Shell-model calculations for ^{51}Ti and ^{52}V

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Employing ^{48}Ca as the core and the renormalized matrix elements of Kuo and Brown, the nuclear energy levels of ^{51}Ti and ^{52}V are calculated within the spherical shell-model framework. For ^{51}Ti all the $0f$-$1p$ configurations are included but the ^{52}V nucleus is described within the $(0f_{7/2}, 1p_{3/2})^3_p \times (1p_{3/2}, 0f_{5/2}, 1p_{1/2})^1_n$ configuration. The results are compared with experiment.

[NUCLEAR STRUCTURE ^{51}Ti, ^{52}V; calculated energy levels, spectroscopic factors for ^{50}Ti$(d, p)^{51}$Ti; magnetic moment for ^{51}Ti.]

In recent years the nuclear shell model and its various approximations have been widely applied to study the ground-state properties and level structure of medium mass nuclei. Instead of employing simple phenomenological residual interactions, the matrix elements of realistic nucleon-nucleon interactions, renormalized for the particle-hole excitations of the core, are used. Though many calculations have been reported for nuclei having only one type of particles outside an inert core,[1] the realistic matrix elements have not been tested well enough for nuclei which have neutrons and protons in well-separated shells. In this note, the Kuo-Brown[2] renormalized matrix elements for the Hamada-Johnston interaction have been employed to study the level structure of ^{51}Ti and ^{52}V. Both of these nuclei have neutron number $N=29$ and for shell-model purposes can be regarded as having three and four nucleons outside the ^{48}Ca core. There is ample experimental evidence[3-5] that ^{48}Ca forms a good closed core.

Investigations of the $N=29$ isotones have been made by a number of workers. Ramavataram[6] studied these nuclei on the basis of the unified model by coupling the valence neutron in the $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ shell-model states to a quadrupole vibration core. Maxwell and Parkinson[7] carried out shell-model calculations restricting the protons to the $(0f_{7/2})^m$ configuration but permitting the valence neutrons to occupy the $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ orbitals. For the residual interaction, Maxwell and Parkinson chose a Gaussian radial dependence. Pellegrini[8] took the residual interaction deduced from the experimental spectra of ^{50}Ti and ^{56}Co. Ohnuma[9] carried out several calculations by assuming central interactions between proton and neutron and by changing the single-neutron energies of the $1p_{1/2}$ and $0f_{5/2}$ orbitals relative to that of the $1p_{3/2}$ orbital, as free parameters, for each nucleus. In the investigation of Horie and Ogawa[10] the matrix elements of the effective interactions between the $0f_{7/2}$ proton and the $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ neutrons are determined by a least-square fitting to the observed energy levels of the $N=29$ isotones. Divadeenam and Beres[11] carried out calculations by including some neutron core-excitation states in the model space. All these calculations involved adjustable parameters. In their paper Divadeenam and Beres suggested that a calculation like the one of Raj, Rustgi, and Singh[12] be carried out for ^{51}Ti.

In this shell-model calculation, the ^{51}Ti nucleus is considered to have two protons and one neutron outside the ^{48}Ca core; the two protons are allowed to populate the $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals, and the valence neutron is restricted to the neutron $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals. All possible three-particle configurations are included in setting up the Hamiltonian matrix for a given total spin and parity. The single-particle energies used are listed in the Kuo-Brown (KB) paper.

In view of the fact that there are no adjustable parameters, Fig. 1 indicates that the KB effective interaction reproduces the energy spectrum of ^{51}Ti fairly well. The spin and level energy of the ground and the first excited state is well reproduced. Although the energies of the other lower excited states are slightly higher than the experimental values, the differences are less than 0.4 MeV. In the calculation of Pellegrini, where only the pure $(0f_{7/2})^m 1p_{3/2}$ configurations were taken into account, the first $\frac{1}{2}^-$ and second $\frac{5}{2}^-$ states were not reproduced. Horie and Ogawa included in addition the $(0f_{7/2})^m 1p_{3/2}$ and $(0f_{7/2})^m 1p_{1/2}$ configurations and their results reproduced the two missing levels in Pelligrini's calculation; but the order of the second $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states was

