaction is taken into account, the kets of Eq. (6a) are no longer eigenkets of the total Hamiltonian H. However, the complete nuclear-state ket $|IM\rangle$ may be expanded on this basis:

$$|IM\rangle = \sum_{K\nu} |I, KM; \nu\rangle\langle K\nu|I\rangle, \tag{7}$$

which implies that K, the projection of the total angular momentum $\vec{I} = \vec{j} + \vec{R}$ ($\vec{R}$ being the collective rotational angular momentum) on the nuclear symmetry axis, is no longer a constant of the motion, even in the adiabatic limit. We admit that K remains approximately a good quantum number.

To the first order in perturbation theory, the eigenkets of the total Hamiltonian take the form

$$|IK\rangle = |I, KM; \nu\rangle + \sum_{K'=K\pm1} |I, K'M; \nu\rangle\langle K'\nu|I\rangle, \tag{8}$$

where

$$\langle K'\nu|I\rangle = \frac{\langle I, K'M; \nu|H'|I, KM; \nu\rangle}{E^{\pi NK}_{I\nu} - E^{\pi NK'}_{I\nu}} R.$$

The constant R depends on the precise choice of the intrinsic state $|\nu K\rangle$. For the eigenkets of the Nilsson Hamiltonian H^π of Eq. (1), R equals 1, and for $|\text{BCS}\rangle$ or $|\text{SBCS}\rangle$ states, R is equal to R_{BCS} and R_{SBCS}, respectively. The energies $E^{\pi NK}_{I\nu}$ are in each case the eigenvalues corresponding to the chosen intrinsic states $|\nu K\rangle$.

The electric dipole transition probability

$$P(E1) = \frac{16\pi}{9\hbar}\left(\frac{\omega}{c}\right)^3 B(E1),$$

$$B(E1) = \left|\sum_{K'\mu} \langle IK'|\mathfrak{M}^\mu_1|IK\rangle\right|^2, \tag{9}$$

where

$$\mathfrak{M}^\mu_1 = e^\pi_{\text{eff}} r Y^\mu_1(\theta\varphi)$$

and

$$e^\eta_{\text{eff}} = -e\frac{Z}{A}, \quad e^p_{\text{eff}} = e\frac{N}{A}$$

is expressed as follows, if calculated with the kets of Eqs. (6)–(8):

$$B(E1) = \langle I_f, K_f M_f; \nu_f|\mathfrak{M}^{K_f - K_i}_1|I_i, K_i M_i; \nu_i\rangle + \sum_K \langle I_f, K_f M_f; \nu_f|\mathfrak{M}^{K_f-K}_1|I_i, KM_i; \nu_i\rangle\langle K\nu_i|I_i\rangle$$
$$+ \sum_{K'} \langle K'\nu_f|I_f\rangle\langle I_f, K'M_f; \nu_f|\mathfrak{M}^{K'-K_i}_1|I_i, K_iM_i; \nu_i\rangle + \sum_{KK'} \langle K'\nu_f|I_f\rangle\langle I_f, K'M_f; \nu_f|\mathfrak{M}^{K'-K}_1|I_i, KM_i; \nu_i\rangle\langle K\nu_i|I_i\rangle, \tag{10}$$

where

$$\langle I', K'M'; \nu'|\mathfrak{M}^\mu_1|I, KM; \nu\rangle = \langle IK, 1K'-K|I'K'\rangle\langle\nu'K'|\mathfrak{M}^{K'-K}_1|\nu K\rangle + \text{phase}\langle I-K, 1K'+K|I'K'\rangle$$
$$\times \langle\nu'K'|\mathfrak{M}^{K'+K}_1|\nu -K\rangle. \tag{11}$$

The $E1$ transition probability depends (as well as the undefined phases) on the precise choice of the one-particle state $|\nu K\rangle$. We propose to compare the theoretical transitions obtained: (i) with the pure usual Nilsson single-particle wave functions with phase $\equiv (-)^{I+K}$; (ii) with BCS wave functions; (iii) with sharp number-projected BCS wave functions (SBCS approximation). The elaboration of these functions and the calculation of the corresponding reduction factors are the subject of the following section.

III. STRICT-PARTICLE-CONSERVING WAVE FUNCTIONS; HINDRANCE FACTORS

One of the present tendencies of the theory of the nuclear structures consists in polishing and refining the residual nuclear interaction. It is now well established that the residual two-body interaction must have, at low energies, a pairing character which is generally incorporated phenomenologically via the BCS pairing interaction. This BCS pairing effect causes strong configuration mixing in the

ground-state wave function and produces the well-known energy gap. But because of the nonstrict conservation of the number of particles, nonphysical effects may appear. One of the aims of this study is to analyze its importance in the transitions of electric dipole type.

A. Odd-mass nuclei SBCS wave functions

The pairing-type Hamiltonian for a system of P pairs of particles (neutrons or protons) has the form:

$$H^{\text{int}} = \sum_{\nu} E_\nu (a_\nu^\dagger a_\nu + a_{\tilde{\nu}}^\dagger a_{\tilde{\nu}}) - G \sum_{\nu\nu'} a_\nu^\dagger a_{\tilde{\nu}}^\dagger a_{\tilde{\nu}} a_\nu , \quad (12)$$

where G designates the pairing-force strength; $a_\nu^\dagger$ and a_ν are, respectively, a creation and an annihilation operator of a particle of energy E_ν in a quantum state $|\nu\rangle$. The states $|\nu\rangle$ and $|\tilde{\nu}\rangle$ are conjugate by time reversal. In the traditional BCS theory the trial ket describing an odd-mass nuclei with the last single-particle in the state $|\mu_0\rangle$ is written

$$|\varphi_{\mu_0}\rangle = a_{\mu_0}^\dagger \prod_{\nu\neq\mu_0} (u_\nu + v_\nu a_\nu^\dagger a_{\tilde{\nu}}^\dagger)|0\rangle . \quad (13)$$

(The vacuum $|0\rangle$ is defined by $a_\nu|0\rangle = 0$, $\forall\nu$.) This ket is not an eigenstate of the particle-number operator

$$N = \sum_{\nu} (a_\nu^\dagger a_\nu + a_{\tilde{\nu}}^\dagger a_{\tilde{\nu}}) ;$$

only its mean value is equal to the real number of particles. The Eq. (13) describes merely a superposition of states of several neighboring nuclei, of which the particle number differs by an even number of particles.