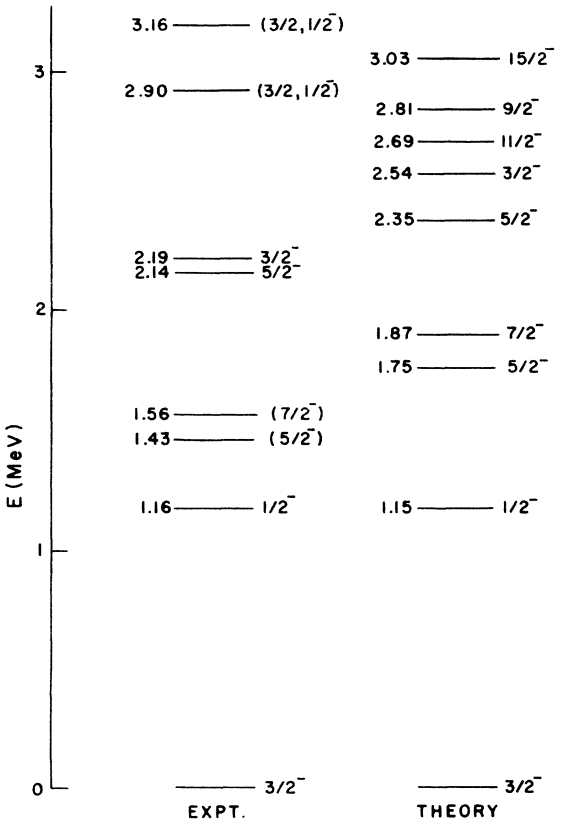

FIG. 1. Calculated and observed energy levels of ^{51}Ti.

TABLE I. Spectroscopic factors S for the reaction ^{50}Ti$(d,p)^{51}$Ti.

E (MeV)	J^π	S_{expt} [a]	S_{theory}
0.0	$\frac{3}{2}^-$	0.736	0.736
1.16	$\frac{1}{2}^-$	0.589	0.467
1.43	$\frac{5}{2}^-$	0.110	0.172
1.56	$\frac{7}{2}^-$	0.0	0.0
2.14	$\frac{5}{2}^-$	0.269	0.144
2.19	$\frac{3}{2}^-$	0.055	0.051

[a] The experimental values of the spectroscopic factor, known only in their relative values, have been normalized to the calculated one for the transition to the ground state.

reversed. Our calculation reproduces the order of all levels below 2.5 MeV. Besides the $(0f_{7/2})^2$ configuration which is dominant in the lower levels, the other important proton configurations for the ground and first excited states are $(0f_{7/2}1p_{3/2})$, $(1p_{3/2})^2$, and $(0f_{5/2})^2$. The total contribution to the wave function, from configurations other than the pure $(0f_{7/2})^2$, is 12% for the ground state and 30% for the first excited state. The agreement between the theoretical and experimental spectroscopic factors for the reaction ^{50}Ti(d,p) ^{51}Ti is quite good, except for the second $\frac{5}{2}^-$ state for which the calculated value is too small.

The ground-state ($\frac{3}{2}^-$) magnetic moment in units of Bohr magneton μ_N, with effective charges $e_p=1$ and $e_n=0$, is found to be -2.18. Using the same effective charges, the calculated magnetic moments for $J^\pi=\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ are 2.13, 5.39, and $4.25\mu_N$, respectively.

The low-lying spectrum[13] of ^{52}V is characterized by two doublets above the ground state $J^\pi=3^+$. The first doublet 2^+ (0.01 MeV) and 5^+ (0.02 MeV) is very close the the ground state and the second doublet 1^+ (0.142 MeV) and 4^+ (0.148 MeV) occurs just below 0.15 MeV. It therefore appears that it would be difficult to obtain good agreement with the observed spectrum of ^{52}V unless an interaction specially fitted for this nucleus is used, since uncertainties in matrix elements and single-particle energies could easily give results in which the order of levels is not reproduced. We believe that it is for this reason that in the calculation of Vervier[14] the 2^+ state turned out to be the ground state. Horie and Ogawa[10] found that the 5^+ state was the lowest. The spin sequence of other low-lying levels was not reproduced in the calculations. With a very carefully chosen interaction, Gersch, Riedel, and Rudolph[15] reproduced the ground state but the order of the 1^+ and 4^+ states was reversed.

In our calculation, the neutron is allowed to occupy the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals and the protons are restricted to the $0f_{7/2}$ and $1p_{3/2}$ orbitals. As in the calculation of Vervier, the lowest state turns out to be the $J^\pi=2^+$ state. The next few states, 3_1^+, 5_1^+, 4_1^+, and 1_1^+, are found to be at 0.07, 0.19, 0.21, and 0.29 MeV, respectively. The uncertainties in the Kuo-Brown matrix elements may be responsible for the reverse order of the 2_1^+ and 3_1^+ states.

It may be mentioned that our calculations for ^{51}Ti and ^{52}V suffer from the drawback that no attempt has been made to construct states of good isospin. However, the isospin mixing will not change the present results for the low-lying states significantly following the reasoning of Kuo and Brown.

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