The sharp number-projected ket (see Ref. 23) is then

$$|\Psi_{\mu_0}(n)\rangle = C\left[\sum_{k=0}^{n+1} \epsilon_k \eta_k^{-P} a_{\mu_0}^\dagger \prod_{\nu\neq\mu_0} (u_\nu + \eta_k v_\nu a_\nu^\dagger a_{\tilde{\nu}}^\dagger) + \text{C.C.}\right]|0\rangle \quad (14)$$

with

$$\eta_k = e^{i[k\pi/(n+1)]}, \quad \epsilon_k = \begin{cases} 1 & \text{if } k=0 \text{ or } n+1 \\ \frac{1}{2} & \text{if } 0<k<n+1 \end{cases} .$$

n is a nonnegative integer and C is a normalization constant. The only nonvanishing components in Eq. (14) are those belonging to a number of pairs of particles equal to $P \pm 2l(n+1)$ with l a nonnegative integer. If the integer n is chosen so that $2(n+1) > \text{Max}(P, \Omega - P)$ (Ω being the total pair degeneracy of the nucleus), then the projection is exactly performed and the ket of Eq. (14) describes a state having exactly P pairs of particles, and a single particle in the state $|\mu_0\rangle$. In most of numerical applications[23] the convergence is in fact obtained for $n \sim 2$ or 4. In this is found precisely one of the essential interests of this method.

B. Reduction factor

For calculating the matrix elements of a one-body particle-conserving operator O (like the electromagnetic moments) it may be advantageous to go over to the quasiparticle representation defined by the Bogoliubov-Valatin transformation: The vacuum is the pure BCS state. Let us define the following operator for quasiparticle pairs:

$$A_\nu = \alpha_{\tilde{\nu}} \alpha_\nu \quad \text{and} \quad A_\nu^\dagger = \alpha_\nu^\dagger \alpha_{\tilde{\nu}}^\dagger ,$$

where $\alpha_\nu^\dagger$ and α_ν are the quasiparticle creation and annihilation operators.

Then we obtain the following expansion:

$$a_{\mu_0}^\dagger \prod_{\nu\neq\mu_0} (u_\nu + \eta_k v_\nu a_\nu^\dagger a_{\tilde{\nu}}^\dagger)|0\rangle = \prod_{\nu\neq\mu_0} (u_\nu^2 + \eta_k v_\nu^2) \Bigg\{ 1 + (\eta_k - 1) \sum_{\nu\neq\mu_0} \frac{u_\nu v_\nu}{(u_\nu^2 + \eta_k v_\nu^2)} A_\nu^\dagger + \frac{(\eta_k - 1)^2}{2} \times \sum_{\nu\neq\mu\neq\mu_0} \frac{u_\nu v_\nu u_\mu v_\mu}{(u_\nu^2 + \eta_k v_\nu^2)(u_\mu^2 + \eta_k v_\mu^2)} A_\nu^\dagger A_\mu^\dagger + \cdots \Bigg\} |\varphi_{\mu_0}\rangle . \quad (15)$$

Still in the same representation, any one-body operator O takes the form

$$O = \sum_{i,j=0}^{2} O_{ij} \quad (16)$$

with O_{ij} representing the sum of components of O, having i creation operators and j annihilation operators of quasiparticles. Further, O connects only components with an equal number of particles. Consequently, we obtain

$$\langle \Psi_{\mu_0}(n)| O \eta_k^{-P} a_{\mu_0}^\dagger \prod_{\nu\neq\mu_0} (u_\nu + \eta_k a_\nu^\dagger a_{\tilde{\nu}}^\dagger)|0\rangle = \langle \Psi_{\mu_0}(n)| O |\varphi_{\mu_0}\rangle \quad (17a)$$

or, in a more general manner

$$\langle \Psi_{\nu_0}(n)|O|\Psi_{\mu_0}(n)\rangle = 2(n+1)C\langle \Psi_{\nu_0}(n)|O|\varphi_{\mu_0}\rangle . \tag{17b}$$

When operators O act on the ket $|\varphi_{\mu_0}\rangle$ only the components O_{11} and O_{02} of Eq. (16) with, respectively, one and two quasiparticle annihilation operators give a nonzero contribution.

From the sum of the preceding remarks, it results that, after elimination of the imaginary part:

$$\langle \Psi_{\nu_0}(n)|O|\Psi_{\mu_0}(n)\rangle = \frac{\sigma(\mu_0)}{\sigma(\nu_0)} O_{11}^{\nu_0\mu_0} - \frac{1}{\sigma(\nu_0)\sigma(\mu_0)} \sum_{k=0}^{n+1} \epsilon_k R_k(\mu_0)[2\delta_{\nu_0}(\mu_0)\cos\Psi_k(\mu_0)\sin^2 x_k + \sin\Psi_k(\mu_0)\sin(2x_k)]$$

$$\times \frac{\gamma_{\nu_0}(\mu_0)}{\rho_{\nu_0 k}^2(\mu_0)} O_{02}^{-\nu_0\mu_0} \tag{18}$$

with the following notation (when the state $|\nu\rangle$ is blocked):

$$\sigma(\nu) = \left[\sum_{k=0}^{n+1} \epsilon_k R_k(\nu)\cos\Psi_k(\nu)\right]^{1/2},$$

$$\gamma_\mu(\nu) = 2u_\mu(\nu)v_\mu(\nu), \quad \delta_\mu(\nu) = u_\mu^2(\nu) - v_\mu^2(\nu), \quad x_k = \frac{k\pi}{2(n+1)}, \quad R_k(\nu) = \prod_{\mu(\neq\nu)} \rho_{\mu k}^{(\nu)},$$

$$\rho_{\mu k}^{(\nu)} = [1 - \gamma_\mu^2(\nu)\sin^2 x_k]^{1/2}, \quad \Psi_k(\nu) = \sum_{\mu(\neq\nu)} \phi_{\mu k}^{(\nu)} + (\Omega - 2P)x_k, \quad \tan\phi_{\mu k}^{(\nu)} = -\delta_\mu(\nu)\tan x_k, \quad \left(|\phi_{\mu k}(\nu)| \leq \frac{\pi}{2}\right).$$

In Eq. (18) the quantities $O_{11}^{\nu_0\mu_0}$ and $O_{02}^{-\nu_0\mu_0}$ are proportional to the matrix element $\langle \nu_0|O|\mu_0\rangle$ and depend in addition on the properties of the operators O in respect to the time reversal.

In general, for all one-body operators O, the matrix element of a transition from the state $|\nu\rangle$ to the state $|\mu\rangle$ satisfies either the property

$$\langle \nu|O|\mu\rangle = \langle \bar{\mu}|O|\bar{\nu}\rangle \tag{19a}$$

(this is the case of operators inducing an electric transition, for example) or the property

$$\langle \nu|O|\mu\rangle = -\langle \bar{\mu}|O|\bar{\nu}\rangle \tag{19b}$$

(this is the case of operators inducing magnetic transitions, angular momentum, $\cdots$).

TABLE II. Convergence of the pairing-reduction factor for both electric (E) and magnetic (M) type transitions versus the degree n of extraction of the unphysical components. Note that the term of order 0 of the sequence is always close to the BCS reduction factor. The convergence is in all cases physically satisfactory for $n = 2$ or 3.

Nucleus	Initial state	Final state	$R_{BCS}^{E/M}$	$R_0^{E/M}$	$R_1^{E/M}$	$R_2^{E/M}$	$R_3^{E/M}$	$R_4^{E/M}$	$R_5^{E/M}$	$R_6^{E/M}$	$R_7^{E/M}$
$^{153}_{63}Eu_{90}$	$\frac{5}{2}\frac{3}{2}^+(411)$	$\frac{5}{2}\frac{5}{2}^-(532)$	5.8×10^{-4}	6.06×10^{-4}	0.092	0.111	0.114	0.115	0.117	0.118	0.118
			0.889	0.915	0.720	0.708	0.705	0.703	0.702	0.701	0.701
$^{161}_{65}Tb_{96}$	$\frac{7}{2}\frac{7}{2}^-(523)$	$\frac{5}{2}\frac{5}{2}^+(413)$	0.022	0.023	0.032	0.044	0.046	0.047	0.047	0.047	0.047
			0.795	0.838	0.524	0.502	0.497	0.495	0.494	0.493	0.493
$^{175}_{71}Lu_{104}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	0.324	0.326	0.068	0.040	0.039	0.038	0.038	0.038	0.038
			0.953	0.961	0.754	0.717	0.715	0.714	0.714	0.714	0.714
$^{179}_{73}Ta_{106}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	9×10^{-4}	1×10^{-3}	0.049	0.066	0.067	0.068	0.068	0.068	0.068
			0.890	0.908	0.803	0.796	0.795	0.795	0.795	0.795	0.795
$^{157}_{64}Gd_{91}$	$\frac{5}{2}\frac{5}{2}^+(642)$	$\frac{3}{2}\frac{3}{2}^-(521)$	0.515	0.525	0.600	0.606	0.608	0.608	0.608	0.608	0.608
			0.926	0.944	0.833	0.827	0.827	0.827	0.827	0.827	0.827
$^{171}_{70}Yb_{101}$	$\frac{5}{2}\frac{5}{2}^-(512)$	$\frac{7}{2}\frac{7}{2}^+(633)$	0.233	0.338	0.001	0.006	0.008	0.009	0.011	0.011	0.011
			0.384	0.555	0.016	0.008	0.006	0.005	0.005	0.005	0.005
$^{175}_{72}Hf_{103}$	$\frac{7}{2}\frac{7}{2}^+(633)$	$\frac{5}{2}\frac{5}{2}^-(512)$	0.088	0.162	0.818	0.882	0.903	0.913	0.920	0.920	0.920
			0.300	0.553	0.963	0.999	1	1	1	1	1
$^{177}_{72}Hf_{105}$	$\frac{9}{2}\frac{9}{2}^+(624)$	$\frac{7}{2}\frac{7}{2}^-(514)$	0.847	0.847	0.619	0.623	0.624	0.625	0.625	0.625	0.625
			0.996	0.997	0.947	0.949	0.950	0.951	0.951	0.951	0.951

More precisely we write

$$O_{11}^{\nu\mu} = [u_\nu(\mu)u_\mu(\nu) + (-1)^T v_\nu(\mu)v_\mu(\nu)]\langle\nu|O|\mu\rangle, \tag{20a}$$

$$O_{02}^{-\nu\mu} = [v_\nu(\mu)u_\mu(\nu) - (-1)^T u_\nu(\mu)v_\mu(\nu)]\langle\nu|O|\mu\rangle, \tag{20b}$$

where $T = +1$ if O satisfies Eq. (19a) and $T = -1$ if O satisfies Eq. (19b).

Finally, the presence of pairing forces without fluctuation of the particle number (SBCS approximation) manifests itself in the transition probabilities by a multiplication factor R_n given by

$$R_n = \Big\{ \frac{\sigma(\mu_0)}{\sigma(\nu_0)} [u_{\nu_0}(\mu_0)u_{\mu_0}(\nu_0) + (-1)^T v_{\nu_0}(\mu_0)v_{\mu_0}(\nu_0)] - \frac{1}{\sigma(\nu_0)\sigma(\mu_0)} \times \sum_{k=0}^{n+1} \epsilon_k R_k(\mu_0)[2\cos\Psi_k(\mu_0)\sin^2 x_k \delta_{\nu_0}(\mu) + \sin\Psi_k(\mu_0)\sin(2x_k)] \times \frac{\gamma_{\nu_0}(\mu_0)}{\rho_{\nu_0 k}^2} [v_{\nu_0}(\mu_0)u_{\mu_0}(\nu_0) - (-1)^T u_{\nu_0}(\mu_0)v_{\mu_0}(\nu_0)] \Big\}^2, \tag{21}$$

where $T = \pm 1$ according to whether transitions of electric or magnetic type are considered.

IV. NUMERICAL RESULTS : DISCUSSION

The theory of the preceding section, together with the single-particle field outlined in Sec. II, has been applied to numerous dipole electric transitions in deformed odd-mass nuclei in the rare-earth region $153 < A < 180$.

A numerical study of the convergence of the pairing factor R_n, given by Eq. (21), versus the degree of extraction n of the nonphysical components, brings to light the importance of the particle-number projection. Table II shows the evolution of R_n ($n = 0, 7$) for four neutron transitions and four proton transitions, of the electric type R_n^E or of the magnetic type R_n^M. One can note that in the majority of cases given in this table, the two types of pairing factors, and especially the R_n of the electric type, differ considerably according to whether they are calculated in the BCS or SBCS approximation. We also notice the rapidity of the convergence of the sequence R_n: for n close to 3 to 5 one can estimate that the projection is realized. We note also, in passing, that $R_{n=0}$ is always close to $R_{\rm BCS}$ (see Ref. 23).

A numerical analysis of the evolution of the reduction factor of the electric type as a function of the two parameters of the model, the pairing strength G, and the nuclear deformation δ shows that the reduction factor can vary in considerable proportions as a function of G and δ.

Figure 1 shows the great sensitivity of $R_{\rm BCS}^E$ and $R_{\rm SBCS}^E$ in relation to G and δ for a neutron transition in the nucleus $^{169}_{70}Yb_{99}$ and a proton transition in the nucleus $^{157}_{65}Tb_{92}$. Figures 1(a) and 1(b) denote an exponential behavior of $R_{\rm SBCS}$ versus G as soon as $GA > 2.2$ and Figs. 1(c) and 1(d) show that the pairing factor, traced versus the deformation parameter δ, is approximately Gaussianlike in the neighborhood of the equilibrium deformation.

The half-lives, calculated with the wave functions of Nilsson (T_N), BCS ($T_{\rm BCS}$), and SBCS ($T_{\rm SBCS}$) with or without Coriolis interaction, are collected in Table III and compared to the experimental values $T_{1/2}$.[31] If one takes account of the pairing correlations, the square of each Coriolis matrix element is multiplied by $R_{\rm BCS}^M$ or $R_{\rm SBCS}^M$ (according to whether or not the conservation of the number of nucleons is imposed), the interaction H' [Eq.(5)] satisfying the law of Eq. (19b) of time reversal.

In all the transitions considered, the reduction factor $R_{\rm BCS}^M$ is of the order of the unit and entails practically no modification of the Coriolis matrix elements in the calculation of the half-lives $T_{\rm BCS}$. On the other hand the $R_{\rm SBCS}^M$ factor remains generally far below the unit and therefore renders the half-lives $T_{\rm SBCS}$ more sensitive to the matrix elements of Coriolis when the strict conservation of the number of nucleons is imposed. The lowest values of $R_{\rm SBCS}^M$ met with in the calculations have been obtained for the transitions: $\frac{5}{2}\frac{3}{2}^+(411) \rightarrow \frac{5}{2}\frac{5}{2}^-(532)$ in $^{153}_{63}Eu_{90}$; $\frac{7}{2}\frac{7}{2}^-(523) \rightarrow \frac{5}{2}\frac{5}{2}^+(411)$ in $^{161}_{65}Tb_{96}$; $\frac{9}{2}\frac{9}{2}^-(514) \rightarrow \frac{7}{2}\frac{7}{2}^+(404)$ in $^{177}_{71}Lu_{106}$; and $\frac{5}{2}\frac{5}{2}^-(512) \rightarrow \frac{7}{2}\frac{7}{2}^+(633)$ in $^{169}_{70}Yb_{99}$ and $^{171}_{70}Yb_{101}$, for which the values of $R_{\rm SBCS}^M$ are, respectively, 0.7, 0.49, 0.64, 0.34, and 0.067, while the corresponding values of $R_{\rm BCS}^M$ are 0.94, 0.90, 0.96, 0.90, and 0.74. Table III shows that in the majority of cases the SBCS theory constitutes a net improvement over the the pure BCS theory. It is well known that the relatively good agreement of T_N with the experience of transitions between intrinsic states tends to be destroyed as a result of the inclusion of the pairing force.[31,43] According to Table III this seems to be due partly to the nonphysical effects induced by the fluctuation of the particle number.

The hindrance factors calculated relative to the

theoretical Nilsson, BCS, and SBCS estimates,

$$F_N = \frac{T_{1/2}(\text{exp})}{T_N}, \quad F_{\text{BCS}} = \frac{T_{1/2}(\text{exp})}{T_{\text{BCS}}}, \quad F_{\text{SBCS}} = \frac{T_{1/2}(\text{exp})}{T_{\text{SBCS}}}$$

are compared in Table IV for ten neutron transitions and four proton transitions to the Nilsson hindrance factors of Löbner and Malmskog[31] (LM). This table speaks for itself.

The transitions taking place from an intrinsic initial state to different states of a rotational band ($I_i = K_i, I_f = K_f + 1$ or $K_f + 2$) are less hindered and the theoretical predictions are in better agreement with the experience. The hindrance factors are all between 0.1 and 10 except for the transitions of the intrinsic state $\frac{5}{2}\frac{5}{2}^-(532)$ to the states $\frac{5}{2}\frac{3}{2}^+(411)$ and $\frac{7}{2}\frac{3}{2}^+(411)$ in the $^{159}_{65}\text{Tb}_{94}$.

The evolution of the hindrance factor versus the mass number of the nuclei is illustrated in Fig. 2. This figure shows as well that on the average the SBCS approximation fits the experimental results better than BCS approximation, the most noticeable exception being $^{173}_{70}\text{Yb}$.

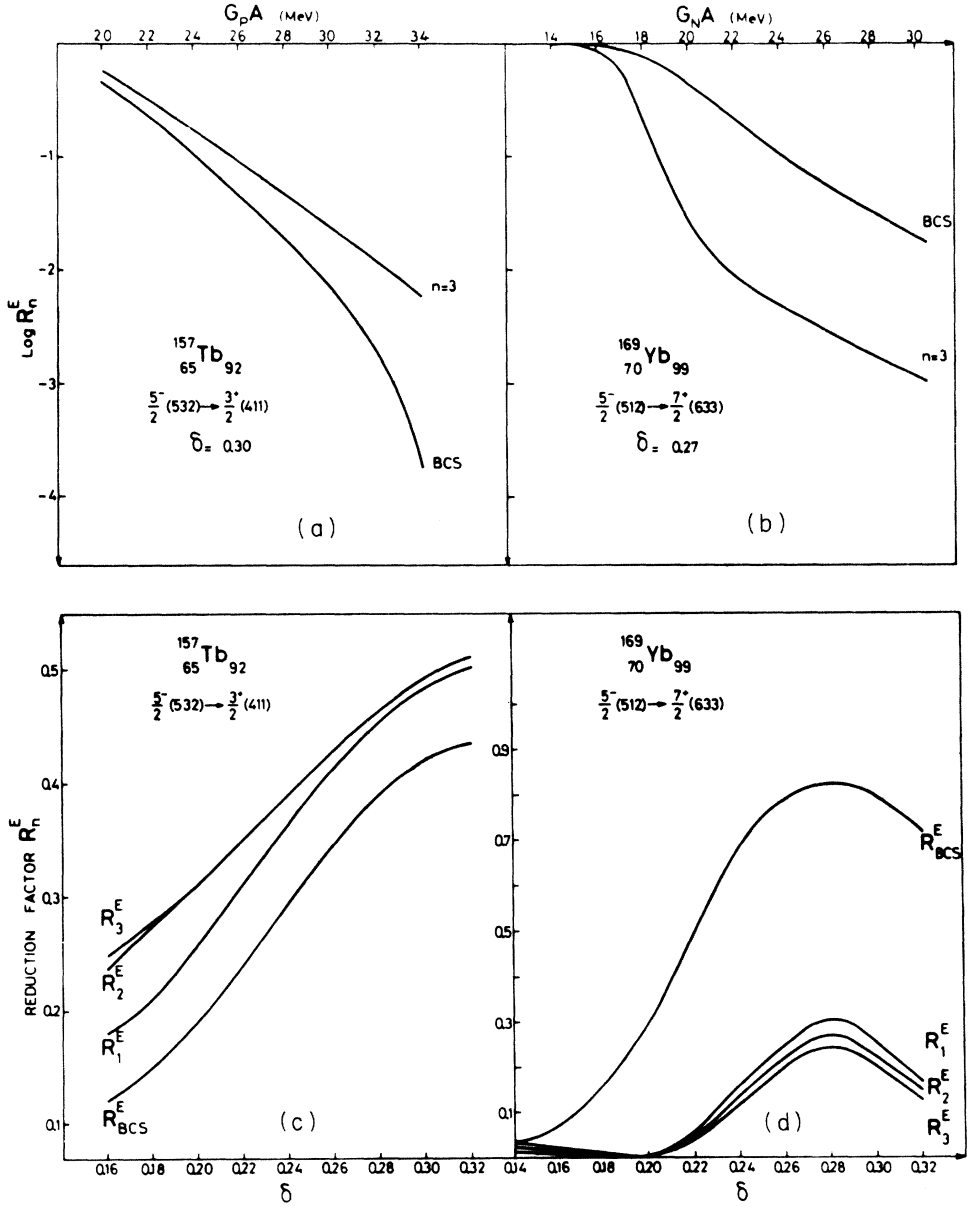

FIG. 1. Variation of the pairing-reduction factor versus the pairing-energy strength [(a) and (b)] and the nuclear deformation parameter δ [(c) and (d)].

TABLE III. Electric and magnetic type (R^E or R^M) pairing reduction factors calculated with BCS and with sharp particle-number-projected BCS (SBCS) wave functions. The corresponding theoretical half-lives (T_{BCS} and T_{SBCS}) together with the Nilsson estimate T_N are compared with experimental half-lives $T_{1/2}$(exp.). All nuclei are supposed to have the same deformation δ in the initial and final states. The Coriolis interaction in most cases spoils the agreement between theory and experiment. With a RPC strength of about 60 to 80% of the theoretical values one would obtain better agreement between theory and experiment.

Nucleus	States Initial	States Final	Transition energy (keV)	$T_{1/2}$(exp.)	Without Coriolis T_N	Without Coriolis T_{BCS}	Without Coriolis T_{SBCS}	With Coriolis T_N	With Coriolis T_{BCS}	With Coriolis T_{SBCS}	δ	R^E_{BCS}	R^M_{BCS}	R^E_{SBCS}	R^M_{SBCS}
$^{153}_{63}Eu_{90}$	$\frac{5}{2}\frac{3}{2}^+(411)$	$\frac{5}{2}\frac{5}{2}^-(532)$	75.4	3.3×10^{-8}	5.21×10^{-9}	8.87×10^{-6}	4.40×10^{-8}	1.74×10^{-9}	1.02×10^{-8}	5.28×10^{-8}	0.30	5.80×10^{-4}	0.8900	0.1182	0.7010
$^{155}_{63}Eu_{92}$	$\frac{3}{2}\frac{3}{2}^+(411)$	$\frac{5}{2}\frac{5}{2}^-(532)$	141	5.3×10^{-9}	2.25×10^{-10}	1.21×10^{-6}	1.91×10^{-9}	5.14×10^{-11}	2.09×10^{-11}	7.01×10^{-10}	0.30	1.84×10^{-4}	0.8755	0.1177	0.6933
$^{159}_{65}Tb_{94}$	$\frac{5}{2}\frac{5}{2}^-(532)$	$\frac{3}{2}\frac{3}{2}^+(411)$	363	1.7×10^{-10}	1.90×10^{-11}	6.76×10^{-11}	4.14×10^{-11}	4.20×10^{-12}	6.76×10^{-11}	1.58×10^{-10}	0.31	0.2807	0.8834	0.4580	0.8227
$^{159}_{65}Tb_{94}$	$\frac{5}{2}\frac{5}{2}^-(532)$	$\frac{5}{2}\frac{3}{2}^+(411)$	305	2.4×10^{-8}	7.47×10^{-11}	2.66×10^{-10}	1.63×10^{-10}	7.84×10^{-12}	3.65×10^{-11}	5.06×10^{-11}	0.31	0.2807	0.8834	0.4580	0.8227
$^{159}_{65}Tb_{94}$	$\frac{5}{2}\frac{5}{2}^-(532)$	$\frac{7}{2}\frac{3}{2}^+(411)$	225	7.6×10^{-9}	1.11×10^{-9}	3.97×10^{-9}	2.43×10^{-9}	3.94×10^{-10}	8.66×10^{-8}	3.60×10^{-7}	0.31	0.2807	0.8834	0.4580	0.8227
$^{161}_{65}Tb_{96}$	$\frac{7}{2}\frac{7}{2}^-(523)$	$\frac{5}{2}\frac{5}{2}^+(413)$	102.4	6.9×10^{-9}	1.18×10^{-9}	5.33×10^{-8}	2.45×10^{-8}	2.81×10^{-10}	9.99×10^{-10}	4.38×10^{-9}	0.31	0.0221	0.7953	0.0480	0.4932
$^{161}_{65}Tb_{96}$	$\frac{5}{2}\frac{5}{2}^-(532)$	$\frac{3}{2}\frac{3}{2}^+(411)$	482	3.0×10^{-10}	7.90×10^{-12}	2.80×10^{-11}	1.71×10^{-11}	1.74×10^{-12}	2.80×10^{-11}	6.59×10^{-11}	0.31	0.2820	0.8849	0.4598	0.8232
$^{175}_{71}Lu_{104}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	396.3	6.6×10^{-9}	5.25×10^{-11}	1.62×10^{-10}	1.37×10^{-9}	1.50×10^{-11}	2.75×10^{-10}	7.92×10^{-9}	0.28	0.3241	0.9539	0.0380	0.7140
$^{175}_{71}Lu_{104}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{9}{2}\frac{7}{2}^+(404)$	282.6	9.3×10^{-9}	6.38×10^{-10}	1.96×10^{-9}	1.66×10^{-8}	1.35×10^{-10}	2.16×10^{-10}	4.52×10^{-10}	0.28	0.3241	0.9539	0.0380	0.7140
$^{175}_{71}Lu_{104}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{11}{2}\frac{7}{2}^+(404)$	144.8	9.7×10^{-8}	4.74×10^{-8}	1.46×10^{-7}	1.23×10^{-6}	2.66×10^{-8}	5.89×10^{-8}	2.07×10^{-7}	0.28	0.3241	0.9539	0.0380	0.7140
$^{177}_{71}Lu_{106}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	147	1.7×10^{-7}	1.07×10^{-9}	4.74×10^{-9}	1.97×10^{-7}	3.07×10^{-10}	6.22×10^{-10}	1.80×10^{-9}	0.26	0.2254	0.9306	0.0054	0.6496
$^{179}_{73}Ta_{106}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	30.7	7.3×10^{-6}	1.25×10^{-7}	1.26×10^{-4}	1.83×10^{-6}	3.70×10^{-8}	3.27×10^{-6}	7.23×10^{-5}	0.25	9.92×10^{-4}	0.8900	0.0680	0.7944
$^{181}_{73}Ta_{108}$	$\frac{9}{2}\frac{9}{2}^-(514)$	$\frac{7}{2}\frac{7}{2}^+(404)$	6.3	3.1×10^{-5}	1.53×10^{-5}	8.45×10^{-4}	1.31×10^{-4}	4.58×10^{-6}	3.75×10^{-5}	1.00×10^{-4}	0.23	0.0181	0.8691	0.1171	0.7786
$^{155}_{64}Gd_{91}$	$\frac{5}{2}\frac{5}{2}^+(642)$	$\frac{3}{2}\frac{3}{2}^-(521)$	105.3	1.7×10^{-9}	1.67×10^{-9}	1.74×10^{-8}	2.81×10^{-9}	3.82×10^{-10}	4.04×10^{-9}	3.98×10^{-9}	0.38	0.0961	0.8684	0.5959	0.9999
$^{155}_{64}Gd_{91}$	$\frac{5}{2}\frac{5}{2}^+(642)$	$\frac{5}{2}\frac{3}{2}^-(521)$	45.3	2.4×10^{-8}	4.91×10^{-8}	5.10×10^{-7}	8.24×10^{-8}	3.48×10^{-9}	9.61×10^{-9}	1.01×10^{-8}	0.38	0.0961	0.8684	0.5959	0.9999
$^{157}_{64}Gd_{93}$	$\frac{5}{2}\frac{5}{2}^+(642)$	$\frac{3}{2}\frac{3}{2}^-(521)$	64.0	1.8×10^{-6}	7.73×10^{-9}	1.50×10^{-8}	1.26×10^{-8}	1.83×10^{-9}	1.78×10^{-8}	1.47×10^{-8}	0.38	0.5160	0.9260	0.6121	0.8266
$^{161}_{66}Dy_{95}$	$\frac{3}{2}\frac{3}{2}^-(521)$	$\frac{5}{2}\frac{5}{2}^+(642)$	74.5	2.9×10^{-8}	3.35×10^{-9}	4.13×10^{-9}	3.63×10^{-9}	7.41×10^{-10}	4.65×10^{-9}	4.09×10^{-9}	0.37	0.8118	0.9846	0.9235	0.9999
$^{169}_{70}Yb_{99}$	$\frac{5}{2}\frac{5}{2}^-(512)$	$\frac{7}{2}\frac{7}{2}^+(633)$	191.4	4.2×10^{-9}	4.49×10^{-10}	8.17×10^{-10}	4.97×10^{-9}	1.08×10^{-10}	1.58×10^{-10}	5.27×10^{-10}	0.34	0.5498	0.8146	0.0904	0.3533
$^{171}_{70}Yb_{101}$	$\frac{5}{2}\frac{5}{2}^-(512)$	$\frac{7}{2}\frac{7}{2}^+(633)$	27.0	8.1×10^{-6}	1.65×10^{-7}	7.09×10^{-7}	1.43×10^{-5}	3.97×10^{-8}	1.30×10^{-7}	1.25×10^{-4}	0.34	0.2336	0.3844	0.0115	0.0046
$^{173}_{70}Yb_{103}$	$\frac{7}{2}\frac{7}{2}^+(633)$	$\frac{5}{2}\frac{5}{2}^-(512)$	351.2	2.3×10^{-8}	1.04×10^{-10}	2.64×10^{-9}	9.97×10^{-9}	2.42×10^{-9}	8.63×10^{-10}	2.42×10^{-9}	0.34	0.0394	0.2582	0.9999	0.9999
$^{173}_{70}Yb_{103}$	$\frac{7}{2}\frac{7}{2}^+(633)$	$\frac{7}{2}\frac{5}{2}^-(512)$	272.4	6.2×10^{-10}	7.52×10^{-10}	1.91×10^{-8}	7.21×10^{-10}	1.03×10^{-10}	1.70×10^{-9}	1.03×10^{-10}	0.34	0.0394	0.2582	0.9999	0.9999
$^{173}_{70}Yb_{103}$	$\frac{7}{2}\frac{7}{2}^+(633)$	$\frac{9}{2}\frac{5}{2}^-(512)$	171.5	2.9×10^{-9}	2.41×10^{-8}	6.12×10^{-7}	2.31×10^{-8}	5.90×10^{-10}	1.54×10^{-9}	5.90×10^{-10}	0.34	0.0394	0.2582	0.9999	0.9999
$^{177}_{70}Yb_{107}$	$\frac{7}{2}\frac{7}{2}^-(514)$	$\frac{9}{2}\frac{9}{2}^+(624)$	104	6.6×10^{-9}	2.81×10^{-9}	2.97×10^{-9}	3.18×10^{-9}	6.91×10^{-10}	2.51×10^{-9}	2.66×10^{-9}	0.34	0.9449	0.9970	0.9999	0.8836
$^{175}_{72}Hf_{103}$	$\frac{7}{2}\frac{7}{2}^+(633)$	$\frac{5}{2}\frac{5}{2}^-(512)$	207.4	2.1×10^{-9}	4.97×10^{-10}	5.62×10^{-9}	5.37×10^{-10}	1.11×10^{-10}	4.93×10^{-9}	1.66×10^{-10}	0.34	0.0883	0.3003	0.9258	0.9999

TABLE III (*Continued*)

Nucleus	States Initial	States Final	Transition energy (keV)	$T_{1/2}$(exp.)	Without Coriolis T_N	Without Coriolis T_{BCS}	Without Coriolis T_{SBCS}	With Coriolis T_N	With Coriolis T_{BCS}	With Coriolis T_{SBCS}	δ	R^E_{BCS}	R^M_{BCS}	R^E_{SBCS}	R^M_{SBCS}
$^{175}_{72}Hf_{103}$	$\frac{7}{2}^+_2$ (633)	$\frac{7}{2}^-_2$ (512)	125.9	4.9×10^{-9}	7.50×10^{-9}	8.49×10^{-8}	8.10×10^{-9}	1.03×10^{-9}	1.87×10^{-8}	1.32×10^{-9}	0.34	0.0883	0.3003	0.9258	0.9999
$^{177}_{72}Hf_{105}$	$\frac{9}{2}^+_2$ (624)	$\frac{7}{2}^-_2$ (514)	321.4	1.2×10^{-7}	1.15×10^{-10}	1.35×10^{-10}	1.83×10^{-10}	2.65×10^{-11}	8.87×10^{-11}	3.36×10^{-11}	0.33	0.8469	0.9968	0.6245	0.9514
$^{177}_{72}Hf_{105}$	$\frac{9}{2}^+_2$ (624)	$\frac{9}{2}^-_2$ (514)	208.4	7.3×10^{-10}	1.84×10^{-9}	2.18×10^{-9}	2.95×10^{-9}	4.88×10^{-10}	5.31×10^{-10}	4.30×10^{-10}	0.33	0.8469	0.9968	0.6245	0.9514
$^{177}_{72}Hf_{105}$	$\frac{9}{2}^+_2$ (624)	$\frac{11}{2}^-_2$ (514)	71.6	9.8×10^{-8}	4.55×10^{-7}	5.37×10^{-7}	7.29×10^{-7}	1.43×10^{-7}	1.56×10^{-7}	1.87×10^{-7}	0.33	0.8469	0.9968	0.6245	0.9514
$^{179}_{72}Hf_{107}$	$\frac{7}{2}^-_2$ (514)	$\frac{9}{2}^+_2$ (624)	215	1.6×10^{-9}	3.19×10^{-10}	4.77×10^{-10}	4.77×10^{-10}	7.85×10^{-11}	3.75×10^{-10}	3.44×10^{-10}	0.32	0.6696	0.8843	0.6702	0.9999

In summary the pairing-reduction factors, hindrance factors, and half-lives for $E1$ transitions with $|\Delta K| = 1$ are considerably modified and on an average improved if one projects out that part of the BCS wave function corresponding to the correct number of particles.

The method used to do this leads to a sequence of functions $\Psi_{\mu_0}(n)$ [Eq. (14)] which converges with a surprising rapidity towards the projected SBCS wave function conserving strictly the number of nucleons. The Coriolis interaction also modifies, and not always in the right way, the whole of the theoretical results.

The quality of the results obtained with the pure Nilsson model seems to us worthy of notice. The greatly improved model and parameters we use (see Sec. II) give us probabilities of $E1$ transitions in better agreement with the experience than those obtained by the majority of other authors.

Possible improvements to the work presented here can be obtained by lifting the restrictive hypotheses we have made, namely:

(i) We have only taken account of H' [Eq. (5)] by a first-order perturbation calculation, and this could well not be valid in every case.

(ii) We have assumed that the pairing interaction is negligible between unlike nucleons. But this question remains still open.

(iii) The octupole vibration-particle coupling strength is certainly weak for $\Delta K = \pm 1$ transitions, but perhaps not always negligible.[44]

(iv) We have admitted, for the lack of precise experimental information, that the deformation of the nuclei does not change in the course of the transition. But, evidently, the transition probability can depend on the difference of deformation between the initial and final states.[6,7,32]

(v) We have admitted in Sec. II that the mean potential had an axial symmetry. But the nonaxiality

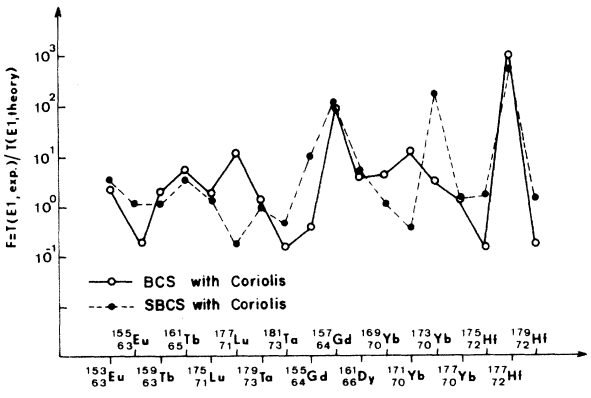

FIG. 2. Variation of the BCS and SBCS hindrance factors versus the mass number of the nuclei.

TABLE IV. Nilsson, BCS, and SBCS hindrance factors for a few $E1$ transitions. The last column refers to a Nilsson estimate (without Coriolis) given by Löbner and Malmskog (Ref. 31). This table shows the importance of the errors due to particle-number fluctuations. Again a reduction of the Coriolis matrix elements would improve most of the hindrance factors.

	With Coriolis			Without Coriolis			$L\,M$
	F_N	F_{BCS}	F_{SBCS}	F_N	F_{BCS}	F_{SBCS}	F_N
			$\frac{5}{2}\frac{5}{2}^+(642) \rightarrow \frac{3}{2}\frac{3}{2}^-(521)$				
$^{155}_{64}\mathrm{Gd}_{91}$	4.45	0.42	0.43	1.0	0.10	0.60	3.8
$^{157}_{64}\mathrm{Gd}_{93}$	985	100.8	122.50	232	120	142	920
$^{161}_{66}\mathrm{Dy}_{95}$	39.1	6.22	7.07	9.0	7.0	8.0	31
			$\frac{7}{2}\frac{7}{2}^+(633) \rightarrow \frac{5}{2}\frac{5}{2}^-(512)$				
$^{169}_{70}\mathrm{Yb}_{99}$	11	6.3	1.17	9.3	5.0	0.8	36
$^{171}_{70}\mathrm{Yb}_{101}$	58	14	0.5	48.8	11.4	5.6	180
$^{173}_{70}\mathrm{Yb}_{103}$	256	5.56	191	220	8.7	163	880
$^{175}_{72}\mathrm{Hf}_{103}$	5.4	0.2	2.8	4.2	0.37	3.9	18
			$\frac{7}{2}\frac{7}{2}^-(514) \rightarrow \frac{9}{2}\frac{9}{2}^+(624)$				
$^{177}_{72}\mathrm{Hf}_{105}$	1242	1067	806	1047	887	654	3000
$^{177}_{70}\mathrm{Yb}_{107}$	2.7	2.62	2.58	2.3	2.2	2	8
$^{179}_{72}\mathrm{Hf}_{107}$	6	0.29	2.16	5	3.4	3.4	16
			$\frac{9}{2}\frac{9}{2}^-(514) \rightarrow \frac{7}{2}\frac{7}{2}^+(404)$				
$^{175}_{71}\mathrm{Lu}_{104}$	93.8	3.67	4.34	125	40.6	4.8	165
$^{177}_{71}\mathrm{Lu}_{106}$	115	17.4	0.35	159	35.8	0.86	207
$^{179}_{73}\mathrm{Ta}_{106}$	39.6	2.2	1.0	58	0.06	4.0	77
$^{181}_{73}\mathrm{Ta}_{108}$	1.3	0.2	0.53	2	0.04	0.3	25

of which one can take account [for example in the unified asymmetrical-rotational model of Chi, Davidson, and Newton[45]] can modify considerably the energy levels and the whole of the electromagnetic transition probabilities.[46]

(vi) The nonlocality of the single-particle potential or of the nucleon-nucleon interaction (both neglected in this work) may play a nonnegligible role, for electric dipole transitions, in the first case by reducing configuration mixture,[36] and in the second case because of the noncommutativity of the dipole operator with nonlocal potentials.[17-19]

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